

Timing Jitter in PAM, ASK, and QAM for a Bandlimiting Scheme

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Abstract—Timing jitter in M -PAM, M -ASK, and M^2 -QAM with AWGN is calculated for a cosine roll-off bandlimiting scheme applying a newly derived closed-form representation in the frequency domain of the jitter power spectral density (jitter PSD). Jitter PSD consists of SS , SN , and NN components. Each component of jitter variance is obtained by integrating the corresponding jitter PSD over a finite frequency domain in a bandlimiting scheme. The total jitter variance is obtained as the sum of the three components. The dependencies of the calculated jitter variance on the system parameters, i.e., SNR , ρ , alphabet size, M , transmission scheme, PLL normalized noise bandwidth, b_L , and roll-off factor, α , are evaluated. Specifically, the SS component increases with increasing M . However, this increase is limited within a finite range. Since the SN and NN components are independent of M , the dependence of the total jitter variance on M is small. At low and medium $SNRs$, the total jitter variance is independent of M . At high $SNRs$, the total jitter variance increases with increasing M , but the increase is rather small when M is greater than 4. Calculated values of the SS , SN , and NN components of jitter variance in QAM are tabulated. The numerical values in QAM can be used to determine the numerical values in PAM and ASK.

Keywords—PAM, ASK, QAM, timing jitter, jitter power spectral density

I. INTRODUCTION

Timing recovery is one of critical functions that are performed at the receiver of a synchronous digital communication system. A timing recovery circuit extracts a timing wave from the incoming pulse train to achieve self-synchronization. In practice, the phase of the timing wave fluctuates randomly due to the random nature of the received data signal and additive noise. This fluctuation, i.e., timing jitter, gives rise to BER degradation, as compared to perfect synchronization. In order to evaluate the jitter generation and the jitter performance of a timing recovery circuit, it is important to determine the jitter power spectral density (jitter PSD) [1].

The analytical expressions for the self-noise power spectral densities for the PAM signal were presented at high $SNRs$ and an expression for the jitter variance was provided [2]. A self-noise reduction method for M -PSK was proposed and demonstrated to have an advantage over conventional methods for small M [3]. Although these works are based on different standpoints from ours, there are some links to be interested between them.

The authors analyzed timing jitter in PAM, ASK and QAM in the case of a timing recovery circuit consisting of a square-law device, a pre-filter, and a PLL arranged in tandem when Gaussian noise exists at the receive filter input, and a closed-form representation of the jitter PSD in the frequency domain was derived from signal analysis of a random process through the timing recovery circuit [4]. The jitter PSD consists of SS , SN and NN components. Each component of jitter variance is obtained by integrating the corresponding jitter PSD

over a finite frequency domain in a bandlimiting scheme. The SS component is caused by the cross product of the signal and is often called pattern jitter, self-noise, or data-dependent jitter. The SN component is caused by the cross product of the signal and the noise. The NN component is caused by the cross product of the noise. The total jitter variance is obtained as the sum of these three components.

In the present paper, each component of jitter variance in the transmission schemes of PAM, ASK, and QAM with AWGN is calculated for a bandlimiting scheme applying a closed-form representation of the jitter PSD. The dependencies of total jitter variance and its three components on the system parameters are evaluated in order to clarify jitter performance. The system parameters are the SNR , ρ , at the input of timing recovery circuit, the alphabet size, M , the transmission scheme, the PLL normalized noise bandwidth, b_L , and the roll-off factor, α . The calculated values of the SS , SN , and NN components of jitter variance in QAM are tabulated. The numerical values in QAM can be used to determine the numerical values in PAM and ASK.

II. ASSUMPTIONS

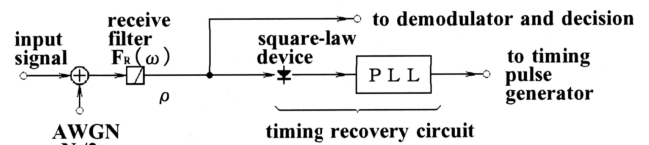


Figure 1. Receiver configuration.

