Perfomances Analysis of Algebraic Space Time Code under Correlated and Uncorrelated Channels

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Abstract— With their very Algebraic-construction based on Quaternionic algebra, Algebraic Space Time Codes (ASTC), called the Golden codes, have a full rate, full diversity and non-vanishing constant minimum determinant for increasing spectral efficiency. They have also uniform average transmitted energy per antenna and good shaping, readily lend themselves to high data rate situations. In this paper, we first analyze the performances of the ASTC codes in correlated Rayleigh channel. We consider a coherent demodulator using different decoding schemes and we analyze the Bit Error Rate (BER). In order to increase the spectral efficiency and to maximize the coding gain, ASTC have been proposed for MIMO flat fading channels. To deal with the frequency selectivity, we use the OFDM modulation. So we analyze the performances of an ASTC-MIMO-OFDM system in terms of BER. Finally, we investigate the impact of spatial correlation on the ASTC code design in terms of BER and capacity.

Keywords— ASTC code, OFDM, MIMO, Rayleigh Channel, spatial correlation, capacity, Bit Error Rate

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been attracting considerable attention because of its robustness against frequency-selective fading [1]. OFDM system has been adopted as a standard for digital audio broadcasting, digital video broadcasting, and broad-band indoor wireless systems thanks to his efficiency combat inter-symbol interference (ISI). In fact, it is considered as an effective method for high-rate communication systems [2]. On the other hand, information theory indicates that a multi-input–multi-output (MIMO) system is able to support enormous capacities [3], provided the multipath scattering of a wireless channel is exploited with appropriate space–time signal-processing techniques. However, the MIMO system requires a complicated channel-equalization technique in a frequency-selective broad-band channel, in order to eliminate the ISI. The use of an OFDM technique for MIMO systems would be desirable to alleviate this problem. Recent studies have shown that high-performance transmission can be provided by combining the OFDM technique with a MIMO system [4]. By providing a temporal and a spatial multiplexing modulation, the space-time codes are used to improve MIMO performances. The Alamouti code [5] and the Golden code [6] represent the most known and used Space-Time Block codes (STBCs). The Golden code, which has been proposed in 2004 for 2*2 MIMO system, is a full-rate and full-diversity space-time code that has a maximal coding gain. Thanks to its algebraic construction, it will be shown in this paper that the ASTC codes outperforms the Alamouti codes in flat fading channels. In a frequency-selective channel, the ASTC codes lose their proprieties due the inter-symbol interference (ISI). The orthogonal frequency division multiplexing (OFDM) modulation can overcome this problem.

MIMO is used for transmitting multiple data streams or increasing the reliability (in terms of BER) assuming that the propagation channels between each pair of transmit and receive antennas are statistically independent and identically distributed. Or in practice, the channels between different antennas are often correlated which is called spatial correlation, interpreted as a correlation between a signal's spatial direction and the average received signal gain. The spatially correlated MIMO channels can substantially reduce system performances. So we propose to study the impact of spatial correlation on the ASTC code design performances.

In this work, we first propose a coded ASTC system in flat fading channel. Then, we analyze the ASTC-MIMO-OFDM system in Rayleigh selective-channel. We use a data aided channel estimation method based on the pilot symbol insertion in the detector to deduce the channel transfer function. Finally, we analyze ASTC-MIMO-OFDM system performances under spatially correlated channel.

This paper is organized as follows. In section II, we present the main criteria of the proposed coded ASTC chain. The third section focuses on ASTC-MIMO-OFDM system. In section IV, we analyze the ASTC-MIMO-OFDM system performances under spatially correlated channel. In section V, we present simulation results. Finally, a conclusion is given in section VI.

II. CODED ASTC SYSTEM

In this section we first propose the system model of a coded ASTC chain, then the channel model is introduced, and finally the decoder structure is described.
A. System Model
At first, equally probable random numbers (the data to be transmitted) are created in Matlab and mapped onto a 4-QAM constellation such that the possible symbol values are \(1+j\), \(1-j\), \(-1+j\), and \(-1-j\). Each information sequence \(S\) is encoded by the ASTC encoder.

In MIMO systems, the general transmission model is
\[
Y = HX + W
\]  
(2)
where \(X\) is the transmitted codeword, \(H\) is the channel matrix and \(W\) is the i.i.d. Gaussian noise matrix.

