

An Adaptive Wideband Beamformer Using Kalman Filter with Spatial Signal Processing Characteristics

Mariel Rivas*, Shuguo Xie*, Donglin Su*

* School of Electronic Information Engineering, Beijing University of Aeronautics and Astronautics, XueYuan Road No.37, HaiDian District, BeiJing, China.

madry17@gmail.com; xieshuguo@buaa.edu.cn; sdl@buaa.edu.cn

Abstract— This paper presents a novel adaptive wideband beamformer for smart antennas capable of effectively steering the main beam to a desired direction, and place nulls to the interfering signals direction in presence of noise. The proposed beamformer uses a fully spatial signal processing configuration, in which a two-dimensional antenna array acts as a spatial filter, therefore the signals received by each element are processed with only weighting schemes, avoiding tape-delay line networks and frequency filters. To obtain the adaptive weights a Kalman filter is applied. Additionally, the algorithm is able to avoid the self-nulling phenomenon, which occurs when there is a mismatch between the desired signal steering vector and the actual steering vector. The simulation results illustrate the satisfactory performance of the proposed wideband spatial beamformer.

Keywords—Wideband smart antennas, adaptive beamforming, fully spatial signal processing, Kalman filter, self-nulling.

I. INTRODUCTION

New applications of wireless communication systems demand more capacity and higher data-rate transmission, therefore an increased operational bandwidth is required. Because of the available spectrum to provide high-data-rate service to all the subscribed users is limited, besides, in densely populated areas the main source of noise in mobile systems is the interference from other users, the attention of latest research has turned to find a technology able to overcome these problems.

In the last few years, smart antenna systems have been gaining much attention, because, based on spatial filtering, they are able to simultaneously increase the useful received signal level and decrease the interference level. In this way, the system capacity and power efficiency can be increased, reducing the overall cost. However, most of the research done on smart antennas has only been concerned to handle narrowband signals [1],[2].

In order to develop wideband smart antenna systems, the adaptation of the radiation pattern should take into account the frequency dependence of the inter-element phase shift. The three main techniques to perform wideband smart antennas are: space-time signal processing, space-frequency signal processing and fully spatial signal processing. Amongst those, the last option is the most attractive one, wherein the signals are processed only in space domain since its configuration

removes tape-delay lines and filters, thus reducing the hardware complexity [3].

The challenge of fully spatial signal processing configuration comes from finding out the optimum weighting coefficients of the beamformer. To date, there are only three beamforming algorithms proposed to solve this challenge, two of them based on rectangular array geometry and one on circular array geometry. However, none of these algorithms is able to work properly in a noisy environment.

In [4] and [5] a second order extended Kalman filter was employed to obtain the adaptive weights for a narrowband case, using a linear array. This narrowband beamformer is able to steer the main lobe and the nulls in presence of noise, moreover, was designed in order to be robust against the possible mismatch that may occur between the desired signal steering vector and the actual steering vector due to signal pointing errors, imperfect array calibration, source local scattering, wavefront distortion, etc. These effects may result in suppression of the desired signal component, phenomenon commonly referred as signal self-nulling.

In this work we apply the Kalman algorithm used in [5] to obtain the optimum weighting coefficients of a wideband circular array in order to effectively steer the main beam to a desired direction while placing nulls directed to the interfering signal direction.

The remainder of this paper is organized as follows. In section II, a configuration of wideband beamformer using fully spatial signal processing in circular arrays is described. In section III, the derivation of the beamforming algorithm based on Kalman filter is explained. Section IV shows simulations results. Finally section V concludes the paper.

II. FULLY SPATIAL SIGNAL PROCESSING BEAMFORMING USING CIRCULAR ARRAYS

Most of the research of smart antennas has mainly involved uniform linear arrays (ULAs) and uniform rectangular arrays (URAs). Another antenna configuration that can be used for pattern beamforming is uniform circular array (UCA) geometry, however, so far, not as much attention has been paid to this topology, even though the advantages that it can offer. For instance, an evident advantage results from the symmetry of the UCA geometry, which does not have edge

elements, thus the directional patterns synthesized by a UCA can be electronically scanned in the azimuthal plane without a significant change in beam shape [6],[7].

