

Mathematical Analysis of Primary Users Characteristics Effects on Cognitive Radios

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Abstract— Cognitive Radios (CRs) have been recently proposed for the problem of spectrum scarcity. The main principle of the operation of these radios is based on the opportunistic access to the frequency spectrum mainly dedicated to Primary Users (PUs). However, statistical characteristics of PUs, either their spatial distribution or time pattern of channel usage can affect the usability of a specific frequency band for CRs. In this paper, based on mathematical analysis, first, the effect of the spatial distribution of PUs on the number of CRs in an Ad-Hoc configuration will be investigated. Analysis of this case shows that there are limits on the number of CRs for full coverage of a given area when they transmit simultaneously with PUs. Then, based on the temporal statistics of PUs, i.e., their arrival rate and channel usage, the usability of a specific frequency band for CRs will be explored in a non-simultaneous scenario. Mathematical analysis shows that for a given temporal specifications of PUs, there is an upper bound for the achievable data rates CRs may obtain.

Keywords— Cognitive Radios, Primary Users, Spectrum Scarcity, Ad-Hoc Network

I. INTRODUCTION

The idea of Cognitive Radios (CRs) based on opportunistic access to the frequency spectrum which is mainly licensed to Primary Users (PUs), has opened new ideas for spectrum utilization [1]. However, for a specific frequency band to be usable by CRs, the statistical characteristics of PUs, either spatial or temporal, seem to be determining factors. For example, if in a given area PUs are so densely distributed that any transmission by CRs would interfere with some PUs, there would be no chance for simultaneous signal transmission of CRs with PUs. On the other hand, when PUs enter and use their channels very frequently, there may not be any opportunity for CRs to non-simultaneously communicate, too.

As the first candidate, TV frequency bands, since they are reported to be busy at most during 25 percent of times, have been announced as the introductory bands for the operation of CRs [2-7]; However, if scenario successes, there would be no obstacle against the extension of the idea to other possible frequency bands. Thus, the usability analysis of a certain

channel in terms of statistical parameters of the corresponding PUs is of importance. In this paper, based on mathematical analysis, the effect of the spatial and temporal statistics of PUs on the usability of their channels for CRs will be investigated. To do so, the structure of the paper will be as follows. In the next section, stating the problem, the situation in which CRs are allowed to transmit will be reviewed. Then, in section III and IV, the effect of the spatial and temporal statistics of PUs on the structure of CRs' network in Ad-Hoc configuration and their available spectrum resources, respectively, will be investigated through mathematical analysis. Finally, section V concludes the paper.

II. PROBLEM STATEMENT

When CRs are allowed to coexist with PUs, two situations are expected: Simultaneous and non-simultaneous utilization of the channel with respect to PUs. When CRs and PUs work simultaneously, four situations can be imagined:

- Both sides cause serious interference for the other sides
- None interfere with the other sides
- Only CRs' transmitters interfere with PUs' receivers
- Only PUs' transmitters interfere with CRs' receivers

The cases 1 and 3 are not permitted, thus CRs should improve their behaviours in order to avoid interfering with PUs. On the other hand, they may want to improve their situations so that they themselves find opportunity to transmit. Case 2, of course, is ideal for both sides. In case 4, if CR is to receive data, it should change its behaviour for reliable reception, too.

If CRs intend to transmit simultaneously with PUs, they should make sure that no excessive interference will be caused for PUs. This situation may be achieved in two ways:

- PUs know CRs' signals very well and are able to cancel them at their receivers
- PUs don't know anything about CRs' signals and thus, it's the CRs' task to avoid interfering with PUs.

In the second situation, one way for CR to be allowed for transmission, if appropriate, is changing its geographical position so that not interfere with PUs' receivers. This capability implies the mobility of CRs if PUs' positions are not

fixed. Also, in a given minimum signal to noise-interference ratio (SNIR) for PUs, this dictates an Ad-Hoc network if they need to have an acceptable coverage. In this case, some conditions on CRs' network in terms of their minimum number and maximum distance must be determined. On the other hand, if CRs decide to transmit only when PUs are absent, based on the temporal statistics of PUs, time holes granted to CRs may be usable or not. To render a specific frequency band with its corresponding PUs as usable or unusable for CRs, thus, conditions under which CRs are allowed to transmit must be analysed.

III. EFFECT OF SPATIAL DISTRIBUTION OF PUS ON THE CONFIGURATION OF AD-HOC CRs

When CRs intend to transmit simultaneously with PUs, they should make sure that their transmissions would not cause severe interference for PUs. Of course, this would not be possible when density of PUs within an area is high. In this case, any transmission of CRs will cause interference for PUs' and depending on the PUs' density, changing the geographical position of CRs may or may not help them to find opportunity for transmission. Here, thus, the density of PUs in a given area is the key factor for possibility of CRs operation.

