

Comparative Analysis of Piecewise-Polynomial of Local Bases

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Abstract— In this work, the results of the comparative analysis of possibilities piecewise-polynomial local bases on three parameters are shown: by quantity of indispensable operations, by efficiency of compression of the information and by error.

Keywords— Algorithm, basis, Haar's function, Harmut's function, piecewise linear, piecewise quadratic, piecewise parabolic.

I. INTRODUCTION

For construction of patterns of the signals received from actual objects, traditional harmonious functions are widely applied. It explains that many signals received from actual objects can be easily presented by a set of sine wave and cosine fluctuations for which the device of Fourier analysis is used. The result is the transition from time to frequency functions. However, representation of time function sine and cosine functions is only one of many representations. Any full system of orthogonal functions can be applied for the decomposition in numbers which correspond to Fourier series.

II. SYSTEM OF HAAR AND HARMUT FUNCTIONS

Orthogonal system of basic functions which are set on a valid axis, for which there also exist algorithm of fast transformation have received wide circulation in technical appendices. They can be split into two classes:

- Global basic functions - such whose values are not equal to zero on one subinterval. Walsh Functions [1], [2], numerical [1], [4], [5],
- Allocative basic functions, nonzero values which are set on the enclosed segment. Examples are Haar functions [1], [2], [4] and Harmut's [2], [4], [6].

Splitting of the valid axis is usually double rational. Later we shall consider the interval [0,1] or [0,1) and will use the concept of a binary piece, which is as a result of division of a given interval $2p$ equal parts ($P=1, 2, \dots$):

$$h_k = h_{pj} = \left[\frac{j}{2^{p-1}} \quad \frac{j+1}{2^{p-1}} \right] \quad (1)$$

Where $j = 0, 1, \dots, 2^{p-1}$, $k = j + 2^{p-1}$

For example, binary pieces can serve as intervals

[0;1); [1/2; 3/4], [3/8; 4/8]

The length of a binary piece h_{pj} equals

$$|h_{pj}| = |h_{pj}^+| + |h_{pj}^-| = 2^{1-p};$$

Where h_{pj}^- , h_{pj}^+ represent its left and right half respectively and also represent binary pieces:

$$h_{pj}^+ = \left[\frac{j-1}{2^{p-1}}; \frac{2j-1}{2^p} \right], \quad h_{pj}^- = \left[\frac{2j-1}{2^p}; \frac{j}{2^{p-1}} \right] \quad (2)$$

The system of un-ratoned Haar functions to the form is defined as:

$$har_k(x) = har_{pj}(x) = \begin{cases} +1 & x \in h_{pj}^- \\ -1 & x \in h_{pj}^+ \\ 0 & x \in h_{pj} \end{cases} \quad (3)$$

It is necessary to note, that $har_0(x) \equiv 1$

Number P is the order of Haar's function and it is known that Haars series can provide as uniform (including the uniform best), and mean-square approximation. All depends on the method of calculating the factors.

$$f(x) = \sum_{k=0}^{\infty} C_k \cdot har_k(x) \quad (4)$$

Haar and Harmut's bases draws the attention of experts for two reasons:

- Reduction of the number of factors necessary for approximation (with a given accuracy) in relation to the general number of binary pieces.
- Absence of "long" operations in expression (2). Operations of addition, subtraction and shift are used only.

The study of methods for reducing the volume of tables of coefficients, improvement of parameters of "smoothness" by obvious means leads to systems of piecewise- polynomial basic functions of higher degree. Piecewise-linear basic functions (Shauder's function) are most easily got as a result of integration with a variable top limit of Haar's orthogonal piecewise-constant functions [5] [7]:

$$Shd_k(x) = 2^p \int_0^x har_k(r) dr \quad (5)$$

It is necessary to consider also, that $Shd_0(x) \equiv 1$ and $Shd_0(x) = x$.

