

Robust Joint Transceiver Power Allocation for Multi-User Downlink MIMO Transmissions

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Abstract– We consider the problem of robust joint transmitter and receiver power allocation for downlink multiple input multiple output (MIMO) transmissions. The channel model is assumed to be Rayleigh frequency flat fading. The objective of power allocation is to minimize the total mean square error under total transmit power constraint. Under robustness issue, we consider joint power allocation with imperfect channel state information (CSI), where the CSI error is assumed to have Gaussian distribution. We show that this problem is formulated as a convex optimization problem. Numerical results indicate the BER performance improvement obtained by considering the robustness into account in the joint power allocation process.

Keywords: robust power allocation, downlink, convex optimization problem, MIMO

I. INTRODUCTION

The downlink of multiuser MIMO wireless communications system has attracted considerable attention recently because of the potential for improving reliability and capacity of the system. A practical approach to this system is using linear signal processing at both ends of the transmitter and receiver to mitigate the co-channel interference (CCI) so as to achieve the desired performance [1]. Joint transmitter and receiver design is an effective interference management technique for MIMO systems. One objective of such a design is to minimize the mean square error (MSE) between a transmitted signal and its estimate at the receiver. [2]. In [3], the authors investigate the performance of joint transmitter and receiver power allocation under the constraint of zero multiple access interference (MAI). In [4,5], the authors use the Karush-Kuhn-Tucker (KKT) optimality conditions to develop an approximate iterative algorithm that performs joint transmitter and power allocation to minimize the total MSE across all users.

The performance of communication systems depends on the accuracy of CSI. Several designs with joint transmitter and receiver optimization assume that the channel is perfectly known at both ends of the transmissions. In practice, it is difficult to obtain perfect CSI due to reasons such as channel estimation error, quantization error, and feedback delay [6,7]. Several recent works consider transmit power allocation with imperfect CSI where the CSI errors are modeled either statistically or unknown but within certain bounds [7,8].

In this paper, we consider the robustness issue of joint transmitter and receiver power allocation in the downlink direction. We first formulated the optimization problem with perfect CSI. The objective of joint transmitter and receiver power allocation is to minimize the total MSE under total transmit power constraint. We refer to this objective as the minimize total MSE criterion. Second, for robustness issue, we consider imperfect CSI at both transmitter and receiver. The CSI statistic error is assumed to have Gaussian distribution. The optimization problem can be written as a convex optimization problem.

The rest of this paper is organized as follows. Section II describes the transmission system model. Section III presents the problem statement of joint transmitter and receiver power allocation with perfect CSI. Section IV presents robust joint transmitter and receiver power allocation with imperfect CSI. Section V contains numerical results and discussions. Finally, section VI provides a summary.

II. SYSTEM MODEL

The following notations will be used. An upper case boldface letter $\mathbf{X}$ represents a matrix. A vector is written as a lower case boldface letter $\mathbf{x}$. The column vector with entries $x_0, \dots, x_{N-1}$ is written as $(x_0, \dots, x_{N-1})$. $\mathbf{I}_N$ represents the $N \times N$ identity matrix. $\|\mathbf{X}\|_F$ denotes the Frobenius norm of matrix $\mathbf{X}$, which is the square root of the sum of all magnitude squares of the components of $\mathbf{X}$. The superscripts $(\cdot)^T$ and $(\cdot)^\dagger$ represent the transpose and complex conjugate transpose operators. The inverse of matrix $\mathbf{X}$ is denoted by $\mathbf{X}^{-1}$. The trace of $\mathbf{X}$ is denoted by $\text{tr}(\mathbf{X})$. Finally, the expectation of a random variable X is denoted by $E[X]$.

We consider synchronous downlink MIMO transmissions with K users. The system is assumed to operate in the time division duplex (TDD) mode with no inter-symbol interference (ISI). The modulation technique is binary phase shift keying (BPSK). The signal from the basestation k , with $k \in \{0, \dots, K-1\}$, transmits symbol $a^k \in \{1, -1\}$ using N_T transmit antennas and the transmit vector (transmit gains) $\mathbf{t}^k = (t_0^k, \dots, t_{N_T-1}^k)$. As a result, the signal transmitted from the N_T antennas from user k is $\mathbf{s}^k = \mathbf{t}^k a^k$. The total signal from all

K users is transmitted through the channel as shown in Figure 1.

