

Design for the Asymmetric Turbo Codes

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Abstract -- Two types of asymmetric turbo codes are proposed, i.e., the parallel concatenated turbo codes that use the non-identical component encoder with different constraint lengths and mixed types of generator polynomials. The proposed schemes can optimize the BER performance of both water-fall region at low SNR and error-floor region at high SNR. The simulation results show the presented asymmetric turbo codes can achieve better performance than the conventional symmetric system over the entire ranges of SNR with a reduced decoding computational complexity.

Keywords -- Asymmetric turbo codes, bit error rate, water-fall, error-floor.

1. INTRODUCTION

Turbo codes have a near Shannon limit error correction capability through iterative decoding based on soft-input and soft-output(SISO) decoding algorithm. Turbo codes have been widely used as channel coding schemes of the wireless mobile communication system [1]. The conventional symmetric turbo encoder consists of two parallel concatenated recursive systematic convolutional (RSC) constituent encoders separated by an interleaver. Generally, these constituent encoders are the identical component codes which are the same as not only the constraint lengths but also the generator polynomials [1] [2].

The bit error rate (BER) performance curve of turbo codes is divided into two regions. The first region is so called "water-fall" in which the BER curve decreases rapidly at lower signal-to-noise ratio (SNR), the second region is so called "error-floor" where the BER curve flattens at higher SNR [3]. The conventional symmetric turbo codes have either a good water-fall performance or a good error-floor performance, but not both over the entire range of SNR. Asymmetric turbo codes are the better trade-off among the improvement of performance, the overall delay and the computational complexity of decoding algorithm. Several asymmetric turbo codes are investigated to improve BER performance compared with the conventional symmetric turbo codes [3] [4] [5].

In this paper, two types of asymmetric turbo codes are presented, which are consisted of the parallel concatenated turbo codes that use the non-identical component codes with different not only the constraint lengths but also the generator polynomials, the proposed schemes can optimize the BER performance both of the water-fall region at low SNR and the

error-floor region at high SNR. The simulation results show the proposed asymmetric turbo codes can perform significantly better than the conventional symmetric system over the entire ranges of SNR with a reduced decoding computational complexity.

2. ASYMMETRIC TURBO ENCODER AND DECODER

Asymmetric turbo encoder is composed of two different parallel concatenated RSC constituent encoders with different constraint length or different generator polynomial, and separated by an interleaver (Int), as shown in Fig.1. Input information bits u_k are grouped into frames of L bits and encoded with two RSC codes. x_k^s is the systematic information bit, x_k^{1p} and x_k^{2p} are the parity bits generated by RSC1 and RSC2, respectively. Puncturing of parity bits x_k^{1p} and x_k^{2p} can achieve the required code rate. c_k is the coded bits which are consisted of x_k^s and x_k^p by P/S multiplexer.

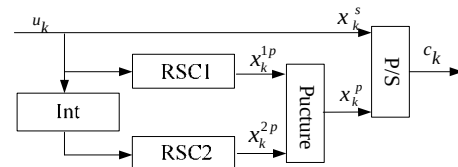


Figure1. Asymmetric turbo encoder

The coded bits c_k are then binary phase shift keying (BPSK) modulated and transmitted through an additive white Gaussian noise (AWGN) channel with a double sided power spectral density of $N_0/2$. At the receiver, the matched filter output y_k is multiplied by the channel reliability value L_c , as shown in Fig.2, $L_c = 4 \cdot (E_b/N_0) \cdot R$, where E_b/N_0 denotes received bit energy to noise power ratio and R represents the code rate [6]. Through S/P de-multiplexer, $L_c y_k$ is separated into $L_c y_k^s$, $L_c y_k^{1p}$ and $L_c y_k^{2p}$ corresponding to the systematic bit and two parity bits, respectively. At first iteration for decoding, a priori Log Likelihood Ratio (LLR) value of SISO1 decoder $L_{ai}(u_k)$ is set to 0, the systematic bit LLR value $L_c y_k^s$ and parity bits LLR value $L_c y_k^{1p}$ are inputted into SISO1 decoder. The extrinsic LLR value $L_{ei}(u_k)$ produced by SISO1 decoder is interleaved

and sent to SISO2 decoder as a priori value $L_{a2}(u_k)$, L_{c2}^{2p} and interleaved L_{c2}^{1p} are sent to SISO2 decoder simultaneously to produce extrinsic value $L_{e2}(u_k)$, then $L_{e2}(u_k)$ is de-interleaved (De-int) and fed back to SISO1 decoder as a priori value for the next decoding iteration. The iterative decoding will keep on working until either a stopping criteria is met or a preset maximum number of iterations is reached. When the iterative decoding is finished, the output value $L_2(u_k)$ of SISO2 decoder is de-interleaved and sent to hard decision (HD) to produce estimated value $\hat{u}_k$.

