

Statistical Analysis of Network Traffic Inter-arrival

Arup Bhattacharjee

Dept. of Computer Science and Engineering
National Institute of Technology
Silchar, India 788010
Email: b_arup@iitg.ernet.in

Sukumar Nandi

Dept. Computer Science and Engineering
Indian Institute of Technology
Guwahati, India 781039
Email: sukumar@iitg.ernet.in

Abstract—Self-similarity with Long Range Dependence (LRD) in network traffic has been modeled using aggregation from On-Off sources. The model requires that each of the On and Off period distribution follow Power-tail distribution like Pareto distribution. This paper characterizes of burst inter-arrival time of network traffic using Bellcore Morristown Laboratory Data [1] and an Academic Institute LAN trace. It is shown that, Log-Normal distribution is a better fit for the data compared to Pareto. Current work analyzes statistically both the data from spatial as well as temporal view point.

I. INTRODUCTION

Self-similarity with Long Range Dependence (LRD) in network traffic was established by Taqqu et.al. [2]. The work used Ethernet data of Bellcore Morristown Research facility [1]. However, later studies have shown that the phenomena is not limited to the link layer alone but spans across other layers in OSI reference model and WAN [3]. LRD introduces difficulties not only in traffic characterization, but in resources management and scheduling as well. Traffic behaviour, observed in [2], was characterized and modeled by Willinger et. al. [4], [5] and authors have proposed use of power-tailed distribution for such modeling.

Numerous studies have found that performance of the network devices in terms of buffer overflow probability, queue length, degrades in presence of self-similar traffic. Kim et.al. [6] have demonstrated that queueing delay is directly proportional to the degree of self-similarity. Sikdar et.al. [7] have shown that indirect LRD manifestation is lower in Random Early Drop (RED) queue at light or moderate load compared to other queueing techniques. Quan et.al. [8] have derived an expression for buffer overflow probability for each queue in a multi-class priority queue fed with Fractional Brownian Motion (fBM) traffic and an analytical expression for packet loss was presented in [9] for priority queue scheduling. Gorrasi et.al. [10] have shown that in addition to increased packet delay, Weighted Fair Queue (WFQ) scheduling do not remain fair under LRD traffic input.

In this paper we make deeper inspection into traffic characteristics and their associated assumptions following the model proposed in [4], [5]. Long term objective of the work is to ascertain if LRD can be considered as a suitable phenomena for incorporation into network traffic models [11]. Current paper characterizes inter-arrival time of frames/packets bursts in Ethernet/IP. It is shown that Log-Normal distribution better models traffic.

Rest of the paper is organized as follows. Section II highlights similar works on traffic characterization and section III discusses relevant mathematical details. Section IV presents probabilistic analysis of the data. In section V proposal of new test methods and their analysis on our data is presented. Section VI shows presence of trend-stationarity in data and section VII compares these findings with Institute LAN trace. Finally, conclusion about the work is drawn in section VIII.

II. RELATED WORK

Pioneering work in [2] motivated Norros et. al. [12] to model network traffic using fractional Brownian Motion (fBM) approach. Author have also shown that, high variability of traffic can not be modeled always by superposition of many i.i.d. random variables. On the other hand, On-Off source model [5] provides an opportunity for a different model based on Stable Lévy Motion [13] depending on order of consideration of limits. Also, researchers used α -stable distribution [14], [15], Chaotic Map [16], [17], Fractional Autoregressive Integrated Moving Average (fARIMA) [18], [19] and Fractional Lévy Motion (fLM) [20] for modeling of network traffic. All these network traffic models are based on Non-Markovian approach. Robert et.al. [21] have proposed a model for network traffic based on Markov Chain. Anderson et.al. [22] have proposed a two state Markov Modulated Poisson Process (MMPP) to model network traffic.

Non-stationarity issue of Internet traffic was raised by Duffield et. al. [23] initially. Cao et. al. [24] have formally shown that Internet traffic (HTTP traffic) exhibits non-stationarity in terms of shifting of marginal distribution from Weibull to Poisson as utilization increases. A different model of non-stationary traffic was given in [25] where authors have shown that LRD phenomena is observable over a very long time scale but on a shorter time scale, traffic behaves more like time-dependent Poisson process. Fukuda et. al. [26] have used detrending on Internet traffic to show that even the Hurst parameter, which measures self-similarity, varies over time.

It is clear from above discussion that, some authors from the non-stationarity school of thought have also identified existence of trends manifested into the arriving traffic [26]. Also, model to account for non-linearity and non-stationarity of the arrival traffic, is an important requirement. Keeping these observations into mind and with long term objective of more detailed characterization of network traffic, current

work statistically analyzes Bellcore data and a very recent academic institute LAN data, which establishes generality and applicability of our work even in today's switched Ethernet environment.

III. MATHEMATICAL BACKGROUND

In this section we discuss briefly about the mathematics underlying the theory of self similarity, LRD, and their relation to the On-Off source model. We also highlight on the working of Ethernet and IP briefly.