Figure 1 shows the receiver configuration. The transmission schemes are assumed to be M -PAM, M -ASK, and M^2 -QAM with AWGN. A cosine roll-off bandlimiting scheme is assumed (Appendix A). The timing recovery circuit is composed of a square-law device and a PLL. The pre-filter transfer function is assumed to be unity. The PLL is assumed to be of second order, and its loop filter is composed of an active filter. The damping factor, ζ , of the PLL is assumed to be $1/\sqrt{2}$. The normalized noise bandwidth, $b_L (=B_L T=2\pi B_L/\omega_r)$, values of the PLL are assumed to range from 10^{-4} to 10^{-2} . Here, B_L is the noise bandwidth, T is the symbol period, and ω_r is the clock radian frequency. The roll-off factor ranges from 0.1 to 1.0, and the SNR , ρ , at the input of the timing recovery circuit is assumed to be from -5 to 40 dB. The alphabet size, M , is assumed to be even. Modulation data a_n and b_n are assumed to be i.i.d. random variables that take M values between $\pm(M-1)$.

III. CALCULATION of JITTER VARIANCE

The total jitter variance σ_{ϕ}^2 is obtained as the sum of the three components SS ($\sigma_{\phi SS}^2$), SN ($\sigma_{\phi SN}^2$), and NN ($\sigma_{\phi NN}^2$) as follows:

$$\sigma_{\phi}^2 = \sigma_{\phi SS}^2 + \sigma_{\phi SN}^2 + \sigma_{\phi NN}^2 \quad (1)$$

Each component varies with the change of each system parameter.

A. Jitter Dependency on SNR, ρ

Figure 2 shows the calculated results of total jitter variance and its three components as a function of ρ . The transmission scheme is PAM. The alphabet size M is 4. The PLL normalized noise bandwidth, b_L , is 10^{-3} , and the roll-off factor, α , is 0.5.

The SS component is independent of ρ . The SN and NN components are proportional to ρ^{-1} and ρ^{-2} , respectively. The total jitter variance is independent of ρ at high SNRs because SS is dominant. At medium SNRs, the total jitter variance is proportional to ρ^{-1} because SN is dominant. At low SNRs, the total jitter variance is proportional to ρ^{-2} because NN is dominant.

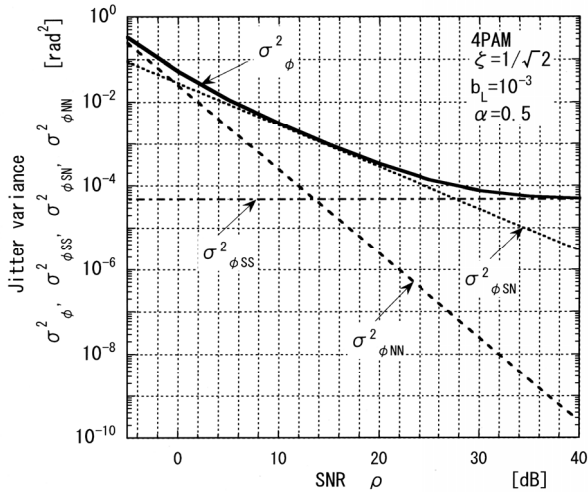


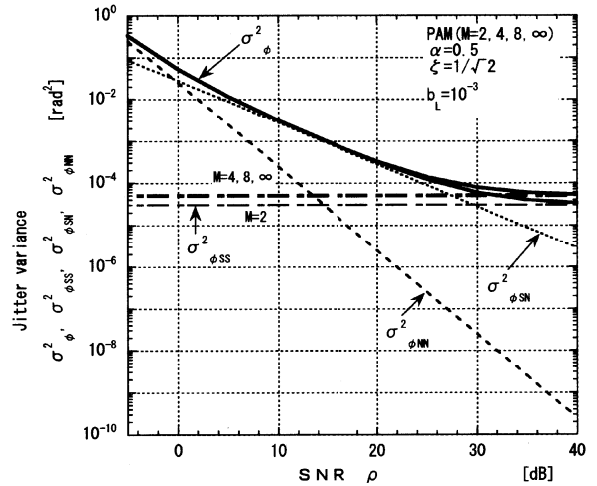
Figure 2. Total jitter variance and its three components as a function of SNR, ρ .