Another interesting property of UCA is its ability to form beam patterns that are relatively invariant with frequency [6]. Because this property makes UCA attractive to be implemented in wideband smart antennas, we selected a UCA topology to work with, and, as we will show later, the results obtained are favourable.

A uniform circular array (UCA) for wideband operation, located on the $x-y$ plane, is shown in Figure 1 [3]. The beamformer structure use fully spatial signal processing configuration in which the received signals are processed only in space domain using weighting schemes, with no delay lines or filters [1],[8].

The array is integrated by N elements arranged in a circular geometry with radius r . The direction of the arriving signal is determined by the azimuth and the elevation angles φ and θ , respectively. As in most practical cases, it is assumed that the elevation angles of the incoming signals to the antenna array are almost constant and without loss of generality we may consider $\theta \approx 90^\circ$ [9]. The inter-element distance along the circumference is l .

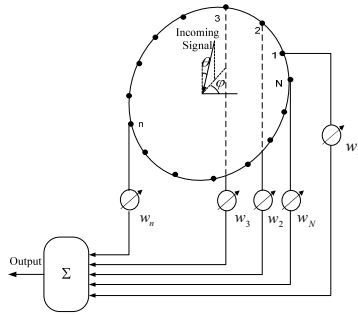


Figure 1. Wideband beamformer constituted by a circular array of N antenna elements.

Assuming that the phase reference point is located at the center of the array, the phase at the n_{th} element is given by:

$$\Phi = (2\pi f/c)r \cos(\varphi - \varphi_n) \quad (1)$$

where $\varphi_n = 2\pi n/N$, $n=1,2,...,N$. The spacing is chosen as half-wavelength at the highest frequency of a desired frequency band.

III. KALMAN FILTER-BASED BEAMFORMING ALGORITHM

In order to obtain the optimum weights, a second order extended Kalman filter is employed; in this section we describe the derivation of the algorithm.

A. Optimization Problem Definition

The output of the wideband beamformer, conformed by a circular array of N antenna elements (Figure 1), is given by:

$$y(k) = x^H(k)w \quad (2)$$

where k is the time index, $x(k) = [x_1(k), ..., x_N(k)]^T \in C^N$ is the array observation vector, and $w = [w_1, ..., w_N]^T \in C^N$ is the complex vector of beamformer weights.

The observation vector can be written as:

$$x(k) = s(k)d + i(k) + n(k) \quad (3)$$

where $s(k)$ is the desired signal waveform, d is the desired signal steering vector, and $i(k)$ and $n(k)$ are the interference, and noise components, respectively. The desired signal and interferers are assumed to be uncorrelated.

The SINR maximization can be obtained by minimizing the total variance of $y(k)$ under the constraint that the target response from the desired look angle φ_0 is unity [10]. It can be expressed as the following optimization problem:

$$\min_w w^H \hat{R}_{xx} w \text{ subject to } w^H a = 1 \quad (4)$$

where $R_{xx} = E\{x(k)x^H(k)\}$ is the $N \times N$ correlation matrix, and a is the presumed desired signal steering vector. The solution to the above optimization problem is commonly referred to as the minimum variance distortionless response (MVDR) beamformer and is given by [4],[5]:

$$w_{opt} = \frac{R_{xx}^{-1}a}{a^H R_{xx}^{-1}a} \quad (5)$$

In practice, the exact covariance matrix R_{xx} is not available whereby has to be estimated from the received data samples as:

$$\hat{R}_{xx} = \frac{1}{M} \sum_{k=1}^M x(k)x^H(k) \quad (6)$$

where M is the number of snapshots available. In this case, the optimization problem (4) should be rewritten as:

$$\min_w w^H \hat{R}_{xx} w \text{ subject to } w^H a = 1 \quad (7)$$

B. Robust Constraint for Signal Self-Nulling Problem

Note that the presumed steering vector a may be different from the actual steering vector d , if this is the case the desired signal is interpreted by the beamformer as an interference signal and is cancelled out instead of being enhanced. This phenomenon is commonly known as signal self-nulling[5]. In practical applications, there may exist arbitrary unknown mismatches between the actual steering vector and the presumed one, yet the norm of the steering vector distortion can usually be bounded by some known

constant $\varepsilon > 0$ [4],[5]. Then, the actual desired signal steering vector belongs to the set:

$$A(\varepsilon) \triangleq \{b \mid b = a + e, \|e\| \leq \varepsilon\} \quad (8)$$

where the vector e represents the mismatch between the presumed and the desired steering vectors, therefore $b = d$. Since d can be any vector in (8), in [4] a constraint is imposed in order to for all vectors that belong to $A(\varepsilon)$, the absolute value of the array response should not be smaller than one, thus the robust formulation of adaptive beamformer can be written as:

$$\min_w w^H \hat{R}_{xx} w \text{ subject to } |w^H b| \geq 1 \text{ for all } b \in A(\varepsilon) \quad (9)$$

The infinite number of nonconvex constraints in (9) was reformulated in [4] as a single convex constraint corresponding to the worst-case mismatch, expressed as:

$$\min_w w^H \hat{R}_{xx} w \text{ subject to } w^H a \geq \varepsilon \|w\| + 1 \quad (10)$$

where $\|\cdot\|$ is the vector Euclidean norm.

C. Optimization problem solution using Kalman Filter

In order to solve the optimization problem (10), Kalman filter theory has been implemented in [5] for the narrowband case. In this section we describe the model presented in [5] but implemented for the wideband case.

The mean square error (MSE) between the zero signal and the beamformer output is defined as

$$MSE = E[|0 - x^H(k)w(k)|^2] = w^H R_{xx} w \quad (11)$$

According to (11) minimizing the beamformer output power is equivalent to minimizing the MSE, therefore, the constraint in (10) can be expressed as

$$h_2(w(k)) = 1 \quad (12)$$

where

$$\begin{aligned} h_2(w(k)) &= \varepsilon^2 w^H(k)w(k) - w^H(k)aa^H w(k) \\ &+ w^H(k)a + a^H w(k) \end{aligned} \quad (13)$$

Then, the robust beamforming problem can be expressed as

$$\min_w MSE \text{ subject to } h_2(w(k)) = 1 \quad (14)$$

As in [5], the aim is express (14) as a dynamic system and use Kalman filter to solve it. An unknown dynamic system

can be modelled as a filter whose state vector w undergoes a first-order Markov process [5] as

$$w(k+1) = \gamma w(k) + v_s \quad (15)$$

where γ is a fixed parameter of the model, v_s is the process noise vector, which is assumed to be white Gaussian with zero mean with a covariance matrix $Q = \sigma_s I$, I is the identity matrix, and the subscript “s” refers to the state equation. Hence, the process equation of the optimal weight vector w is given by (15), while the measurement equation is given by

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x^H(k)w(k) \\ h_2(k) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} \quad (16)$$

which, in matrix notation, can be expressed as

$$z(k) = h(w(k)) + v_m(k) \quad (17)$$

where $v_1(k)$ and $v_2(k)$ are the residual and the constraint errors, respectively, and the subscript “m” refers to the measurement equation. They can be modelled as zero-mean, independent white noise sequences with the covariance matrix

$$R = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \quad (18)$$

Because of the nonlinearity of the measurement equation (16), the second-order extended Kalman filter [5],[11] is used to find a recursion for the estimated weight vector $\hat{w}(k)$. The process begin by the evaluation of the Jacobian $H_w(k, w(k))$ of $h(w(k))$ and the Hessian matrices $H_{ww}^{(1)}$ and $H_{ww}^{(2)}$ of its components as

$$\begin{aligned} H_w(k, w(k)) &= \{\nabla_w h^T(w(k))\}^T \\ &= \begin{bmatrix} x^H(k) \\ \varepsilon^2 w^H(k) - (aa^H w(k))^H + a^H \end{bmatrix} \end{aligned} \quad (19)$$

$$H_{ww}^{(1)} = \nabla_w \nabla_w^H \{x^H(k)w(k)\} = 0 \quad (20)$$

$$H_{ww}^{(2)} = \nabla_w \nabla_w^H \{h_2(w(k))\} = \varepsilon^2 I - aa^H \quad (21)$$