A. Self Similarity

The term Self Similar was coined by Manderbolt for explaining water level pattern of river Nile observed by Hurst, also known as *Hurst Effect* and the degree of self similarity is expressed by *Hurst parameter* or *Hurst exponent*, denoted by H .

Definition I: A real valued stochastic process $Z = \{Z(t)\}_{t \in \mathbb{R}}$ is self similar with $H > 0$ (H-ss), if for any $a > 0$,

$$\{Z(at)\}_{t \in \mathbb{R}} \stackrel{d}{=} \{a^H Z(t)\}_{t \in \mathbb{R}} \quad (1)$$

where $\stackrel{d}{=}$ denotes equality in finite dimensional distribution sense.

Above definition clearly shows that self similar processes are scale invariant. Scale invariance of a self similar process allows one to infer finite dimensional distribution of the stochastic process, $Z = \{Z(t)\}_{t > 0}$, on the positive real line from the distribution of the process on any interval. If the stochastic process is H-ss with stationary increments, then the stochastic process is also denoted as H-sssi. Another related definition in this connection is,

Definition II: A Gaussian H-sssi process $\{Z(t)\}_{t \in \mathbb{R}}$ with $0 < H < 1$ is called *Fractional Brownian Motion* (fBM) and its increment process is called *Fractional Gaussian Noise* (fGN). Further, $\sigma^2 = \text{Var}(Z(1)) = 1$.

One can therefore say that, H-sssi process exhibit high variability and variance of the aggregated series decay very slowly. Suppose $Z(t)$ is a H-sssi process with mean μ and variance σ^2 ; a stationary sequence is defined as $X^m = (X_k^m : k = 1, 2, 3, \dots)$, where $X_k^m = \frac{1}{m}[Z_{km-m+1} + \dots + Z_{km}]$, $k \geq 1$ for each $m = 1, 2, \dots, X^m$. Then $\text{Var}(X^m) \sim \sigma^2 m^{2(H-1)}$ as $m \rightarrow \infty$. A Brownian Motion has $H = 0.5$ and so $\text{Var}(X^m)$ decays hyperbolically, but for H-sssi process decay has power-tail. One can conclude that, averaging of data can not remove high variability and so probability of occurrence of events far-away from the mean value is significant (*Noah Effect*).

B. Long Range Dependence

Long Range Dependence (LRD) of data refers to the temporal affinity present in the data in contrast to self similarity which refers to the distributional property of data. Following equation shows that autocovariance of an LRD increment process,

$$\gamma(k) = \frac{\sigma^2}{2} [|k+1|^{2H} - |k|^{2H} + |k-1|^{2H}] \quad (2)$$

and when $\frac{1}{2} < H < 1$, $\gamma(k) > 0$ for $k \neq 0$. Further, if $H \neq \frac{1}{2}$, then $\gamma(k) \sim \sigma^2 H(2H-1)|k|^{2H-2}$ as $k \rightarrow \infty$.

This implies that, Autocorrelation Function (ACF) of an LRD process decays very slowly, somewhat like power-tail. Further, Hurst parameter H can be estimated in a number of ways. One of the method is called *Aggregated Variance*, where aggregated sequence is created as discussed in case of self-similarity. This is plotted after taking logarithm of variance of the expression. Finally, a linear polynomial fit gives the estimate based on *Law of Large Number*. Another technique of estimation is called *R-S Plot*, which is based on *Central Limit Theorem*. Both the above techniques are time-domain based; in frequency domain, using *Periodogram* in log-Log scale, one can estimate H .

C. Ethernet and IP Framework

Ethernet, a link layer protocol proposed by Xerox Corp. to operate at 10 Mbps initially, has been designed to treat communication channel as bus topology. Any ethernet frame must be of minimum 64 byte length to avoid loss of collision signal and frame length can vary with maximum up to 1518 bytes, except for Jumbo frames. In case of collision, sender waits for a random amount of time, guided by *Binary Exponential Backoff Algorithm*, and then retries. An inherent problem of this algorithm is known as *Capture Effect* [27], allowing biasness towards current sender.

IV. ANALYSIS OF DATA

Data used in present study comprises of Ethernet traffic data of Bellcore Morristown Research facility which is publicly available for research purpose [1] and data captured from an Academic Institute LAN. All the analysis in the current study has been primarily done using Bellcore Data but in the later part of the paper it is shown that second set of data, which is very recent, also complies to these findings. Bellcore data is a subset of the data used for the analysis in [2], which is labeled as *BC-pAug89* containing 1,000,000 frames. Other dataset has been collected from an Academic Institute LAN, the trace lasted for about 2654 seconds and approximately 1,700,000 packets are contained. This trace was started on 13-07-2009 at 16:15 hours and it ended on 13-07-2009 at 17:30 hours. All packets present in the trace correspond to the traffic between approximately 200 Desktop PC, 16 Servers/Workstation and Internet. In the current study, we use inter-arrival time of the frames.