B. Jitter Dependency on Alphabet Size, M

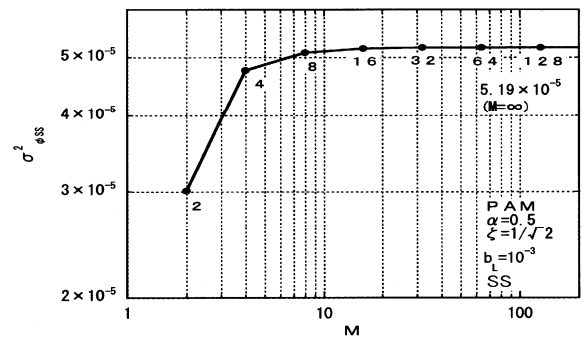
Figure 3(a) shows the calculated results of total jitter variance and its three components as a function of ρ with parameter M . The transmission scheme is PAM, and the alphabet size M is 2, 4, 8, or ∞ . The PLL normalized noise bandwidth b_L is 10^{-3} , and the roll-off factor is 0.5. Figure 3(b) shows the calculated results of the SS component of jitter variance as a function of M . The following equation holds with roll-off factor α ranging from 0.1 to 1.0.

$$\frac{\sigma_{\phi SS}^2(M = \infty)}{\sigma_{\phi SS}^2(M = 2)} = 1.23 \sim 4.17 \quad (\alpha = 0.1 \sim 1.0) \quad (2)$$

The SS component increases with increasing M . However, this increase is limited within a finite range, as in (2). Since the SN and NN components are independent of M , the dependence of the total jitter variance on M is small. At low and medium SNRs, the total jitter variance is independent of M because SN and NN are dominant. At high SNRs, the total jitter variance increases with increasing M because SS is dominant, but the increase is rather small when M is greater than 4.



(a) Total jitter variance and its three components as a function of SNR, ρ with parameter M .



(b) SS component of jitter variance as a function of M .
Figure 3. Dependency of jitter variance on M .

C. Jitter Dependency on the Transmission Scheme

Figure 4 shows the calculated values of total jitter variance in PAM, ASK, and QAM as a function of SNR, ρ . In Figures 4(a), 4(b), and

4(c), $M = 2, 4$, and ∞ , respectively. In Figure 4(d), $M=4$ with roll-off factors of 0.1, 0.5, and 1.0.

The total jitter variance of PAM is twice that of QAM because the SS , SN , and NN components of PAM are twice those of QAM. The total jitter variance of ASK is equal to that of PAM at high $SNRs$ and is equal to that of QAM at low $SNRs$ because SS of ASK is equal to that of PAM, and SN and NN of ASK are equal to those of QAM. At medium $SNRs$, the total jitter variance of ASK varies gradually from that of QAM to that of PAM with increasing SNR .

D. Jitter Dependency on PLL Normalized Noise Bandwidth, b_L

Figure 5 shows the total jitter variance and its three components as a function of b_L . In Figures 5(a), 5(b), and 5(c), $\rho=0, 20$, and 40 dB, respectively.

The SS , SN , and NN components are proportional to b_L . However, the SS component is proportional to the square of b_L . Such dependency of each component of jitter variance on b_L holds for all parameters considered in the present paper. This may be a result of the relationship between the jitter source PSD at the PLL input and the extractor function of the PLL characteristics [4]. This will be discussed in detail in the future.

E. Jitter Dependency on Roll-off Factor, α

Figure 6 shows the total jitter variance and its three components as a function of α . In Figures 6(a), 6(b), and 6(c), $SNR \rho=0, 20$, and 40 dB, respectively.

The SS , SN , and NN components increase with decreasing α and diverge when α approaches zero. The SS component increases more rapidly than the SN and NN components for small α . This is because the timing recovery circuit is of non-decision-directed type. When a timing recovery circuit of decision-directed type is assumed, such divergence characteristics in the case of small α may be improved. This will be discussed in detail in the future.