Then, the recursion for the estimated weight vector starts with an initial weight vector estimate $\hat{w}(0)$ with the associated covariance matrix $P(0|0)$, and subsequently updates the weight vector estimate as

$$\hat{w}(k) = \hat{w}(k-1) + K(k)[z - \hat{z}(k|k-1)] \quad (22)$$

where the predicted measurement $\hat{z}(k|k-1)$ and the filter gain $K(k)$ are given by:

$$\hat{z}(k|k-1) = \begin{bmatrix} \gamma x^H(k) \hat{w}(k-1) \\ h_2(\gamma \hat{w}(k-1)) + \frac{1}{2} \text{tr}\{H_{ww}^{(2)} P(k|k-1)\} \end{bmatrix} \quad (23)$$

and

$$K(k) = P(k|k-1) H_w^H(k, \gamma \hat{w}(k-1)) S(k)^{-1} \quad (24)$$

respectively, and $\text{tr}\{\cdot\}$ denotes the matrix trace. The innovation covariance $S(k)$ and the predicted weight vector covariance $P(k|k-1)$ are given by

$$S(k) = H_w(k, \gamma \hat{w}(k-1)) P(k|k-1) H_w^H(k, \gamma \hat{w}(k-1)) + \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \text{tr}\{H_{ww}^{(2)} P(k|k-1) H_{ww}^{(2)} P(k|k-1)\} + R \quad (25)$$

$$P(k|k-1) = \gamma^2 P(k-1|k-1) + Q \quad (26)$$

respectively. The updated weight vector covariance can be expressed as

$$P(k|k) = \{I - K(k) H_w(k, \gamma \hat{w}(k-1))\} P(k|k-1) + \{I - K(k) H_w(k, \gamma \hat{w}(k-1))\}^H + K(k) R K^H(k) \quad (27)$$

The parameters γ and σ_s^2 of the state equation should be chosen so that the model can track changes in the optimal weight vector due to changes in the operating environment. From equation (25) can be seen that when increasing these parameters, the Kalman filter tends to assign more weight to recent data, enabling better tracking of the environment. For stationary environments, the optimal weight vector does not change with time, and therefore, $\gamma=1$ and $\sigma_s^2=0$ can be chosen in this case.

From the first line of the measurement equation, it follows that the parameter σ_1^2 should be chosen of the same order as the optimal output power of the array. Note that the beamformer is no sensitive to the choice of σ_1^2 because the norm of the weight vector is chosen by the filter so that the output power of the beamformer matches the value of σ_1^2 . The value of the parameter σ_2^2 should be chosen very small, for example 10^{-12} , so that the robustness constraint is satisfied with a high accuracy.

IV. SIMULATIONS

For the simulations, we assume a uniform circular array (UCA) with $N=16$ omnidirectional antenna elements. The

desired operating frequency band is 1.8GHz to 2.4GHz, with a bandwidth $B=600\text{MHz}$, providing a fractional bandwidth (FB) of 28.5%.

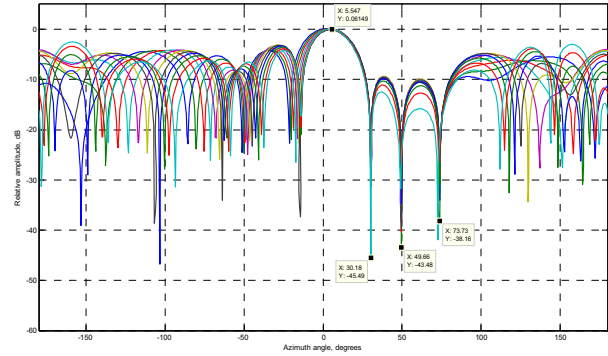


Figure 2. Array Beampatterns for frequencies distributed from 1.8GHz to 2.4GHz.