For the purpose of analysis, assume that PUs have determined the limit IT_{max} for the maximum tolerable Interference Temperature (IT). For a given distance from PUs' receivers, this limits the maximum transmission power of CRs. In mathematical form, we have:

$$IT \leq IT_{max} \Rightarrow P_{r-CR} = \frac{P_{t-CR}}{(4\pi f R)^2} \leq P_{max} = kBIT_{max} \quad (1)$$

thus,

$$P_{t-CR} \leq (4\pi f R)^2 kBIT_{max} = P_{t-CR-max} \quad (2)$$

where, P_{r-CR} is the received power of a CR's transmitter at a PU's receiver, P_{t-CR} is the corresponding transmitted power of CR and R is the distance between CR and PU. For simplicity, here, it is assumed that any CR and PU work as both receiver and transmitter. Equation (1) shows the maximum permissible transmitted power of CR for a distance R from PU. On the other hand, if CR has to transmit with power P_{t-CR} , the minimum distance from PU would be:

$$R \geq \sqrt{\frac{P_{t-CR}}{(4\pi f)^2 kBIT_{max}}} = R_{min} \quad (3)$$

To determine the IT_{max} for PU, one suitable alternative is defining IT_{max} based on the minimum required SNIR with P_{r-CR} as the received interference at PU. Thus, we have:

$$SNIR_{PU} = \frac{S_{PU}}{N + kBIT} = \frac{S_{PU}}{N + \frac{kBP_{t-CR}}{(4\pi f R)^2}}$$

$$\geq SNIR_{PU-min} = \frac{S_{PU}}{N + kBIT_{max}} \quad (4)$$

thus,

$$P_{t-CR} \leq \frac{(4\pi f R)^2}{kB} \left(\frac{S_{PU}}{SNIR_{PU-min}} - N \right) \quad (5)$$

where S_{PU} is the received power of PU from its own corresponding transmitter. On the other hand, CRs themselves need a minimum SNIR in order to complete their communication. This means that for a given distance D from their corresponding receivers, CRs need to use the minimum transmission power P_{t-min} . That is:

$$\begin{aligned} SNIR_{CR} &= \frac{P_{t-CR}}{N(4\pi f D)^2} \geq SNIR_{CR-min} = \frac{P_{t-min}}{N} \\ &= \frac{P_{t-min}}{N(4\pi f D)^2} \\ \Rightarrow P_{t-CR} &\geq N(4\pi f D)^2 SNIR_{CR-min} \end{aligned} \quad (6)$$

Taking the minimum distances R_{min} and D_{min} instead of R and D and combining (5) and (6) yields:

$$\begin{aligned} N(4\pi f D_{min})^2 SNIR_{CR-min} \\ \leq \frac{(4\pi f R_{min})^2}{kB} \left(\frac{S_{PU}}{SNIR_{PU-min}} - N \right) \end{aligned} \quad (7)$$

This gives rise to:

$$\left(\frac{D_{min}}{R_{min}} \right)^2 \leq \frac{1}{kB N \times SNIR_{CR-min}} \left(\frac{S_{PU}}{SNIR_{PU-min}} - N \right) \quad (8)$$

If PU's transmitter at distance L from CR causes interference for it, too, (8) will be changed into:

$$\left(\frac{D_{min}}{R_{min}} \right)^2 \leq \frac{\left(\frac{S_{PU}}{SNIR_{PU-min}} - N \right)}{kB \left(N + \frac{S_{PU}}{(4\pi f L)^2} \right) SNIR_{CR-min}} \quad (9)$$

Now, assume that N_{CR} and N_{PU} , the number of CRs and PUs respectively, are uniformly distributed in an area $A = 4\pi R^2$. In order for CRs to not interfere with PUs, no CR should be present in a circle with radius R_{min} . With the uniform distribution of CRs and furthermore, assuming a large enough N_{CR} , the probability of k CRs be present in an area of radius R_{min} follows a binomial distribution and is equal to [8]:

$$\begin{aligned} P(X_{CR} = k) &= \binom{N_{CR}}{k} \left(\frac{R_{min}^2}{R^2} \right)^k \left(1 - \frac{R_{min}^2}{R^2} \right)^{N_{CR}-k} \\ &\xrightarrow{\text{yields}} \frac{e^{-\left(N_{CR} \frac{R_{min}^2}{R^2} \right)} \left(\frac{R_{min}^2}{R^2} \right)^k}{k!} \end{aligned} \quad (10)$$

where X_{CR} is the random variable (R.V.) of number of CRs present in the area of radius R_{min} . To derive (10), the Poisson approximation has been used for binomial distribution when N_{CR} is large. If transmission of CRs is to not interfere with PUs, we should have:

$$P(X_{CR} < 1) = P(X_{CR} = 0) = e^{-\left(N_{CR} \frac{R_{min}^2}{R^2}\right)} \quad (11)$$

