

Statistical Analysis of Modulation Selection Strategies in Cognitive Radios

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Abstract- The new upper bound interference temperature (IT) has provided Cognitive Radios (CRs) with new resources of frequency spectrum. The principle is that CRs are allowed to use channels mainly licensed to some Primary Users (PUs) provided that their operation will not violate the IT limit determined by PUs. In this paper, through statistical analysis, the effect of the modulation selection strategies of active CRs on the probability of blocking new CRs in one hand and on the excessive interference created for PUs on the other will be analyzed. The results, as expected, show that when CRs mostly use higher power modulations, both probability of blocking new CRs and interfering with PUs increase. Also, when CRs use the average power instead of maximum power as their metric for avoiding interfering with PUs, their interference with PUs are more likely, although they possibly obtain higher data rates.

Keywords— Cognitive Radios, Primary Users, Interference Temperature, Modulation Selection Strategy

I. INTRODUCTION

Cognitive Radios (CRs) based on opportunistic spectrum access have recently been proposed for the problem of spectrum scarcity [1]. The main principle of their operation is to use channels mainly dedicated to Primary Users (PUs) whenever it is idle. However, the recently proposed upper limit Interference Temperature (IT) allows CRs to even simultaneously transmit with PUs provided that their transmission would not violate the maximum interference temperature limit IT_{max} at PUs' receivers [2]. Based on such behavioural characteristics of CRs as modulation selection strategy, the amount of the spectrum resources available to them may vary. For example, when most of the CRs use higher power modulations, the probability for the request of a new CR to be blocked may become large. On the other hand, if most of the CRs use lower power modulations, there would be higher probabilities for new CRs to find opportunity for transmission. In this paper, based on some statistical analysis, the effect of the modulation selection strategy on the probability of blocking new CRs and also on the interference created to PUs will be analysed. To do so, the structure of the paper is as follows. In section II the probability of excessive

interference for PUs and also blocking new CRs will be derived through mathematical modelling. Section III compares two strategies for modulation selection, i.e., based on average power and maximum power. Finally, section IV concludes the paper.

II. MATHEMATICAL MODEL FOR PROBABILITY OF INTERFERING WITH PU AND BLOCKING NEW CRs

In presence of PUs, CRs may operate in either of two modes: simultaneous or non-simultaneous with PUs. When CRs work non-simultaneously with PUs, there would be no worry about interference as long as they leave the channel upon the arrival of PUs. This method works well for PUs with deterministic behaviour. For stochastic behaviours of PUs, however, more care should be taken. On the other hand, when CRs are allowed to transmit simultaneously with PUs provided that the interference caused to PUs would not exceed the upper limit IT_{max} , the situation will be somewhat different. In this case, limits should be put on the transmission power of CRs. Since the size of modulation affects directly the transmission power of CRs, its effect on the blocking probability of request of newcomer CRs for transmission and creating excessive interference for PUs will be analysed in the following.

According to IEEE.802.22 [3], the standard modulation of CRs is one of the BPSK, QPSK, QAM-16 and QAM-64 which will be used under an OFDM plan. Assuming equi-probable symbols, the average power of these modulations are [4]:

$$\begin{aligned} P_{BPSK} &= \frac{E_g d}{2}, & P_{QPSK} &= \frac{E_g d}{2}, \\ P_{16-QAM} &= \frac{15E_g d}{6}, & P_{64-QAM} &= \frac{63E_g d}{6} \end{aligned} \quad (1)$$

where E_g , d are pulse shape energy and AGC tuning factor, respectively. For the purpose of simplicity, assume that the arrivals of an individual CR follow a Poisson process with mean rate λ , which is assumed to be the same for all CRs. In a time instance, an individual CR is either present or absent

where the probability of absence is $p = e^{-\lambda}$. Thus, its average power will be:

$$P_{average} = P_i \times (1 - e^{-\lambda}) + 0 \times e^{-\lambda} = P_i(1 - e^{-\lambda}) \quad (2)$$

where P_i stands for the average power of i^{th} modulation type and takes either values of (1).