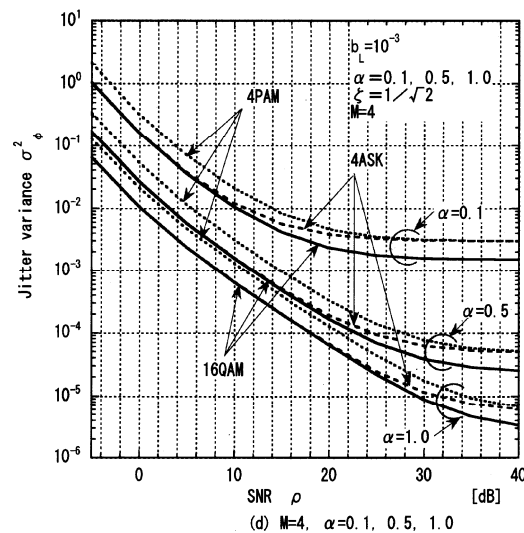
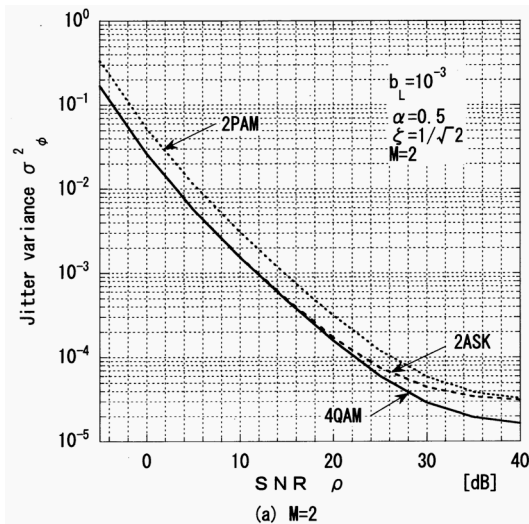
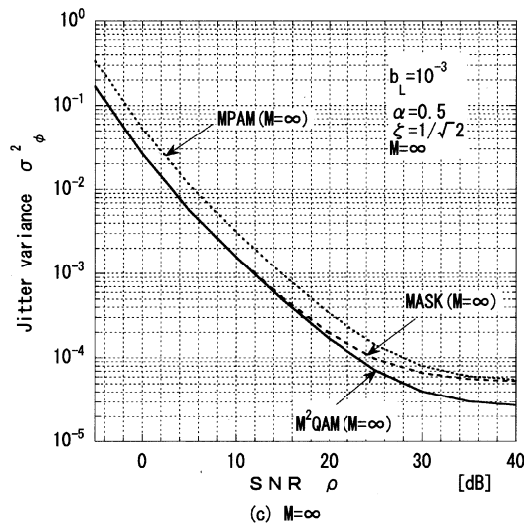
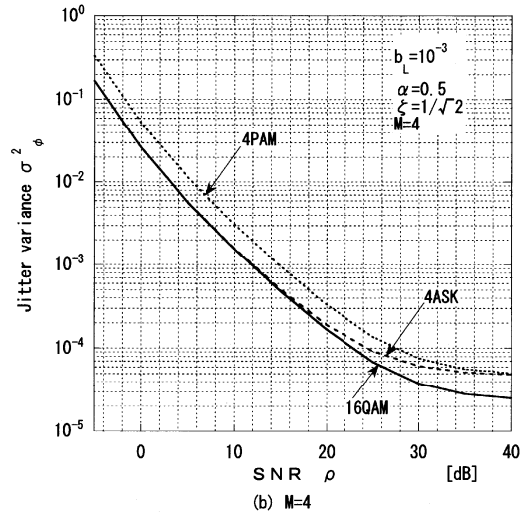


Figure 4. Total jitter variance in PAM, ASK, and QAM as a function of SNR, ρ .