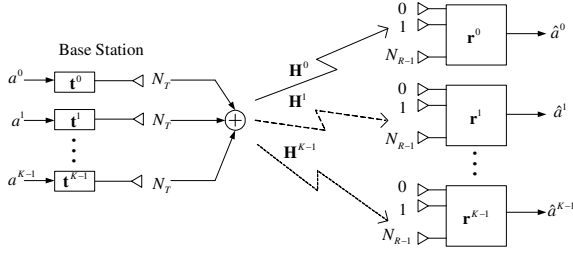


Figure 1: MIMO transmission downlink system model.

The receiver has N_R receive antennas. The received signals on these antennas are given by

$$\mathbf{y} = \sum_{k=0}^{K-1} \mathbf{H}^k \mathbf{s}^k + \mathbf{n} = \sqrt{E_b} \sum_{k=0}^{K-1} \mathbf{H}^k \mathbf{t}^k a^k + \mathbf{n} \quad (1)$$

where the energy per bit E_b is assumed to be 1, $\mathbf{H}^k$ is the $N_R \times N_T$ channel gain matrix for user k , and $\mathbf{n} = (n_0, \dots, n_{N_R-1})$ is the additive white Gaussian noise (AWGN) at the receive antennas. The entries of $\mathbf{H}^k$ are assumed independent and identically distributed (IID) according to the Rayleigh distribution with zero mean and unit variance, while the entries of $\mathbf{n}$ are IID zero mean Gaussian with variance σ^2 . In this work, the statistics of $\mathbf{H}^k$ only influences the numerical results, but not the process of power allocation.

The receiver recovers symbol a^k by linear processing of $\mathbf{y}$ using the receive vector $\mathbf{r}^k = (r_0^k, \dots, r_{N_R-1}^k)^T$. In particular, the decision statistics for a^k is $\hat{a}^k = \mathbf{r}^k \mathbf{y}$. Finally, the total transmit power constraint, which is denoted as $\sum_{k=0}^{K-1} \text{tr}(\mathbf{T}^k \mathbf{T}^{k\dagger}) \leq P$, where P is the power available for all users.

III. JOINT POWER ALLOCATION WITH PERFECT CSI

For signal detection, we use the MSE as the performance criterion, where the MSE for user k is $\mathbb{E}[(\hat{a}^k - a^k)^2]$. The total MSE from all K users is [3,5]

$$MSE = \sum_{k=0}^{K-1} \left(\left| \mathbf{r}^k \mathbf{H}^k \mathbf{t}^k - 1 \right|^2 + \sum_{j=0, j \neq k}^{K-1} \left| \mathbf{r}^k \mathbf{H}^j \mathbf{t}^j \right|^2 \right) + \sigma^2 \sum_{k=0}^{K-1} \left\| \mathbf{r}^k \right\|^2. \quad (2)$$

A. Formulation of Optimization Problem

We first formulated the optimization problem for joint transmitter and receiver power allocation with perfect CSI. In this case, the objective is to find the set of optimal transmit and receive vectors $\mathbf{t}^k$'s and $\mathbf{r}^k$'s in order to minimize the total MSE subject to the constraint on total transmit power. From (2), the problem can be expressed as

$$\min_{\mathbf{t}, \mathbf{r}} MSE \text{ subject to } \sum_{k=0}^{K-1} \text{tr}(\mathbf{t}^k \mathbf{t}^{k\dagger}) \leq P, \quad (3)$$

where $\mathbf{t} = \{\mathbf{t}^0, \dots, \mathbf{t}^{K-1}\}$ and $\mathbf{r} = \{\mathbf{r}^0, \dots, \mathbf{r}^{K-1}\}$.

To solve this problem, we form Lagrangian as in (3)

$$L(\mathbf{t}, \mathbf{r}, \mu) = MSE(\mathbf{t}, \mathbf{r}) + \mu \left(\sum_{k=0}^{K-1} \text{tr}(\mathbf{t}^k \mathbf{t}^{k\dagger}) - P \right), \quad (4)$$

where μ denoted lagrange multiplier. In order to achieve the optimum transmitter and receiver vector, we take the first order derivative of (3) with respect to both $\mathbf{t}^k$ and $\mathbf{r}^k$ yielding

$$\mathbf{t}_{opt}^k = \left(\sum_{j=0}^{K-1} \mathbf{H}^{j\dagger} \mathbf{r}^{j\dagger} \mathbf{r}^j \mathbf{H}^j + \mu \mathbf{I} \right)^{-1} \mathbf{H}^{k\dagger} \mathbf{r}^{k\dagger}, \quad (5)$$

$$\text{and } \mathbf{r}_{opt}^k = \mathbf{t}^{k\dagger} \mathbf{H}^{k\dagger} \left(\sum_{j=0}^K \mathbf{H}^j \mathbf{t}^j \mathbf{t}^{j\dagger} \mathbf{H}^{j\dagger} + \sigma^2 \mathbf{I} \right)^{-1}. \quad (6)$$