For simplicity, again, the geometry of the analysis is assumed to be circular in which, PU is located at the centre of the circle and CRs are spread over the area of the circle with uniform distribution. In this case, the probability distribution of the distance between one individual CR and PU will be as follows:

$$f_d(d) = \begin{cases} 0 & d < 0 \\ \frac{2d}{R^2} & 0 \leq d \leq R \\ 0 & d > R \end{cases} \quad (3)$$

where R shows the maximum distance from PU. Depending on the distance, each CR creates interference to PU as much as $P_{di} = \frac{P_{average}}{(4\pi f d)^2}$ whose average would be:

$$\begin{aligned} P_{di-average} &= E[P_{di}] = E\left[\frac{P_{average}}{(4\pi f d)^2}\right] \\ &= \frac{P_{average}}{(4\pi f)^2 E[d^2]} = \frac{P_{average}}{8\pi^2 f^2 R^2} = \frac{P_i(1 - e^{-\lambda})}{8\pi^2 f^2 R^2} \end{aligned} \quad (4)$$

The interfering power at PU would be the sum of the interferences of all CRs. Assuming N for number of CRs is large enough, according to the central limit theorem [5], the received power of CRs at PU will have a normal distribution with mean and variance as follows:

$$\begin{aligned} \mu &= N \times E[P_{di}] = \frac{NP_i(1 - e^{-\lambda})}{8\pi^2 f^2 R^2} \\ \sigma^2 &= N \times (E[P_{di}^2] - E^2[P_{di}]) = 3N \left(\frac{P_i(1 - e^{-\lambda})}{16\pi^2 f^2 R^2} \right)^2 \end{aligned} \quad (5)$$

Therefore, the probability of the received power P_d exceeds a predetermined threshold in term of IT_{max} will be as follows:

$$\begin{aligned} Prob(Excessive\ Interference(EI)) &= \int_{P_{dmax}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(P_d - \mu)^2}{2\sigma^2}} dP_d \\ &= \int_{kBIT_{max}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(P_d - \mu)^2}{2\sigma^2}} dP_d \\ &= Q\left(\frac{kBIT_{max} - \mu}{\sigma}\right) \\ &= Q\left(\frac{16\pi^2 f^2 R^2 kBIT_{max}}{\sqrt{3N}P_i(1 - e^{-\lambda})} - \sqrt{\frac{4N}{3}}\right) \end{aligned} \quad (6)$$

where $Q(.)$ is the well-known Q-function :

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt \quad (7)$$

In deriving (6), we made use of the definition of IT [2]:

$$IT = \frac{P(f, B)}{kB} \quad (8)$$

in which, k stands for the Boltzmann's constant, $P(f, B)$ for the interfering power at PU and B for the bandwidth. When CRs decide to utilize L channels with the same interests, the total interference will be divided into L channels equally. Assuming $N \gg L$, the distribution of the interference in each channel will be again normal with mean $\mu = \frac{NP_i(1 - e^{-\lambda})}{8L\pi^2 f^2 R^2}$ and variance $\sigma^2 = 3 \frac{N}{L} \left(\frac{P_i(1 - e^{-\lambda})}{16\pi^2 f^2 R^2} \right)^2$. Thus, the probability of interference limit violation for in one channel would be:

$$Prob(EI) = Q\left(\frac{\sqrt{L}16\pi^2 f^2 R^2 kBIT_{max}}{\sqrt{3N}P_i(1 - e^{-\lambda})} - \sqrt{\frac{4N}{3L}}\right) = p \quad (9)$$

and probability of no capacity be available for servicing new CRs would be:

$$Prob(Blocking\ new\ CRs) = p^L \quad (10)$$

Accordingly, the probability of at least one channel has capacity for new CRs would be $1 - p^L$. The variations of $Q(.)$ as a function of IT_{max} , R , P_i , N , L and λ has been depicted in figs. 1-6, where in each figure only one parameter has been taken as the independent variable with all other parameters as constant. It can be seen that when each of the following variables, namely, number of CRs, their arrival rates and their transmission power increases, then the probability of blocking new CRs' requests and excessively interfering with PUs increases. Also, when each of the following variables, that is, IT_{max} , CRs' distance from PU and number of channels granted to CRs increases, then the probability of blocking new CRs or making excessive interference for PU decreases. Since for a new CR to have permission of using the marginal interference temperature the total received power including that of the new CR must not violate the IT_{max} at PU, either the transmission power of the new CR must be low enough or its distance from PU must be large enough. Furthermore, (9) and (10) show that when CRs behave selfishly and want all the available resources for themselves through frequently using all channels with high powers, they make the conditions worse both for their partners and PUs.