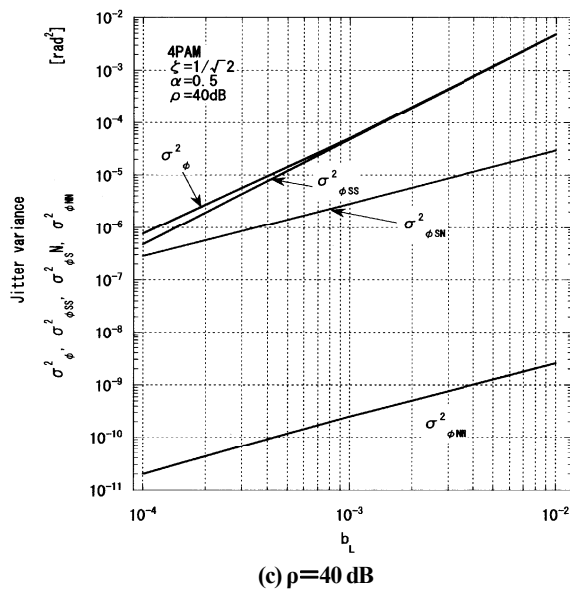
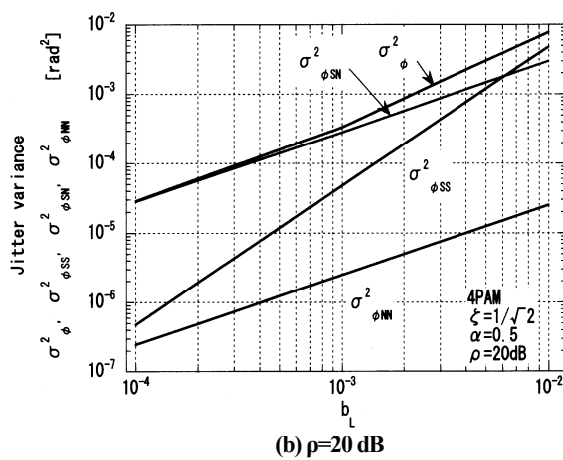
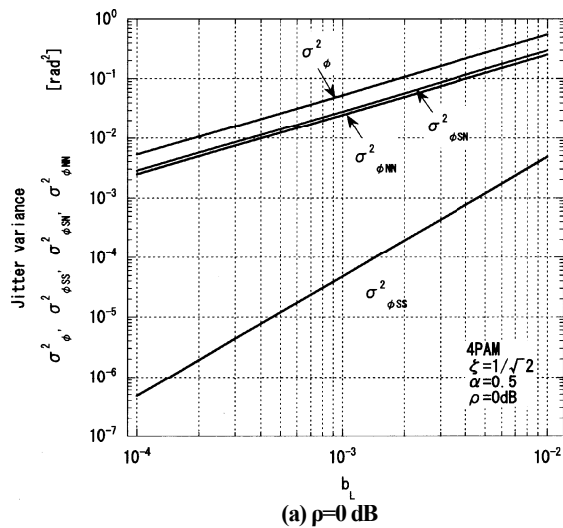


Figure 5. Total jitter variance and its three components as a function of PLL normalized noise bandwidth, b_L .