We observe that the value of Lagrange multiplier can be computed as follow. From (5), matrix $\mathbf{A}$ is defined as

$$\mathbf{A} = \sum_{j=0}^{K-1} \mathbf{H}^{j\dagger} \mathbf{r}^{j\dagger} \mathbf{r}^j \mathbf{H}^j. \quad (7)$$

Therefore, equation (5) can be written as

$$\mathbf{t}_{opt}^k = (\mathbf{A} + \mu \mathbf{I})^{-1} \mathbf{H}^{k\dagger} \mathbf{r}^{k\dagger}. \quad (8)$$

From the total power constraint $\sum_{k=0}^{K-1} \text{tr}(\mathbf{t}^k \mathbf{t}^{k\dagger}) \leq P$. Consider for the worse case

$$\begin{aligned} P &= \sum_{k=0}^{K-1} \text{tr}(\mathbf{t}^k \mathbf{t}^{k\dagger}) \\ &= \text{tr} \left(\sum_{k=0}^{K-1} (\mathbf{A} + \mu \mathbf{I})^{-1} \mathbf{H}^{k\dagger} \mathbf{r}^{k\dagger} \mathbf{r}^k \mathbf{H}^k (\mathbf{A} + \mu \mathbf{I})^{-1} \right) \\ &= \text{tr} \left((\mathbf{A} + \mu \mathbf{I})^{-1} \mathbf{A} (\mathbf{A} + \mu \mathbf{I})^{-1} \right), \end{aligned} \quad (9)$$

where matrix $\mathbf{A}$ has a singular value decomposition (SVD) and defined as $\mathbf{U}\mathbf{A}\mathbf{U}^\dagger$. Finally, the total power is rewritten as

$$\begin{aligned} P &= \text{tr} \left(\mathbf{U} (\mathbf{A} + \mu \mathbf{I})^{-1} \mathbf{A} (\mathbf{A} + \mu \mathbf{I})^{-1} \mathbf{U}^\dagger \right) \\ &= \text{tr} \left(\mathbf{U} \mathbf{A} \mathbf{U}^\dagger (\mathbf{A} + \mu \mathbf{I})^{-2} \right) \\ &= \sum_{i=1}^m \frac{\lambda_i}{(\lambda_i + \mu)^2}, \end{aligned} \quad (10)$$

where λ_i is a singular value of matrix $\mathbf{A}$.

B. BER Performance Approximation

To approximate the BER performance, we make an assumption that the interference plus noise follows a Gaussian distribution. With the Gaussian assumption, the BER of user k , denoted by BER^k , can be expressed as a function of the SINR for user k , denoted by $SINR^k$. It was shown in [5] that

$$SINR^k = \frac{1}{MSE^k} - 1, \quad (11)$$

which tells us that maximizing the SINR is equivalent to minimizing the MSE. For BPSK modulation, the bit error probability for user k , denoted by BER^k , is expressed as [9]

$$BER^k = \mathcal{Q} \left(\sqrt{2 SINR^k} \right). \quad (12)$$

where $\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du$ and known as Q-function.

IV. JOINT POWER ALLOCATION WITH IMPERFECT CSI

In this section, we consider robust joint transmitter and receiver optimization when both transmitter and receiver have imperfect CSI. In other words, for each user k , the channel knowledge $\hat{\mathbf{H}}^k$ is different from the actual channel matrix $\mathbf{H}^k$. Denote the CSI error by $\mathbf{E}^k = \mathbf{H}^k - \hat{\mathbf{H}}^k$. The CSI error is assumed to be Gaussian distributed, where $\mathbf{E}$ is the block diagonal matrix that contains all the $\mathbf{E}^k$'s.

To formulate the robust power allocation under the minimum total MSE criterion, we first write the total MSE taking into account the CSI error by defining the $MSE(\mathbf{E})$ as

$$MSE(\mathbf{E}) = \sum_{k=0}^{K-1} \text{tr} \left(\left| \mathbf{r}^k (\hat{\mathbf{H}}^k + \mathbf{E}^k) \mathbf{t}^k - 1 \right|^2 + \left(\sigma_E^2 \text{tr}(\mathbf{t}^k \mathbf{t}^{k\dagger}) + \sigma^2 \right) \mathbf{r}^k \mathbf{r}^{k\dagger} \right). \quad (13)$$