III. MODULATION SELECTION STRATEGY

When CRs use the channels simultaneously with PU provided that they maintain the total IT at PU below IT_{max} , they may transmit based on either of the following strategies:

average Power or maximum power. In the first case, the average transmission power of CR should not violate the IT_{max} , while in the latter, its maximum power must satisfy IT_{max} limit. To quantify these, assume that the modulation under study be the M-QAM. The corresponding average power with unit pulse energy and AGC factor is [6]:

$$P_{ave} = \frac{1}{M} \sum_{m=1}^M P_m = \frac{1}{MT_s} \sum_{m=1}^M (2m-1-M^{1/2})^2 = \frac{M-1}{6T_s} \quad (11)$$

where T_s is the time interval of one symbol. Taking the IT_{max} into consideration, the size of modulation M should satisfy the following condition:

$$\frac{M-1}{6T_s} \leq kB(IT_{max} - IT)(4\pi fR)^2 \Rightarrow M \leq 3kBT_s(IT_{max} - IT)(4\pi fR)^2 + 1 = UB1 \quad (12)$$

In which IT is the measured or estimated interference temperature and $UB1$ is the upper bound for M when average power must not violate the IT_{max} . The measure of the average power means that CR would not violate the IT_{max} on average. In other words, in some occasions it might violate the IT_{max} while other times, it might use the channel far lower than IT_{max} . In fact, for 16-QAM and 64-QAM, this means that $\frac{4}{16}$ and $\frac{26}{64}$ percent of times, respectively, CR possibly violates the IT_{max} , which may be a noticeable figure.

When CR plans based on the measure of maximum power, it would not violate the IT_{max} at PU at all. The maximum power of M-QAM modulation is:

$$P_{max} = \frac{(2M-1-M^{1/2})^2}{T_s} = \frac{2M-4\sqrt{M}+2}{T_s} \quad (13)$$

Considering the IT_{max} limit, this leads to:

$$P_{max} = \frac{2M-4\sqrt{M}+2}{T_s} \leq kB(IT_{max} - IT)(4\pi fR)^2 \Rightarrow M \leq \left(\sqrt{\frac{kBT_s(IT_{max} - IT)(4\pi fR)^2}{2}} + 1 \right)^2 = UB2 \quad (14)$$

The upper bounds $UB1$ and $UB2$, in which $kBT_s(IT_{max} - IT)(4\pi fR)^2$ has been taken as an independent variable x , are shown in fig. 7. Comparing (12) with (14) clearly declares that planning based on maximum power is in fact based on the worst case. Since system will not be in its worst case always, this method is somewhat conservative, but it guarantees a communication free of any interference for PU.

On the other hand, planning based on average power is not as conservative as that of maximum power and it provides CR with higher data rate, but there are occasions in which the probability of interfering with PU might become large. In brief, choosing either of methods involves in some tradeoffs between robustness (no interference for PU) and optimality (higher data rate) that designer must decide which one is more appropriate.

IV. CONCLUSION

In this paper, the effect of modulation selection strategies on the probabilities of creating excessive interference for PUs and blocking new CRs was investigated through statistical analysis. As expected, results show that when CRs use higher power modulation and behave selfishly, the probability of finding opportunity to transmit for CRs decreases while at the same time, the probability of excessively interfering with PUs increases. Also, for a certain interference temperature, operation frequency, bandwidth and distance from PUs, when CRs plan based on average power to avoid interfering with PUs, although they possibly access to higher data rate, they probably interfere with PUs more.