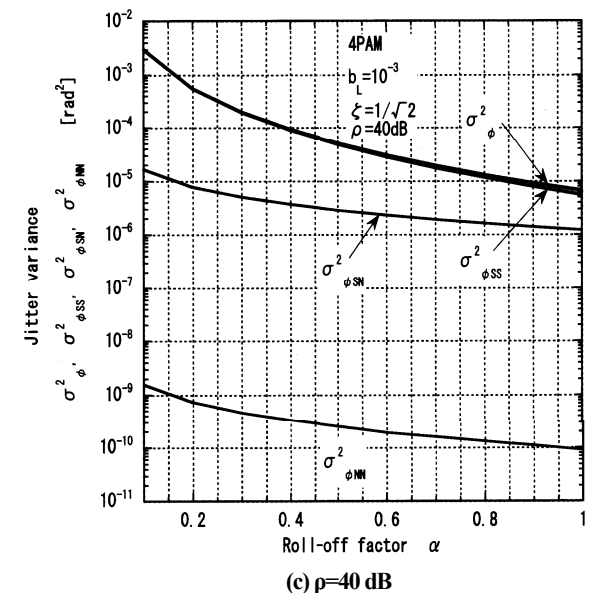
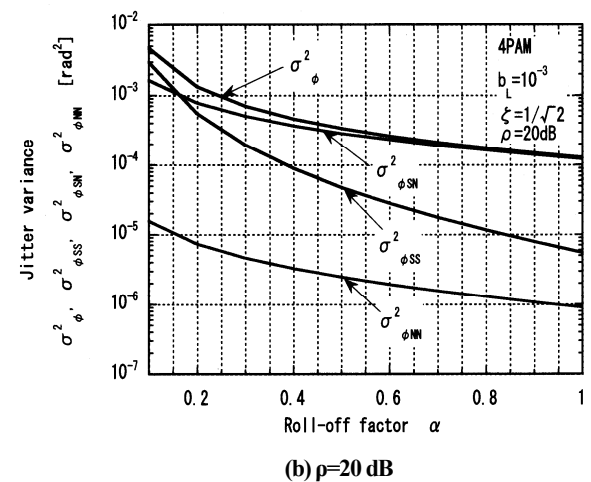
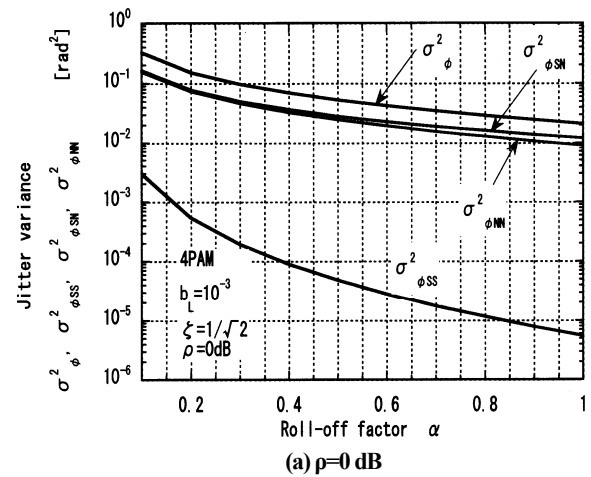


Figure 6. Total jitter variance and its three components as a function of roll-off factor, α .

Table 1. SS Component of Jitter Variance in QAM.

(a) M=2

α	$\sigma^2_{\phi_{SS}}$		
	$b_L=10^{-4}$	$b_L=10^{-3}$	$b_L=10^{-2}$
0.1	1.25×10^{-5}	1.25×10^{-3}	1.24×10^{-1}
0.2	2.16×10^{-6}	2.16×10^{-4}	2.15×10^{-2}
0.3	7.13×10^{-7}	7.13×10^{-5}	7.10×10^{-3}
0.4	3.06×10^{-7}	3.06×10^{-5}	3.05×10^{-3}
0.5	1.51×10^{-7}	1.50×10^{-5}	1.50×10^{-3}
0.6	8.00×10^{-8}	8.00×10^{-6}	7.97×10^{-4}
0.7	4.44×10^{-8}	4.44×10^{-6}	4.42×10^{-4}
0.8	2.51×10^{-8}	2.51×10^{-6}	2.50×10^{-4}
0.9	1.41×10^{-8}	1.41×10^{-6}	1.41×10^{-4}
1	7.74×10^{-9}	7.75×10^{-7}	7.73×10^{-5}

(d) M= ∞

α	$\sigma^2_{\phi_{SS}}$		
	$b_L=10^{-4}$	$b_L=10^{-3}$	$b_L=10^{-2}$
0.1	1.54×10^{-5}	1.54×10^{-3}	1.52×10^{-1}
0.2	2.86×10^{-6}	2.86×10^{-4}	2.85×10^{-2}
0.3	1.02×10^{-6}	1.02×10^{-4}	1.02×10^{-2}
0.4	4.79×10^{-7}	4.79×10^{-5}	4.78×10^{-3}
0.5	2.60×10^{-7}	2.60×10^{-5}	2.59×10^{-3}
0.6	1.55×10^{-7}	1.55×10^{-5}	1.54×10^{-3}
0.7	9.83×10^{-8}	9.83×10^{-6}	9.81×10^{-4}
0.8	6.55×10^{-8}	6.55×10^{-6}	6.53×10^{-4}
0.9	4.53×10^{-8}	4.53×10^{-6}	4.52×10^{-4}
1	3.23×10^{-8}	3.23×10^{-6}	3.22×10^{-4}