Often in practical appendices, series of piecewise-linear functions with the purpose of getting the amplitudes of all basic functions that are equal to unit, are convenient to operate with "normalized" systems:

$$\begin{aligned} \tilde{Shd}_k(x) &= 2^p \int_0^x har_k(r) dr \\ P &= 0, 1, \dots; \quad k = 0, 1, 2, \dots \end{aligned} \quad (6)$$

Figure 1a shows piecewise-constant bases of Haar, Fig. 1b shows Shauder bases and Fig. 1c piecewise-parabolic bases of Haar.

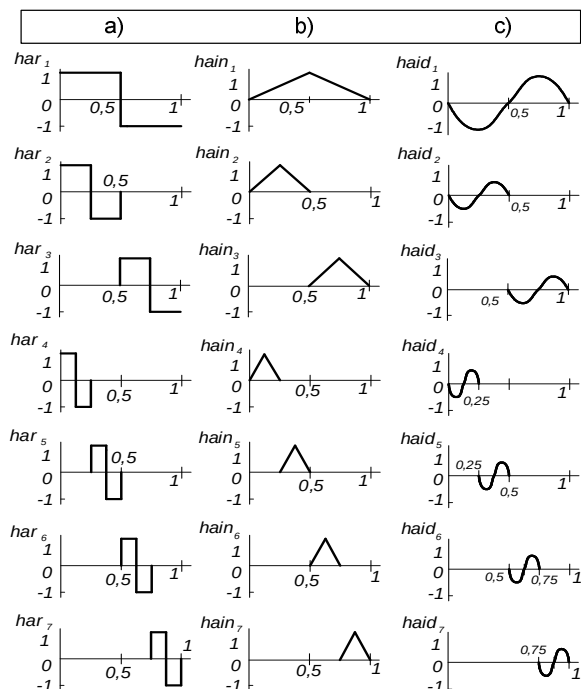


Figure 1 a) Piecewise-constant, b) Piecewise-linear and c) Piecewise-quadratic bases of Haar.

$har_k(x)$ functions can be placed in an orderly manner (Walsh's functions can analogically be arranged) in three ways:

- To arrange binary fractions k in the natural order of increment Adamarov's ordering;
- Arrange k in ascending order of inverted code - dyadic ordering;
- Grouping k in group with number of categories after a comma 1,2,3, etc., and inside of each group to arrange fractions in ascending order - classical ordering;

Adamars ordering of Haar functions leads to the reception of the optimal FTH algorithms, but practical use of spectra in adamars ordering is inconvenient ordering, since such spectrum does not provide uniform convergence of the partial sums for continuous functions which are usually considered as signals.

In [11], it is shown that the approximation of continuous signals with partial sums of a spectrum provides uniform convergence with classical ordering functions of Haar. As dyadic ordering differs from classical only by the arrangement of functions inside groups, it also has this property. Dyadic arrangement of Haar functions provides greater uniformity of approximation as rejection of members of some occurs not on one hand, and in regular intervals on all range of definition.

Harmut [13] [14] examined a system of piecewise-constant orthogonal basic functional occupying the position between Walsh and Haar systems. The most simple are functions of system $\{harm_k(x)\}$, obtained from Haar functions by double accession precisely same under the form, but on an adjacent binary piece on the right, and one of them joins with a "+" sign, and another with a "-" sign. As a result, a system of evens and odds relative to the middle of the binary interval functions, whose integration leads to a system of piecewise-linear functions and also even and odds:

$$hin_k(x) = 2^p \int_0^x hrm(r) dr \quad (7)$$

Charts of functions of system $\{harm_k(x)\}$ are shown in Figure 2a and in Figure 2b charts of system $\{hin_k(x)\}$ are shown. It can be seen that they are related to localized category.