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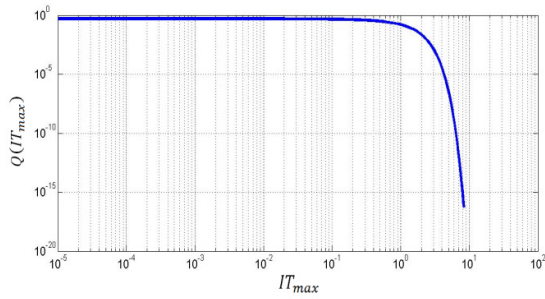


Figure 1. $Q(IT_{\max})$ versus upper bound of Interference Temperature.

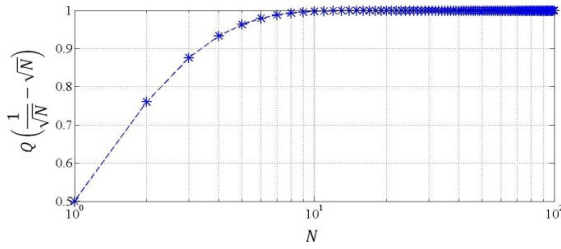


Figure 2. $Q\left(\frac{1}{\sqrt{N}} - \sqrt{N}\right)$ versus the number of CRs .

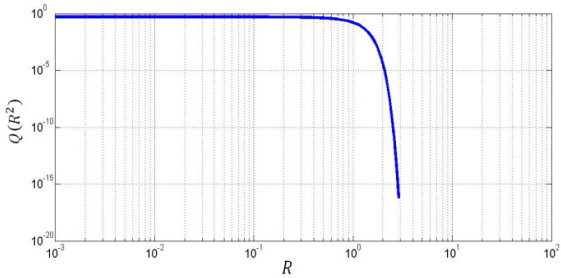


Figure 3. $Q(R^2)$ versus the distance of CRs from PU.

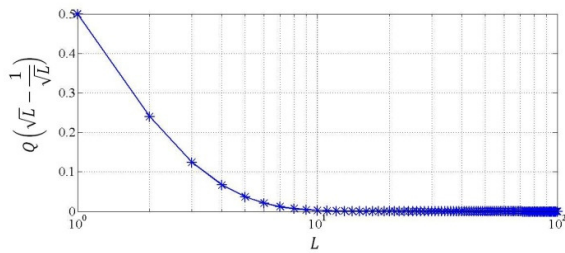


Figure 4. $Q\left(\sqrt{L} - \frac{1}{\sqrt{L}}\right)$ versus number of alternative channels .

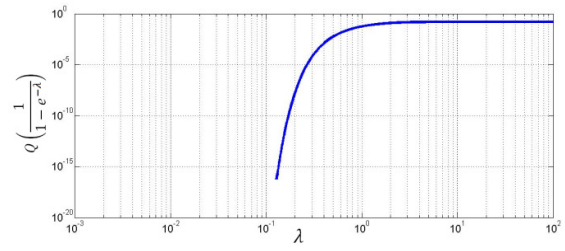


Figure 5. $Q\left(\frac{1}{1-e^{-\lambda}}\right)$ versus arrival rate of CRs.

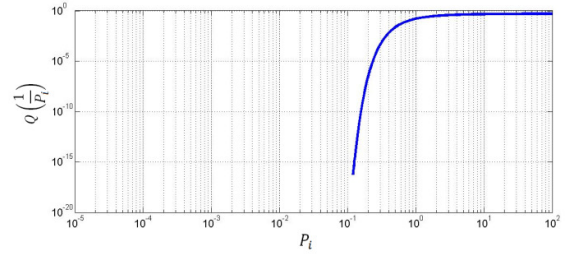


Figure 6. $Q\left(\frac{1}{P_t}\right)$ versus transmission power of CRs.

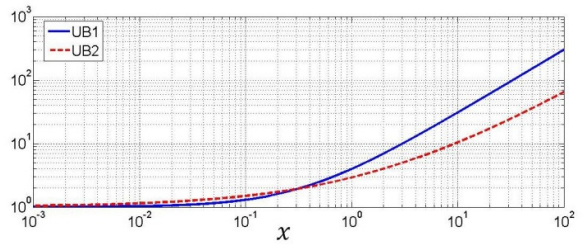


Figure 7. Comparison of possible M for CRs when they operate based on average power and maximum power.