It follows that the robust joint transmitter and receiver power allocation problem with CSI error subject to total transmit power constraint can be formulated as the following optimization problem,

$$\min_{\mathbf{t}, \mathbf{r}} MSE(\mathbf{E}) \text{ subject to } \sum_{k=0}^{K-1} \text{tr}(\mathbf{t}^k \mathbf{t}^{k\dagger}) \leq P. \quad (14)$$

A. Robust Receive gain vector

We consider the design of the optimum receive gain vector. In order to compute the optimum receive gain vector, we differentiate (13) with respect to $\mathbf{r}^k$ and set the result to zero. So the optimum robust receive gain vector is given by

$$\mathbf{r}^k = \mathbf{t}^{k\dagger} \mathbf{H}^{k\dagger} \left(\hat{\mathbf{H}}^k \left(\sum_{j=k+1}^{K-1} \mathbf{t}^j \mathbf{t}^{j\dagger} \right) \hat{\mathbf{H}}^{k\dagger} + \sigma^2 \mathbf{I} + \sigma_E^2 \|\mathbf{t}^k\|^2 \right)^{-1} \quad (15)$$

where $0 \leq k \leq K-1$.

B. Robust Transmit gain vector

As described the design of the receiver gain vector for a given transmit gain vector $\mathbf{t}$. We now present the design of the robust transmitter for a given receiver gain vector. We express the robust transmitter and receiver power allocation problem as

$$\min_{\mathbf{t}, \mathbf{r}} \sum_{k=0}^{K-1} \left\| \mathbf{r}^k (\hat{\mathbf{H}}^k + \mathbf{E}^k) \mathbf{t}^k - 1 \right\|^2 + \left(\sigma_E^2 \|\mathbf{t}^k\|^2 + \sigma^2 \right) \|\mathbf{r}^k\|^2 \quad (16)$$

$$\text{subject to } \sum_{k=0}^{K-1} \|\mathbf{t}^k\|^2 \leq P. \quad (17)$$

For a given $\mathbf{r}$, the problem is a convex optimization problem and can be expressed as

$$\min_{\mathbf{t}} \sum_{k=0}^{K-1} \left\| \mathbf{r}^k (\hat{\mathbf{H}}^k + \mathbf{E}^k) \mathbf{t}^k - 1 \right\|^2 + \sigma_E^2 \|\mathbf{t}^k\|^2 \|\mathbf{r}^k\|^2 + \sigma^2 \|\mathbf{r}^k\|^2 \quad (18)$$

$$\text{subject to } \sum_{k=0}^{K-1} \|\mathbf{t}^k\|^2 \leq P. \quad (19)$$

The last term in the objective function in (18) does not affect the optimum value of $\mathbf{t}$, we drop this term in the formulation of the transmit gain vector optimization problem. Dropping this term and introducing the dummy variables γ^k and η^k for $0 \leq k \leq K-1$, the optimization problem in (18) and (19) is formulated as the following convex optimization problem,

$$\min_{\mathbf{t}, \gamma, \eta} \sum_{k=0}^{K-1} \gamma_k + \sigma_E^2 \|\mathbf{r}^k\|^2 \eta^{k^2} \quad (20)$$

$$\text{subject to } \left\| \mathbf{r}^k (\hat{\mathbf{H}}^k + \mathbf{E}^k) \mathbf{t}^k - 1 \right\|^2 \leq \gamma^k \quad (21)$$

$$\sum_{k=0}^{K-1} \left\| \mathbf{t}^k \right\|^2 \leq \eta^k \quad (22)$$

$$\eta^k \leq P. \quad (23)$$

V. NUMERICAL RESULTS

A. Joint Power Allocation with Perfect CSI

In this section, we solve the optimization problem for joint transmitter and receiver power allocation with perfect CSI using the optimization toolbox in the MATLAB software [10]. The channel gain matrices $\mathbf{H}^k$'s are randomly generated based on Rayleigh flat fading with independent zero-mean and unit-variance components. Assume that $N_T = N_R = 4$. Each data point is an average obtained from 10 realizations of channel gain matrices. For the problem with the minimax MSE criterion, we tried with 15 different random initial solutions and observed that the solutions always converge to the same local optimum. From our experience, we observe a unique local optimum in this joint power allocation problem. However, unlike the problem with the minimum total MSE criterion, we do not currently have a proof whether the problem is convex or not.

Figure 2 shows the total and maximum MSEs for the power allocation with the minimum total MSE criterion subject to the total transmit power constraint as a function of the SNR, which is equal to $P/(K\sigma^2)$.