Assuming this probability should be larger than some threshold α , then:

$$e^{-\left(N_{CR} \frac{R_{min}^2}{R^2}\right)} \geq \alpha \Rightarrow N_{CR} \leq -\frac{R^2}{R_{min}^2} \ln(\alpha) \quad (12)$$

On the other hand, in order for CRs to be able to communicate with each other, two conditions must be satisfied. First, interferences from PUs must be low enough. This implies that no PU should be present in a neighbourhood of radius D_{min} . Second, the distances between CRs should be lower than D_{min} in order for their signals to be reliably communicated. Taking the same routine as in (10) for large N_{PU} , two following conditions must be satisfied:

- Number of PUs in a circle with radius D_{min} must be zero such that CRs encounter no interference.
- Number of CRs in a circle with radius D_{min} must be greater than zero such that they can communicate with each other.

Assuming the same threshold probability of α as in (12) for simplicity, we would have:

$$\begin{aligned} P(X_{PU} < 1) &= Prob(X_{PU} = 0) = e^{-\left(N_{PU} \frac{D_{min}^2}{R^2}\right)} \geq \alpha \\ \Rightarrow N_{PU} &\leq -\frac{R^2}{D_{min}^2} \ln(\alpha) \\ OR \quad D_{min}^2 &\leq -\frac{R^2}{N_{PU}} \ln(\alpha) \end{aligned} \quad (13)$$

$$\begin{aligned} P(X_{CR} \geq 1) &= 1 - P(X_{PU} = 0) = 1 - e^{-\left(N_{CR} \frac{D_{min}^2}{R^2}\right)} \geq \alpha \\ \Rightarrow N_{CR} &\geq -\frac{R^2}{D_{min}^2} \ln(1 - \alpha) \\ OR \quad D_{min}^2 &\geq -\frac{R^2}{N_{CR}} \ln(1 - \alpha) \end{aligned} \quad (14)$$

Combining (13) and (14), we have:

$$\begin{aligned} -\frac{R^2}{N_{CR}} \ln(1 - \alpha) &\leq D_{min}^2 \leq -\frac{R^2}{N_{PU}} \ln(\alpha) \\ \Rightarrow N_{CR} &\geq \frac{\ln(1 - \alpha)}{\ln(\alpha)} N_{PU} \end{aligned} \quad (15)$$

and combining (12) and (15) would lead to:

$$\frac{\ln(1 - \alpha)}{\ln(\alpha)} N_{PU} \leq N_{CR} \leq -\frac{R^2}{R_{min}^2} \ln(\alpha) \quad (16)$$

This means that for a given R_{min} and N_{PU} of PUs, there must be limits on the number of CRs if they intend to communicate with each other in an Ad-Hoc configuration. A small R_{min} can be perceived as high sensitivity of PU's receiver and vice versa; thus, when PUs' receivers are not sensitive enough, CRs must not be present everywhere. On the other hand, in a crowded society of PUs, CRs must be present between any pair of PUs in order to provide full coverage. It may not be possible for CRs' nodes to satisfy these conditions always. If so, they have to transmit non-simultaneously with PUs.

IV. EFFECTS OF TEMPORAL STATISTICS OF PUS ON THE AVAILABLE RESOURCE FOR CRS

Now assume the non-simultaneous scenario. This case is appropriate when PUs' density is so high that there is no chance for simultaneous transmission of CRs with PUs. In this case, CRs must wait until PUs leave the channel, then they can transmit. The effects of temporal statistics of PUs may be considered from two points of view: first, the rates at which PUs enter the channel and second, the average times PUs hold their channel. Analysing the effects of these characteristics will determine whether a specific channel is usable by CRs or not.