To have full-rate square codes using QAM constellation, we consider square (2*2) linear dispersion. We can express the code word \(X\) as the result of multiplication of each four consecutive symbols of information sequence \(S\) by the matrix \(\Phi\).

\[
\Phi = \begin{pmatrix}
\alpha & \alpha \theta & 0 & 0 \\
0 & 0 & i\alpha & i\alpha \theta \\
0 & 0 & \alpha & \alpha \theta \\
\bar{\alpha} & \bar{\alpha} \theta & 0 & 0
\end{pmatrix}
\]  
(3)

So at time \((t,t+1)\), we can express the vector \(X_t\) as follow, where the first two lines are transmitted over antenna 1, and the rest two ones are transmitted over antenna 2.

\[
X_t = \begin{pmatrix}
(\alpha(a + \theta b))_{(t,1)} \\
(\bar{\alpha}(c + \theta d))_{(t+1,1)} \\
(\alpha(c + \theta d))_{(t,2)} \\
(\bar{\alpha}(a + \theta b))_{(t+1,2)}
\end{pmatrix}
\]  
(4)

B. Channel Model
In this work, we suppose that the encoded signal is transmitted over a non selective correlated Rayleigh fading channel. We consider here the Clarke channel model. The resulting sequence \(X\) will be transmitted over a non selective channel \(H\). We can express the elementary matrix \(H_t\) at time \((t,t+1)\) as:

\[
H_t = \begin{pmatrix}
h_{11} & h_{12} & 0 & 0 \\
h_{21} & h_{22} & 0 & 0 \\
0 & 0 & h_{11} & h_{21} \\
0 & 0 & h_{12} & h_{22}
\end{pmatrix}
\]  
(5)

We note here that the encoder can transmit 4 symbols on each antenna at the same time, whereas the Alamouti [5] encoder can only code 2 symbols at a time.

C. Decoder structure
We will consider two structures of decoders: We decode the received signal using Brute Force ML Decoding and Sphere Decoding Algorithm.

1) ML decoding
The best performance is given by the brute force ML decoder which searches for the matrix \(X\) which minimizes the overall noise power. i.e. an ML decoder computes an estimate of the transmitted matrix as

\[
\hat{X} = \arg \min_{X} \|Y - HX\|^2
\]  
(6)

But the ML decoder has a very high complexity in MIMO channels. To lower the complexity, a new type of decoding method called sphere decoding can be used. The sphere decoding algorithm has near ML performance with reasonably low complexity [7].

2) The Sphere Decoding Algorithm
The principle of sphere decoding algorithm is to search the closest constellation point to the received signal with in a sphere of some initial radius. If a point is found and if the distance between the centre and the point is less than the radius, the radius is updated to that distance and the process is continued till only one point is left in the sphere. That will be the closest constellation point to the received point. If a point
is not found initially, then the sphere radius is incremented and the same process is followed [8].

III. ASTC-MIMO-OFDM SYSTEM

In this section we first describe the system model of ASTC-MIMO-OFDM chain, then the channel model is introduced, and finally the decoder structure is described.

A. System Model

We consider a coherent system over a frequency-selective correlated Rayleigh fading MIMO channel with two transmit and received antennas \(N_t = N_r = 2\). The overall schematic diagram of ASTC-MIMO-OFDM transceiver is depicted in Fig. 2.

The same stages applied to information data in the first system model (section II) will be applied to this system. Before transmission over the two antennas, a conversion to a serial stream of the ASTC output is done. The \(N_t\) streams are then fed to \(N_t\) OFDM modulators, which uses an IFFT module, with \(N_{fft}\) subcarriers and a cycle prefix (CP) of length \(N_c\). The \(N_t\) vectors of length \(N_{fft} + N_c\) are transmitted over a frequency and time selective MIMO channels. In order to avoid ISI, the CP length \(N_c\) is assumed to be longer than the largest multipath delay spread.