(b) M=4

α	$\sigma^2_{\phi_{SS}}$		
	$b_L=10^{-4}$	$b_L=10^{-3}$	$b_L=10^{-2}$
0.1	1.48×10^{-5}	1.48×10^{-3}	1.47×10^{-1}
0.2	2.72×10^{-6}	2.72×10^{-4}	2.71×10^{-2}
0.3	9.62×10^{-7}	9.61×10^{-5}	9.58×10^{-3}
0.4	4.44×10^{-7}	4.44×10^{-5}	4.43×10^{-3}
0.5	2.38×10^{-7}	2.38×10^{-5}	2.37×10^{-3}
0.6	1.40×10^{-7}	1.40×10^{-5}	1.39×10^{-3}
0.7	8.75×10^{-8}	8.75×10^{-6}	8.73×10^{-4}
0.8	5.74×10^{-8}	5.74×10^{-6}	5.73×10^{-4}
0.9	3.90×10^{-8}	3.90×10^{-6}	3.90×10^{-4}
1	2.74×10^{-8}	2.74×10^{-6}	2.73×10^{-4}

Table 2. S/N Component of Jitter Variance in QAM.

α	$\rho \sigma^2_{\phi_{SN}}$		
	$b_L=10^{-4}$	$b_L=10^{-3}$	$b_L=10^{-2}$
0.1	7.83×10^{-3}	8.18×10^{-2}	1.16
0.2	3.81×10^{-3}	3.87×10^{-2}	4.54×10^{-1}
0.3	2.47×10^{-3}	2.49×10^{-2}	2.74×10^{-1}
0.4	1.80×10^{-3}	1.81×10^{-2}	1.93×10^{-1}
0.5	1.40×10^{-3}	1.41×10^{-2}	1.47×10^{-1}
0.6	1.13×10^{-3}	1.14×10^{-2}	1.18×10^{-1}
0.7	9.42×10^{-4}	9.46×10^{-3}	9.71×10^{-2}
0.8	8.00×10^{-4}	8.02×10^{-3}	8.19×10^{-2}
0.9	6.89×10^{-4}	6.90×10^{-3}	7.02×10^{-2}
1	6.00×10^{-4}	6.01×10^{-3}	6.09×10^{-2}

(c) M=8

α	$\sigma^2_{\phi_{SS}}$		
	$b_L=10^{-4}$	$b_L=10^{-3}$	$b_L=10^{-2}$
0.1	1.52×10^{-5}	1.52×10^{-3}	1.51×10^{-1}
0.2	2.83×10^{-6}	2.83×10^{-4}	2.82×10^{-2}
0.3	1.01×10^{-6}	1.01×10^{-4}	1.01×10^{-2}
0.4	4.71×10^{-7}	4.71×10^{-5}	4.69×10^{-3}
0.5	2.55×10^{-7}	2.55×10^{-5}	2.54×10^{-3}
0.6	1.51×10^{-7}	1.51×10^{-5}	1.51×10^{-3}
0.7	9.57×10^{-8}	9.57×10^{-6}	9.55×10^{-4}
0.8	6.35×10^{-8}	6.35×10^{-6}	6.34×10^{-4}
0.9	4.38×10^{-8}	4.38×10^{-6}	4.37×10^{-4}
1	3.11×10^{-8}	3.11×10^{-6}	3.11×10^{-4}

Table 3. N/N Component of Jitter Variance in QAM.

α	$\rho^2 \sigma^2_{\phi_{NN}}$		
	$b_L=10^{-4}$	$b_L=10^{-3}$	$b_L=10^{-2}$
0.1	7.62×10^{-3}	7.79×10^{-2}	9.45×10^{-1}
0.2	3.61×10^{-3}	3.65×10^{-2}	3.96×10^{-1}
0.3	2.28×10^{-3}	2.29×10^{-2}	2.41×10^{-1}
0.4	1.62×10^{-3}	1.63×10^{-2}	1.68×10^{-1}
0.5	1.22×10^{-3}	1.23×10^{-2}	1.26×10^{-1}
0.6	9.63×10^{-4}	9.65×10^{-3}	9.81×10^{-2}
0.7	7.77×10^{-4}	7.79×10^{-3}	7.89×10^{-2}
0.8	6.40×10^{-4}	6.41×10^{-3}	6.47×10^{-2}
0.9	5.34×10^{-4}	5.34×10^{-3}	5.39×10^{-2}
1	4.50×10^{-4}	4.50×10^{-3}	4.53×10^{-2}