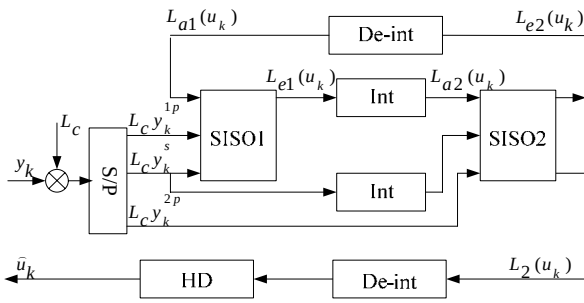


Figure2. Asymmetric turbo decoder

3. PROPOSED ASYMMETRIC TURBO CODES

Two types of asymmetric turbo codes are presented, which consist of the parallel concatenated turbo codes that use the non-identical component codes with the different constraint lengths or mixed types of generator polynomials.

3.1 Type-I of Asymmetric Turbo Codes

Type-I of asymmetric turbo codes consist of the parallel concatenated turbo codes that use the different types of two component codes with the different constraint lengths and generator polynomials.

The constraint length of the constituent encoder for turbo codes plays an important role in determining the performance and decoding computational complexity. Free distance is shorter for turbo codes with shorter constraint lengths, it can cause poorer error floor performance. Turbo codes with longer constraint lengths can achieve longer free distance and better performance, but the computational complexity will increase correspondingly. In order to obtain good performance and reduce decoding computational complexity, it is necessary to decrease the constraint length of one of the constituent encoders of conventional symmetric turbo codes, and then type-I of asymmetric turbo codes can be constructed by two component codes with different constraint lengths.

The generator polynomial of the component encoder for turbo codes is an important factor in affecting the performance of turbo codes. Using the primitive polynomials as the feedback generator polynomials of the component codes is known to be able to lower error-floor at high SNR, but at a cost of degrading water-fall performance at low SNR. If the non-primitive polynomials are used as one of the feedback generator polynomials, water-fall performance would be

improved at low SNR. And then, type-I of asymmetric turbo codes are also introduced that the generator polynomials of the component codes consist of mixed types of the primitive and non-primitive polynomials.

The types of generator polynomial denoted by G (octal) are listed in table 1, the constraint lengths denoted by K are made choice of 3 to 5, respectively. P-G denotes primitive polynomial used as the feedback generator polynomial of the constituent encoder of turbo codes, NP-G denotes non-primitive polynomial.

Table 1. The types of the component codes with the different constraint length and generator polynomial

	P-G	NP-G
K=3	G=[7, 5]	G=[5, 5]
K=4	G=[13,17]	G=[15,17]
K=5	G=[23,35]	G=[37,21]

The constraint length 5 of one of the constituent encoders of conventional symmetric turbo codes in [1] is reduced to 3 and 4, primitive and non-primitive polynomial are chosen as feedback generator polynomials of two constituent codes. According to table 1, sixteen types of asymmetric turbo codes with the different constraint lengths and generator polynomials are constructed, as shown in table 2. K1 and K2 denote the constraint lengths of RSC1 and RSC2, respectively ; G1 and G2 denote the generator polynomials of RSC1 and RSC2, respectively ; P and NP denote primitive and non-primitive polynomial, suffix 1, 2 of P and NP are corresponding to RSC1 and RSC2.

Table 2. sixteen types of asymmetric turbo codes with the different constraint lengths and generator polynomials

	P1-P2	NP1-NP2	NP1-P2	P1-NP2
K1=3	G1=[7, 5]	G1=[5, 5]	G1=[5, 5]	G1=[7, 5]
K2=5	G2=[23,35]	G2=[37,21]	G2=[23,35]	G2=[37,21]
K1=4	G1=[13,17]	G1=[15,17]	G1=[15,17]	G1=[13,17]
K2=5	G2=[23,35]	G2=[37,21]	G2=[23,35]	G2=[37,21]
K1=5	G1=[23,35]	G1=[37,21]	G1=[37,21]	G1=[23,35]
K2=3	G2=[7, 5]	G2=[5, 5]	G2=[7, 5]	G2=[5, 5]
K1=5	G1=[23,35]	G1=[37,21]	G1=[37,21]	G1=[23,35]
K2=4	G2=[13,17]	G2=[15,17]	G2=[13,17]	G2=[15,17]

3.2 Type-II of Asymmetric Turbo Codes

Type-II of asymmetric turbo codes consist of the parallel concatenated turbo codes that use the different component codes with the identical constraint length but the different types of feedback generator polynomial.