B. Channel model

We assume that the ASTC-MIMO-OFDM symbols are transmitted over a time and frequency selective Rayleigh fading channel and that the channel taps remain constant during a packet transmission. Consequently, the channel impulse response (CIR) between \(q^\text{th}\) transmitting antenna and \(p^\text{th}\) receiving antenna is modeled by a tapped delay line as

\[
h_{k}^{p,q}(l) = \sum_{i=0}^{L-1} h_{i}^{p,q}(l) \delta(k - l)
\]

where \(h_{i}^{p,q}(l)\) is the \(i^\text{th}\) path from the \(q^\text{th}\) transmitting antenna to \(p^\text{th}\) receiving antenna at time \(k\) and \(L\) is the largest order among all impulse responses. The channel taps sequence \(\{h_{i}^{p,q}(l)\}\) is a correlated complex Gaussian process with zero mean and the same variance \(\sigma_h^2\) and the autocorrelation function

\[
E[h_{k}^{p,q}(l)h_{k-l}^{m,n}(l')^\ast] = \rho_{rs}^{(m,p)} \rho_{rs}^{(n,q)} J_0(2\pi f_d m') \delta(l - l')
\]

where \(J_0\) is the Bessel function with zero order, \(f_d\) is the normalized Doppler shift, \(\rho_{rs}^{(m,p)}\) and \(\rho_{rs}^{(n,q)}\) refers respectively to the correlation coefficient between the received antenna \((m,p)\) and the transmitted antennas \((n,q)\). To obtain a correlated Rayleigh fading channel, the autocorrelation function of \(\{h_{i}^{p,q}(l)\}\) process is given by

\[
R_h = \sigma_h^2 \exp(j2\pi f_d k) J_0(2\pi f_d m k)
\]

We can thus express the MIMO-OFDM received signal in a matrix notation as

\[
y_k = h_k x_k + w_k
\]

Where \(x_k\) is the \(k^\text{th}\) MIMO-OFDM symbol, \(w_k\) represents the AWGN at time \(k\) with \(N_r N_{fft}\) i.i.d. elements and \(h_k\) is the equivalent channel matrix represented as

\[
h_k = \begin{bmatrix}
h_k(0) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
h_k(L-1) & \cdots & h_k(0)
\end{bmatrix}
\]

where \(h_k(l) = [h_{i}^{p,q}(l)]\) are the \(N_t N_r\) matrices for \(l=0\ldots L-1\).

C. Decoder structure

At the receiver, after removing the CP, the signal is transformed back to the frequency domain by the mean of a DFT process. We can express the received frequency-domain signal as

\[
Y = HX + W
\]

Where \(W\) is the frequency domain noise with zero mean and variance \(\sigma_w^2\), \(X\) is the frequency domain data matrix and \(H\) is a block diagonal matrix with \(N_t N_{fft}\) frequency response of the channel matrix given by

\[
H = \begin{bmatrix}
H(0) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & H(N_t N_{fft} - 1)
\end{bmatrix}
\]

Where \(n^\text{th}\) block \(H(n), n=0\ldots N_t N_{fft}-1\), represents the MIMO-OFDM channel gain at the \(n^\text{th}\) subcarrier and can be written as
We can express the restored useful data $X$ as

$$\hat{X} = H^\dagger Y$$

where $(\cdot)^\dagger$ denotes the pseudo-inverse operator.

As mentioned in equation 11 the restitution of signal needs the knowledge of the channel response which is generally unknown. In this section, we present a channel estimation method for OFDM systems using pilot symbols [9]. For MIMO-OFDM systems, pilots are inserted in both time and a frequency domain as it is shown in Fig. 3. Let us denote $XP$ the vector of length $P$ whose elements are the pilot symbols.

Based on the LS criterion, channel estimation method at pilot location, is given by

$$\overline{H}_p = (X_p)^\dagger Y_p$$

Then channel frequency response estimation at non-pilot positions can be done by interpolating the channel estimates at neighboring pilot symbol positions. Several efficient interpolation techniques for OFDM channel estimation have been investigated in [9]. In this work, we use the linear interpolation for its simplicity.

Once the channel effect is compensated, the decision variable $\hat{X}$ is passed for decoding. Zero Forcing sub-optimum decoder is used in this work, to reduce the numerical complexity without significant performance loss.

A serial to parallel module, at each DFT output, is used to reshape the signal $\hat{X}$. Then, we provide the sequences $\hat{S}$, and finally restitute the information sequence $\hat{S}$ given by

$$\hat{S} = \Phi_{c}^{-1} \hat{X}_{c}$$

**IV. PERFORMANCE OF ASTC-MIMO-OFDM SYSTEM UNDER SPATIALLY CORRELATED MIMO CHANNELS**

In practice, the MIMO channels between different antennas are often correlated which is called spatial correlation. As a result shown by information theory [11], the channel capacity can be substantially reduced for spatially correlated MIMO channels. It has been also shown that more the spatial correlation increases, more the BER decreases for the same SNR. So what is the impact of spatial correlation on the ASTC code design performances?