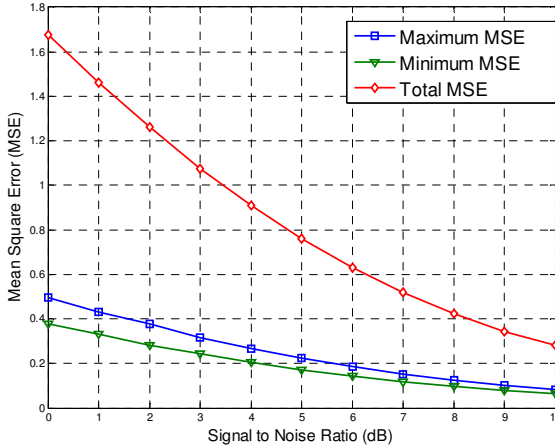


Figure 2: Total and maximum MSEs for power allocation with the minimum total MSE criterion.

Figure 3 shows the BER for all users and the average BER for the power allocation with the minimum total MSE criterion subject to the total transmit power constraint. We observe that the maximum of BER is occurred with user 3 due

to the bad channel quality as compare with the other channels gain and the other users have the good channel. In this figure, we also show the average BER which is computed under the total power constraint. We observe that user 1 and user 4 have roughly the same channel quality.

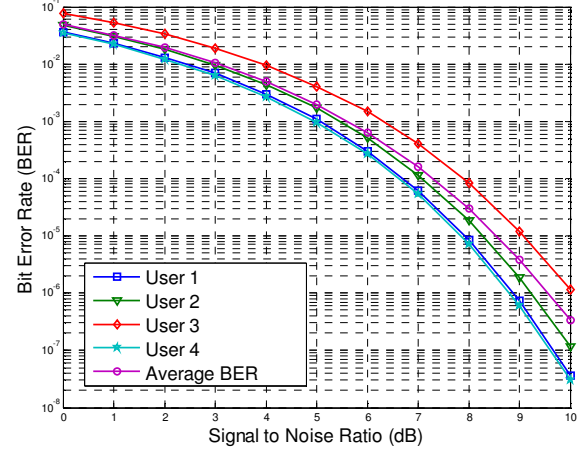


Figure 3: Individual Bit error rate and average BER for power allocation with the minimum total MSE criterions.

B. Joint Power Allocation with Imperfect CSI

To solve the convex optimization problems, we use the MATLAB toolbox YALMIP [11] to define optimization problem involving convex optimization problem.

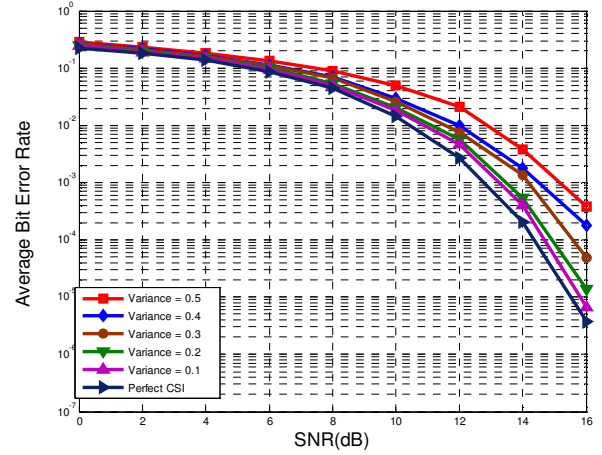


Figure 4: Average BERs for robust power with the minimum total MSE criterions for different value of σ_E^2 .

Figure 4 shows the average BERs for minimum total MSE criterions for robust joint power allocation under the different value of σ_E^2 . For a sufficiently large SNR, we observe that the

higher σ_E^2 or CSI error, the higher BER. We verify that a higher CSI error yields higher average BERs. In addition, at the average BER of 10^{-3} , the perfect CSI requires a lower SNR by approximately 1.1 dB at $\sigma_E^2 = 0.2$ and roughly 2.0 dB at $\sigma_E^2 = 0.5$.

VI. CONCLUSION

We formulated the robust joint transmitter and receiver power allocation problem for synchronous downlink MIMO transmissions. The objective of power allocation is to minimize the total MSE subject to the total transmit power constraint in the presence of CSI error. We assumed that the CSI errors have Gaussian distribution. Assume that the optimum receive gain vector is known, we transformed the power allocation problem into a convex optimization problem. Numerical results indicate that the MSE varied according to the channel quality and the higher value of $\sigma_E^2 = 0.2$, yield a higher BER. In addition, we quantify the reduction of the SNR required for the average BER of 10^{-3} obtained from robust power allocation.

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