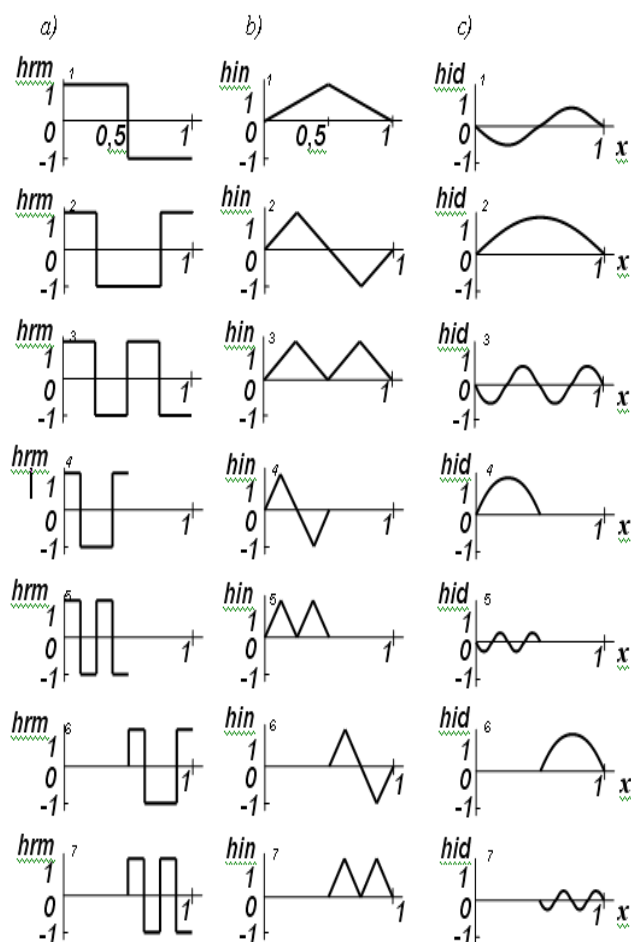


Figure 2 a) Piecewise-constant, b) Piecewise-linear and c) Piecewise-quadratic bases of Harmut.

In many practical appendices connected with recovery of functions between readouts, the possibilities of continuous piecewise-linear bases are not enough. This can be explained by two basic reasons:

- Because of low speed of convergence of the approximations caused by comparatively big error of piecewise-linear interpolation which often leads to significant loss on factors;
- Because of non smoothness of approximation (the first derivative of disruptive functions of basis) – there is the absence of the concept of curvature which causes essential restrictions. For example, it is possible to skip the maxima and minima of functions.

III. RESULTS OF THE COMPARATIVE ANALYSIS

In this work, results of the comparative analysis of possibilities of piecewise -constant, piecewise -linear and piecewise -quadratic local bases are shown. As criterion of comparison the factors were taken into consideration are:

- quantity of necessary operations;
- effect of signal compression;

- approximation error.

Requirements for algorithms of fast spectral transformation consist first of all, of a minimality of quantity of operations, simplicity of each operation and a minimality of the demanded volume of operative memory.

If we compare algorithms of fast transformation of Walsh (FTW) and fast transformation of Haar (FTH) by required quantity of operations of addition and subtraction, then the following can be deduced. If in algorithms FTW for spectral processing of an array from $N=2^n$ values of an initial signal, $n \times 2^n$ operations of addition and subtraction in algorithms of FTH, the required number of operations is reduced up to $2(N-1)$. For fast Harmut transformation this parameter has the value $3N-4$.

In Figures 3 and 4, results of the comparative analysis on effect of compression of a signal in the form of the diagram are shown. Here for research purpose, Haar's piecewise-linear bases (PLB), piecewise-constant basis (PCB) and piecewise-quadratic bases (PQB) were taken. For processing of these bases, functions consisting of the combination of elementary functions (CEF), a signal received as a result of geophysical researches (GR), and also a file received as a result of bench tests of car components (RBCC) are taken. The comparative analysis of all Haar and Harmut's piecewise bases shows that with transition from basis of the lowest degrees to higher degrees, the improvement of effect of compression as in Haar and Harmut's bases is observed.

For Haar bases, the effect of compression for PCB makes from 1.8% up to 2.7%, for PLB makes from 3.2% up to 7.1%, and for PQB this parameter makes from 5.5% up to 10.9 %. In other words the effect of compression constantly grows.

For Harmut bases, the effect of compression for PCB makes from 4.8% up to 12.3%, for PLB makes from 7.3% up to 29.3 %, and for PQB this parameter makes from 17.2% up to 56.2%. The effect of compression including this case constantly grows.

The comparative analysis of Haar and Harmut's bases shows, that from the point of view of effect of compression of a signal, Harmut's bases gives best result.