To analyse this situation, assume that number of arrivals of PUs in a given time interval follows a Poisson distribution with parameter λ . CRs may or may not know the time pattern based on which PUs use their channels. First, assume that CRs only know λ . In this case, a packet of length T_{CR} can be transmitted without causing any interference to PUs with following probability:

$$P(X_{PU} < 1) = P(X_{PU} = 0) = e^{-\lambda T_{CR}} \quad (17)$$

Assume further that a single CR has N packets to transmit, where N is assumed to be large enough. The probability of transmitting k packets without interfering with PUs would be $e^{-k\lambda T_{CR}}$. Thus, the average transmission time without interfering with PUs would be:

$$\begin{aligned} T_{average} &= T_{CR} e^{-\lambda T_{CR}} + 2T_{CR} e^{-2\lambda T_{CR}} + \dots + NT_{CR} e^{-N\lambda T_{CR}} \\ &= \sum_{k=1}^N k T_{CR} e^{-k\lambda T_{CR}} \\ &\cong \sum_{k=1}^{\infty} k T_{CR} e^{-k\lambda T_{CR}} \\ &\xrightarrow{T_{CR} \rightarrow 0} \frac{1}{T_{CR}} \int_0^{\infty} t e^{-\lambda t} dt = \frac{1}{\lambda^2 T_{CR}} \end{aligned} \quad (18)$$

Thus, the average number of packets would be:

$$N_{average} = \frac{T_{average}}{T_{CR}} = \frac{1}{\lambda^2 T_{CR}^2} \quad (19)$$

This implies that for large λ , there is no chance for CRs to transmit. On the other hand, if CRs use the packets of half length, they can transmit packets two times more than before. Another meaning of (19) is that CRs can compensate for large λ through using shorter packets. This is not, of course, true always since within a data packet all the bits don't carry CRs' information.

When CRs know the probability distribution of PUs' time length of channel usage, situation is somewhat different. For the purpose of analysis, take t_{PU} be the R.V. of PUs' channel usage with mean $\mu_{t_{PU}}$. In order to find the available channel time for CR, take the time interval T_A as the analysis interval and assume further that T_A is large enough. Again, in order to find opportunity for transmission, CR must wait for PU to leave the channel. If PU enters k times, then the average time for CR equals $T_A - k \mu_{t_{PU}}$. Using this time, CR can transmit packets as many as:

$$n_k = \left\lfloor \frac{T_A - k \mu_{t_{PU}}}{T_{CR}} \right\rfloor \quad (20)$$

where operator $\lfloor \cdot \rfloor$ stands for the nearest smaller integer. In this case, the maximum number of packets N_{max} would be achieved when $k = 0$. Taking this number to be large enough, then, the average number of packets would be:

$$\begin{aligned} N_{average} &= \sum_{k=0}^{N_{max}} n_k e^{-\lambda T_A} \frac{(\lambda T_A)^k}{k!} \\ &= \sum_{k=0}^{N_{max}} \left\lfloor \frac{T_A - k \mu_{t_{PU}}}{T_{CR}} \right\rfloor e^{-\lambda T_A} \frac{(\lambda T_A)^k}{k!} \\ &\leq \sum_{k=0}^{N_{max}} \frac{T_A - k \mu_{t_{PU}}}{T_{CR}} e^{-\lambda T_A} \frac{(\lambda T_A)^k}{k!} \\ &\cong \sum_{k=0}^{\infty} \frac{T_A - k \mu_{t_{PU}}}{T_{CR}} e^{-\lambda T_A} \frac{(\lambda T_A)^k}{k!} \\ &= \frac{T_A}{T_{CR}} (1 - \lambda \mu_{t_{PU}}) \end{aligned} \quad (21)$$

Now let T_{CR} be the length of a single bit, then $N_{average}$ shows the maximum number of transmitted bits during T_A on average. Therefore, the achievable bit rate R_b would be:

$$R_b = \frac{N_{average}}{T_A} \leq \frac{(1 - \lambda \mu_{t_{PU}})}{T_{CR}} \quad (22)$$

One meaning of (22) can be that there is no chance for CRs to use channels when PUs either enter their channels very frequently or hold it often very long, but there is always

opportunity for enough high speed CRs, of course, if they not violate the channel boundaries. For a channel with bandwidth B , this emphasizes that the symbol duration T_s satisfies:

$$\frac{1}{T_s} = \frac{1}{k T_{CR}} < B \quad (23)$$

thus,

$$R_b < k B (1 - \lambda \mu_{t_{PU}}) \quad (24)$$

where, k shows the number of bits per symbol, which is dependent on the modulation size. Equation (24) shows the dependency of the maximum achievable data rate on the temporal statistics of PUs in one hand, and on the bandwidth and modulation size, on the other. It can be seen that in order for a specific frequency band to be usable by CRs, not only the corresponding PUs must not enter their channels very frequently, but also the time during it they hold the channel must be short enough, something that can be justified conceptually.

V. CONCLUSIONS

In this paper, the usability of a specific channel for CRs' operations with its corresponding PUs was explored through mathematical analysis. The results show that for a certain spatial distribution of PUs, there are limits on the number of CRs in Ad-Hoc configuration if they want to have full coverage, a condition that might not be satisfied always. On the other hand, for a given statistics of PUs in channel arrival rate and channel usage, there is an upper bound for the achievable data rate for CRs.

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