The capacity analysis of an ASTC-MIMO-OFDM system, under correlated Rayleigh frequency-selective channel, has been evaluated in [10]. It has been shown that the capacity increase almost linearly when the normalized Doppler frequency $f_d$ decrease, and the match between the analytic and the simulated capacity, is more perfect for high Eb/No.

In this part we determine the analytic capacity expression of ASTC-MIMO-OFDM system under spatially correlated channel. To introduce the correlation effect in the system model, a correlated matrix should be generated according to [12]

$$vec(H_c) = C * vec(H) = R_{c} * vec(H)$$

where $H_c$ represent the correlated matrix of the MIMO channel. The correlation matrix of the MIMO channel, $R$, is obtained by the Kronecker product of the transmit and receive correlation matrices as follow [13]

$$R = R_{tx} \otimes R_{rx}$$

Using the properties of the Kronecker product, we have

$$H_c = R_{tx}^{1/2} H (R_{rx}^{1/2})^T$$

We can thus re-express the received frequency-domain signal in a matrix notation as

$$Y = H_c X + W$$

$$H_c = \begin{bmatrix} H_c(0) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_c(Nf \ast Nfft - 1) \end{bmatrix}$$

$$H_c(n) = \sum_{l=0}^{L} R_{tx}^{1/2} h_l(l) R_{rx}^{1/2} \exp(-j2\pi nk/Nfft)$$

First of all we should determine the mutual information between transmitted code word $X$ and received vector $Y$. Then we can deduce the analytic expression of the information capacity of ASTC under spatially correlated MIMO channel for a given channel matrix $H_c$. The mutual information is given by
\[ I(X, Y) = H(Y) - H(Y|X) = H(Y) - H(W) \] (23)

Where \( H(Y) \) denotes the entropies of multivariate distribution \( Y \). The capacity is obtained by maximizing the mutual information

\[ C = \max(I(X, Y)) = \max(H(Y)) - H(W) \] (24)

So we have to determine the exact expression of \( H(Y) \) to obtain the capacity expression. It has been shown in [10] that the expression of the capacity can be expressed as

\[ C = \log_2 \det(I_{Nfft+Nt}) + \frac{1}{\sigma_w^2} Q_X H_c^H H_c \ \text{bits/s/Hz} \] (25)

The covariance matrix \( Q_X \) can be expressed as [10]

\[ Q_X = \{\Phi \Phi^H\}_{Nfft+Nt} \] (26)

where \( \Phi \) is a copy of \( \Phi \) (\( Nfft*Nt/4 \) times).

Then the capacity expression will be [10]

\[ C = \log_2 \det(I_{Nfft+Nt}) + \frac{1}{\sigma_w^2} \Phi \Phi^H H_c^H H_c \] (27)

If we derive the exact expression of \( \Phi \Phi^H \) we get \( I_{Nfft+Nt} \).

Exploiting the fact that \( H \) is a block diagonal matrix, we can write

\[ H_c^H H_c = \begin{pmatrix} H_c(0) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_c(Nt*Nfft-1) \end{pmatrix} \] (28)

\[ = \|H_c\|^2 = Nfft*Nt \|H_c(n)\|^2 \forall 0 \leq n \leq Nfft*Nt-1 \]

Whereas

\[ \|H_c(n)\|^2 = \left\| \sum_{l=0}^L R_{l,j}^{1/2} h_l(l) R_{j,l}^{1/2} \exp(-j2\pi \frac{nk}{Nfft}) \right\|^2 \]

\[ \leq \sum_{l=0}^L \left\| R_{l,j}^{1/2} h_l(l) R_{j,l}^{1/2} \exp(-j2\pi \frac{nk}{Nfft}) \right\|^2 \]

\[ \leq \sum_{l=0}^L \rho^2 \|h_l(l)\|^2 = \rho^2.L.P_h^2 \] (29)

\[ \rho^2 = \sum_{l=0}^L \left\| R_{l,j}^{1/2} \right\|^2 \text{ and } P_h \text{ denotes the power to each channel coefficient in time domain.} \]

At this stage, the last capacity expression can be bounded by

\[ C = \log_2 \left[ 1 + \frac{1}{\sigma_w^2} \|H_c\|^2 \right] \]

\[ = \log_2 \left[ 1 + \frac{1}{\sigma_w^2} Nfft.Nt \|H_c\|^2 \right] \] (26)

\[ \leq \log_2 \left[ 1 + \frac{1}{\sigma_w^2} Nfft.Nt.\rho^2.L.P_h^2 \right] \]