F. Tabulated Numerical Values for the Three Components in QAM

The calculated values for the SS , SN , and NN components of jitter variance in QAM are tabulated in Tables 1, 2, and 3, respectively. The SN and NN components are normalized by ρ and the square of ρ , respectively. The numerical values in QAM can be used to determine the numerical values in PAM and ASK. The values of the SS , SN , and NN components in PAM are twice those in QAM and the values of SS in ASK are twice those in QAM. The values of SN and NN in ASK are equal to those in QAM.

IV. CONCLUSIONS

The total jitter variance and its three components, i.e., SS , SN , and NN , were calculated for a cosine roll-off bandlimiting scheme in M -PAM, M -ASK, and M^2 -QAM with AWGN in the case of a timing recovery circuit consisting of a square-law device and a PLL, applying a closed-form representation of the jitter PSD in the frequency domain.

The dependencies of total jitter variance and its three components on system parameters SNR , ρ , alphabet size, M , transmission scheme, PLL normalized noise bandwidth, b_L , and roll-off factor, α , were evaluated in order to clarify the performance of jitter. Specifically, the dependencies on alphabet size, M , PLL normalized noise bandwidth, b_L , and roll-off factor, α , were clarified for the first time based on the calculated results. The calculated values for the SS , SN , and NN components of jitter variance in QAM were tabulated. The numerical values in QAM can be used to determine the numerical values in PAM and ASK. These results are expected to be useful in determining the system parameters while taking jitter performance into consideration because both self-noise and a noise-dependent term are involved.

The effect of a pre-filter on timing jitter, for the case in which a pre-filter is used, in terms of jitter reduction and the dependencies of jitter variance on b_L and α will be discussed in the future. In addition, the effect of timing jitter on the BER characteristics will also be examined.

APPENDIX

A. Cosine Roll-off and Root Roll-off Characteristics[5],[6]

A transmitted pulse waveform is assumed to be $g_T(t)$. An impulse response of the receive filter is assumed to be $g_R(t)$. A pulse waveform at the receive filter output is assumed to be $g(t)$. $G_T(\omega)$ and $G_R(\omega)$ are assumed to be the frequency spectrum of $g_T(t)$ and the transfer function of the receive filter, respectively. $G(\omega)$ is assumed to be the frequency spectrum of $g(t)$ at the receive filter output and is assumed to exhibit cosine roll-off characteristics. Then, the following equations hold:

$$\left. \begin{aligned} g_T(t) &\leftrightarrow G_T(\omega) \\ g_R(t) &\leftrightarrow G_R(\omega) \\ g(t) &\leftrightarrow G(\omega) \end{aligned} \right\} \quad (A.1)$$

$$g(t) = g_T(t) * g_R(t) \quad (A.2)$$

$$G(\omega) = G_T(\omega)G_R(\omega) \quad (A.3)$$

$$\left. \begin{aligned} G(\omega) &= 1 && ; |\omega| \leq \frac{(1-\alpha)\omega_r}{2} \\ G(\omega) &= 0 && ; \frac{(1+\alpha)\omega_r}{2} < |\omega| \\ G(\omega) &= \frac{1}{2} - \frac{1}{2} \sin \frac{\pi(|\omega| - \omega_r/2)}{\alpha\omega_r} && ; \frac{(1-\alpha)\omega_r}{2} < |\omega| \leq \frac{(1+\alpha)\omega_r}{2} \end{aligned} \right\} \quad (A.4)$$

$G_T(\omega)$ and $G_R(\omega)$ in the following equation exhibit root roll-off characteristics.

$$G_T(\omega) = G_R(\omega) = \sqrt{G(\omega)} \quad (A.5)$$

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