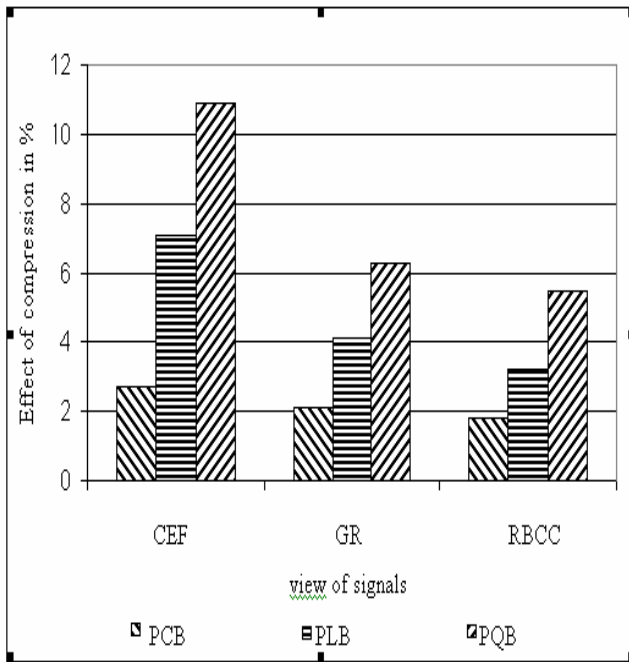


Figure 3. Results of comparison of opportunities of Haar's local bases.

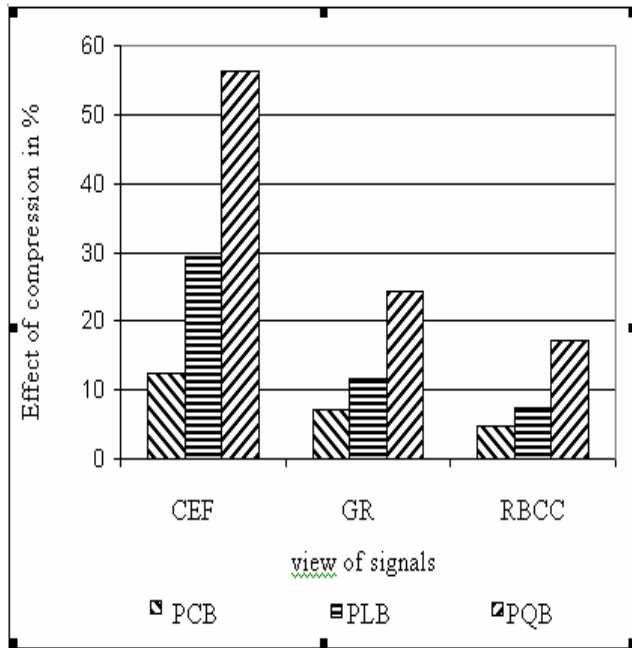


Figure 4. Results of comparison of opportunities of Harmut's local bases.

For an estimation of approximation error, we shall consider the corresponding expressions:

- For piecewise-constant bases:

$$|P(x) - f(x)| \leq \frac{1}{2} \max |f'(x)|h; \quad (8)$$

- For piecewise-linear bases:

$$|P(x) - f(x)| \leq \frac{1}{8} \max |f''(x)|h^2; \quad (9)$$

- For piecewise-quadratic bases:

$$|P(x) - f(x)| \leq \frac{\sqrt{3}}{27} \max |f'''(x)|h^3; \quad (10)$$

In these formulas $P(x)$ represents local Haar and Harmut's bases, $f(x)$ is approximated function, $f'(x)$, $f''(x)$ and $f'''(x)$, respectively, are derivatives of the first, second and third degree from approximated function, and h is the step of approximation.

IV. CONCLUSIONS

Thus, transition to Haar and Harmut's piecewise-polynomial bases allows us to learn the high effect of signal compression. Harmut's basis gives greater effect of compression in comparison with Haar's basis. This fact gives way to an increase in accuracy of approximation and also allows for the achievement high-efficiency computing structures on their basis.

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