The performance of turbo codes at low SNR is determined by the distance spectrum, but the performance at high SNR is determined by the effective free distance. There are a number of key design parameters involved in determining the distance spectrum and free distance for turbo codes, such as the choice of constituent encoders, the types of interleaver and adding tail bits. Several interleavers are designed to improve BER performance of turbo codes [7] and [8]. Here, we focus all attention on using the non-identical component codes with the primitive and non-primitive feedback generator polynomials

to construct asymmetric turbo codes. The component codes with non-primitive generator polynomials may optimize the distance spectrum to improve water-fall performance, and the component codes with primitive generator polynomials may enlarge the free distance to reduce error-floor [3]. The performance of type-II of asymmetric turbo codes is superior to the conventional symmetric system over the entire range of SNR for the same constraint length.

Four types of the asymmetric turbo codes with the same constraint lengths but mixed types of generator polynomials are presented, as shown in table 3. In fact, P1-P2 and NP1-NP2 turbo codes are the conventional symmetric turbo codes. For the ease of comparison, they are treated as the special asymmetric turbo codes.

Table 3. four types of asymmetric turbo codes with the same constraint lengths but the difference generator polynomials

	P1-P2	NP1-NP2	NP1-P2	P1-NP2
K1=5	G1=[23,35]	G1=[37,21]	G1=[37,21]	G1=[23,35]
K2=5	G2=[23,35]	G2=[37,21]	G2=[23,35]	G2=[37,21]

4. SIMULATION RESULTS

For all simulations in the paper, the frame size of the information bits L is set to 378, the constraint lengths K are 3, 4 and 5, maximum number of iteration is 8, one-half code rate R , pseudo-random interleaver, SISO decoder, log-MAP decoding algorithm, BPSK modulation, AWGN channel. The simulation results are shown in Fig.3-5.

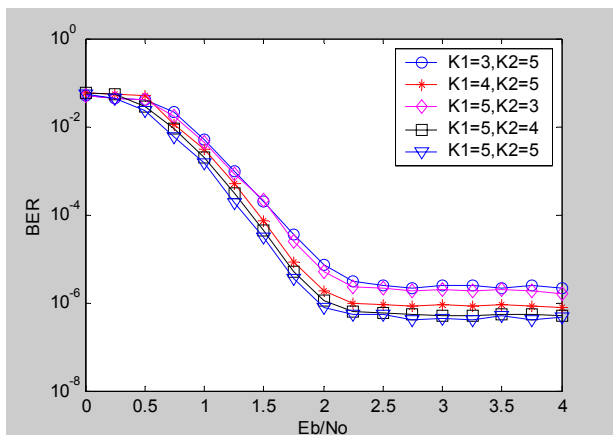


Figure3. BER versus E_b/N_0 for proposed asymmetric P1-P2 turbo codes with the combination of the different constraint lengths and primitive generator polynomials

Fig.3 shows the BER versus E_b/N_0 for proposed asymmetric P1-P2 turbo codes with the different combination of the constraint length $K=3, 4, 5$ and primitive generator polynomials. For the ease of comparison, the BER versus E_b/N_0 for the conventional symmetric P1-P2 turbo codes with the same constraint length $K_1=K_2=5$ and primitive generator polynomials is also shown in Fig.3, the conclusions are drawn:

(1) By decreasing the constraint length of one of the constituent encoders of conventional symmetric turbo codes,

the water-fall and error-floor performances of asymmetric turbo codes are all become bad. It is caused by the smaller free distance of the component codes with short constraint length. However, the computational complexity of decoding algorithm can be reduced for turbo codes with short constraint length. Therefore, this types of asymmetric turbo codes can get a favorable trade-off between the performance and the computational complexity.

(2) The performance of asymmetric turbo codes with $K_1=4, K_2=5$ or $K_1=5, K_2=4$ is slightly inferior to conventional symmetric turbo codes with $K_1=K_2=5$. Especially, the performance of asymmetric turbo codes with $K_1=5, K_2=4$ is nearly close to conventional turbo codes but with a reduced computational complexity.

(3) The performance of asymmetric turbo codes with $K_1=5, K_2=4$ is superior to one with $K_1=4, K_2=5$. Similarly, the performance of asymmetric turbo codes with $K_1=5, K_2=3$ is superior to that with $K_1=3, K_2=5$. It is evident that reducing the constraint length of RSC2 component code is a good scheme when the asymmetric turbo codes with the different constraint lengths are constructed.