By averaging over the capacity of \( Nfft*Nt \) narrowband channels we derive the following capacity

\[ C \leq \log_2 \left[ 1 + \frac{1}{\sigma_w^2} Nfft.Nt.\rho^2.L.P_h^2 \right] \text{ bits/s/Hz} \] (31)

V. SIMULATION RESULTS

To investigate the performance of the proposed space time code, a series of Monte Carlo simulations were carried out. We first present coded ASTC system performances over correlated Rayleigh fading channel. Then we present the performance of the ASTC MIMO-OFDM system.

A. BER performance for coded ASTC system

Fig. 4 presents a comparison of Brute Force ML and Sphere decoding techniques. For Brute Force ML decoding technique, the BER is close to \( 10^{-1} \) for about 10 dB, however, in case of the Sphere decoding technique at 8 dB, the BER is about \( 10^{-1} \) so the gain is around 2dB for the Sphere decoding technique. The ML decoder suffers from a high complexity. These drawbacks can be addressed by the Sphere decoder technique, which helps reducing the BER.
Fig. 4. Comparison of Brute Force ML and Sphere decoding techniques

Fig. 5 compares the BER performances of the Golden code and the classical Alamouti code. We can see that for a high SNR, the Golden code have a good performance in terms of BER. In the case of 2x1 transmission, the Golden BER remains close to Alamouti BER; for BER equal to $10^{-3}$ the gain is around 1dB for Golden code. By comparing this case to the 2x2 transmission case, the Golden code provides a gain of about 4dB for a BER equal to $10^{-1}$. This gain comes from the fact that we are coding 4 symbols at the same time with Golden code, however we code only 2 symbols with Alamouti, then the gain still significant in terms of rate.

These results lead us to deal with real channel conditions mainly if we use a selective Rayleigh channel with unknown channel coefficients.

B. BER performance for ASTC-MIMO-OFDM system

In this sub-section we present a comparison of the BER performances between the ASTC code and the classical Alamouti code combined with MIMO-OFDM system.

Fig. 5 shows that the gain obtained by ASTC code is about 2 dB for a BER of $10^{-3}$. The same raison for this gain: it comes from the fact that we are coding 4 symbols at the same time with ASTC code, however we code only 2 symbols with Alamouti.

C. Performance for ASTC-MIMO-OFDM system under spatially correlated MIMO channels

In this sub-section, we illustrate the results of the effect of spatial correlation of MIMO channel on the performances of ASTC-MIMO-OFDM architecture in terms of BER and capacity.

We can see that the correlation between sub channels degrade considerably the performances of ASTC-MIMO-OFDM system. In fact, for a fixed SNR of 10 dB for example, the BER of ($R_{tx}=R_{rx}=0.99$) is much important than the BER of ($R_{tx}=R_{rx}=0.01$). For a fixed value of correlation, the BER decrease when the value of the signal to noise ratio is
increased. We also notice that for \((R_{Tx}=R_{Rx}=0.99)\), the BER is almost similar to uncoded SISO QAM-4 system. This can be explained by the fact that with strong correlation, the use of ASTC codes become less efficient and hence the importance of the spatial correlation on the hole performances.

The ergodic capacity as a function of SNR per receive antenna, for both the analytic capacity expression, derived in (30) and the simulated expression using (24), is shown in Fig. 8.

It can be observed that the strong correlation decrease considerably the BER and capacity performances of the ASTC code design: the strong spatial correlation has an impact on ASTC code design: the strong spatial correlation decrease considerably the BER and capacity performances of the ASTC-MIMO-OFDM system.

VI. CONCLUSIONS

In this paper, we have proposed a MIMO transmission system, based on Algebraic space time coding which has good properties. Numerical results show that this code has a reasonable BER that outperforms the classical Alamouti code over correlated Rayleigh fading channel. In realistic multihop channel, frequency selectivity can be solved by the use of OFDM modulation. Numerical results show that ASTC codes maintain their properties and achieve good BER performances compared to the classical Alamouti MIMO-OFDM system. Finally, we have demonstrated that spatial correlation has an impact on ASTC code design: the strong spatial correlation decrease considerably the BER and capacity performances of the ASTC-MIMO-OFDM system.

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