The interelement spacing is half wavelength at the highest frequency of operation. The presumed desired angle is $\phi_a = 5^\circ$. The desired signal with $\text{SNR} = 0\text{dB}$ is assumed to impinge on the array from the direction $\phi_d = 7^\circ$. Three interferers are assumed to impinge on the array from the directions $\phi_{i_1} = 30^\circ$, $\phi_{i_2} = 50^\circ$ and $\phi_{i_3} = 75^\circ$, all with interference-to-noise ratio (INR) equal to 30dB . The presumed signal steering vector is assumed to be normalized as $a^H a = N$, and following the procedure of [4] and [5], the robustness parameter is chosen as $\varepsilon = 3$. The parameters of the Kalman filter were selected as $\sigma_s^2 = 0$, $\sigma_1^2 = 750$, $\sigma_2^2 = 10^{-12}$ and $\gamma = 1$.

The directional pattern obtained using the proposed wideband beamformer is shown in Figure 2. We can see that the main beam and the nulls are steered to the desired and interferer directions respectively, and in these points all the frequencies have the same behaviour demonstrating the wideband property of the beamformer. Also is proven that this beamformer does not suffer from self-nulling.

V. CONCLUSIONS

An adaptive wideband beamformer for smart antennas using circular array geometry has been proposed. The beamformer use a fully spatial signal processing configuration in which each antenna element is connected to a single multiplying weight, avoiding delay lines and filters. The result shows that the beamformer operates within a wide frequency band, and efficiently steer the main beam to the desired direction and place nulls in the directions of the interference signals in presence of high level of noise. Also has been proved that the beamformer avoid the self-nulling phenomenon.

REFERENCES

- [1] M. Ghavami, "Wideband Smart Antenna Theory Using Rectangular Array Structures," IEEE Transactions on Signal Processing, vol. 50, no. 9, pp. 2143-2151, Sep. 2002.

- [2] J.C. Libert and T. S. Rappaport, *Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications*, Englewood Cliffs, NJ, Prentice Hall, 1999.
- [3] M. Rivas, S. Xie and D. Su, "A review of adaptive beamforming techniques for wideband smart antennas" 6th International Conference on Wireless Communication Networking and Mobile Computing (WICOM), Chengdu, China, 2010.
- [4] S. A. Vorobyov, A. Gershman, and Z. Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 313-324, Feb, 2003.
- [5] A. El-Keyi, T. Kirubarajan, and A. B. Gershman, "Robust adaptive beamforming based on the Kalman filter," *IEEE Trans. Signal Processing*, vol. 53, pp. 3032-3041, Aug. 2005.
- [6] S. C. Chan and C. K. S. Pun, "On the design of digital broadband beamformer for uniform circular array with frequency invariant characteristics," *IEEE International Symposium on Circuits and Systems (ISCAS)*, vol.1, pp. I-693 - I-696, 2002.
- [7] C. Balanis, *Antenna Theory: Analysis and Design*, John Wiley & Sons, Ltd, Third Edition, 2005.
- [8] M. Uthansakul and M. E. Bialkowski, "Fully spatial wideband beamforming using a rectangular array of planar monopoles," *IEEE Transaction on Antenna and Propagation*, vol. 54, pp. 527-533, February 2006.
- [9] P. Ioannides and C. A. Balanis, "Wideband beamforming using circular arrays," in *IEEE Antennas and Propagation Society International Symposium*, vol. 3, pp. 2627 – 2630, 2004.
- [10] P. Pal and P.P. Vaidyanathan. "Frequency invariant MVDR beamforming without filters and implementation using MIMO radar," *Proc. IEEE International Conference on Acoustic, Speech, and Signal Processing (ICASSP)*, pp 2081-2084, 2009.
- [11] D. Simon, *Optimal State Estimation*, Wiley Interscience, 2006.