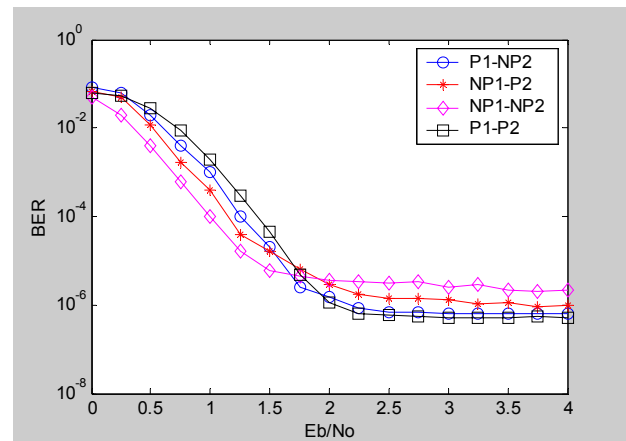


Figure4. BER versus E_b/N_0 for proposed asymmetric turbo codes with the combination of the different constraint length $K_1=5, K_2=4$ and the mixed types of generator polynomials

The BER versus E_b/N_0 for the four types of asymmetric turbo codes with the combination of the different constraint lengths $K_1=5, K_2=4$ and mixed types of generator polynomials is shown in Fig.4, we draw the conclusions:

(1) At low SNR ($E_b/N_0 \leq 1.75\text{dB}$), the performance of NP1-NP2 turbo codes is the best, NP1-P2 the better, P1-NP2 inferior, P1-P2 poor. But at high SNR ($E_b/N_0 \geq 1.75\text{dB}$), the results are in reverse order. The reason is that the non-primitive generator polynomials can optimize the distance spectrum to improve water-fall performance of turbo codes and the primitive polynomials enlarge the free distance to reduce error-floor.

(2) Considering the entire range of SNR ($0\text{dB} \leq E_b/N_0 \leq 4\text{dB}$), NP1-P2 and P1-NP2 turbo codes are the better trade-off between the water-fall and the error-floor performance. Evidently, NP1-P2 and P1-NP2 turbo codes are superior to P1-P2 and NP1-NP2 turbo codes over the entire

range of SNR. Therefore, it is a good choice that one constituent encoder is commonly constructed as a primitive feedback generator polynomial and another as non-primitive polynomial when the asymmetric turbo codes with the mixed types of generator polynomials are designed.

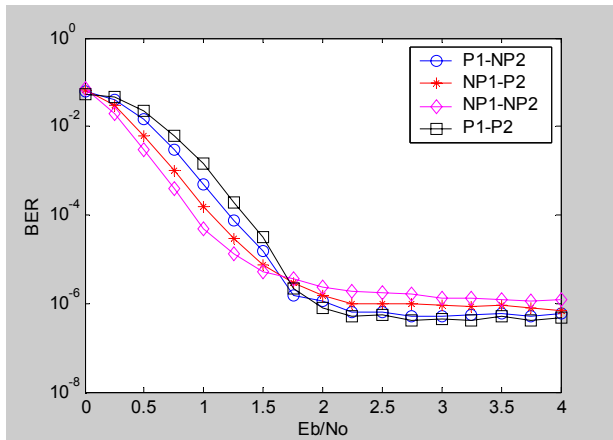


Figure 5. BER versus E_b/N_0 for proposed asymmetric Turbo codes with the combination of the same constraint length $K_1=K_2=5$ and the mixed types of generator polynomials

Fig.5 shows the BER versus E_b/N_0 for the four types of asymmetric turbo codes with the combination of the same constraint length $K_1=K_2=5$ and mixed types of generator polynomials. It was observed from the simulation results in Fig.5:

(1) The error-floor performance of conventional symmetric P1-P2 turbo codes is good at high SNR, but its water-fall is poor at low SNR. On the contrary, the water-fall performance of symmetric NP1-NP2 turbo codes is superior at low SNR, but its error-floor is inferior at high SNR.

(2) At low SNR, the water-fall performance of the proposed asymmetric NP1-P2 and P1-NP2 turbo codes is superior to symmetric P1-P2 turbo codes but inferior to NP1-NP2 turbo codes. However, the error-floor of NP1-P2 and P1-NP2 asymmetric turbo codes is inferior to P1-P2 turbo codes but superior to NP1-NP2 turbo codes at high SNR. These conclusions indicate effectively that the proposed asymmetric turbo codes are superior to the conventional symmetric turbo codes over the entire range of SNR.

5. CONCLUSIONS

In this paper, two types of asymmetric turbo codes are presented, their two component codes are constructed with different constraint lengths and mixed types of generator polynomials. The proposed schemes can improve the waterfall performance at the low SNR and the error floor at the high SNR while reducing the decoding computational complexity. Compared with the conventional symmetric turbo codes, the presented asymmetric turbo codes have the advantages of the superior BER performance over the entire ranges of SNR.

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