A. Distributional Analysis

The bursting mechanism of any network traffic intuitively has a strong 2-way influence, burst duration influences number of bytes exchanged and also number of bytes exchanged influences burst duration. Also, inter-arrival time can have 2-way influence on duration of bursts. However, long-memory phenomena suggests that there shall be far reaching influence

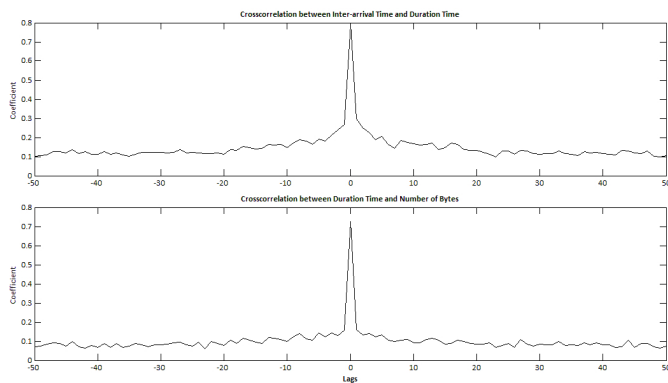


Fig. 1. Long-memory across parameters

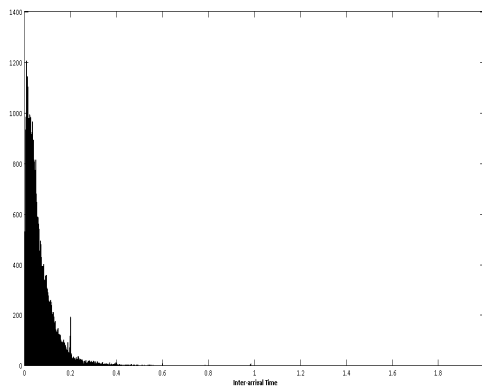


Fig. 2. Histogram of Inter-arrival Time

between these parameters. Figure 1 shows the cross-correlation among these parameters upto ± 50 lags.

Figure 1 suggests that, there is very minimal influence among the parameters across time-scales other than at current lag. We therefore consider inter-arrival time for further analysis as performance of network devices matters more during burst.

Histogram of inter-arrival time provides an insight into the underlying distribution which is shown in figure 2. It can be seen that the data exhibits high degree of skewness (index 6.16) and large excess-kurtosis (index 82.89), which is generally believed to cause heavy-tailedness. Based on the appearance of the histogram, possible candidate distributions for the data are Log-Normal and Pareto distribution, where Log-Normal is selected for peakedness in data and Pareto is selected to account for long-memory and power-tail behaviour. Figure 3 shows the comparison of empirical distribution of the data with respect to these two distributions in log-log scale so as to observe tail behaviour. Both the distributions have been fitted to provide Maximum Likelihood Estimate (MLE) of the corresponding parameters of the distribution and their 95% confidence interval.

Figure 3 has been shown with x-axis in reverse order to depict the fact that Pareto distribution fits poorly at least upto the mean value of inter-arrival data, which contributes 66.33% of the total population. Instead, Log-Normal distribution is

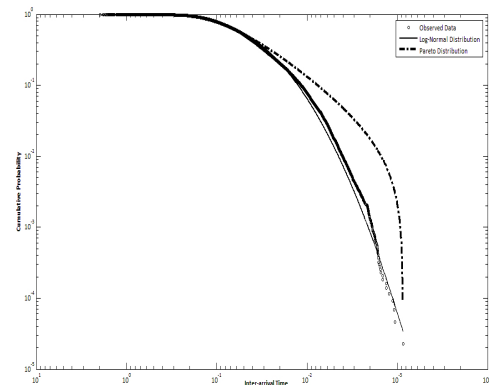


Fig. 3. Empirical Cumulative Distribution of Inter-arrival Time (log-log scale)

found as a very good fit to the data in hand. The matter becomes clearer from figure 4 where upper figure is set in Log-linear scale and the lower figure is set in log-log scale. Figures show that fitted Pareto distribution can predict probability almost immediately after the mean value is crossed but it again fits poorly at upper-tail. Both the plots however confirm that, fitted log-normal distribution provides substantially accurate measure of probability with respect to the data in hand. Validity of the fitting is evaluated by plotting Negative Log-Likelihood surface around the estimated parameters of the fitted distribution and then superimposing the estimated parameters onto the plot. Figure 5 and 6 shows the negative log-likelihood surface for Log-Normal and Pareto distribution respectively. It is clear from these figures again, Log-Normal distribution fitting is better with the estimated parameters, but, fitted Pareto distribution is unable to generate any surface on one side of the estimated parameter which agrees with the finding in lower plot of figure 4.

Further confirmation of the matter is made from the Quantile-Quantile (QQ) plot of the dataset. QQ-plot displays effectiveness of the estimated parameters by generating random numbers from the fitted parameters, estimating quantiles from them and finally regressing it with the quantiles of the original dataset. Figure 7 shows the QQ-plot of the fitted Log-Normal distribution (upper figure) and Pareto distribution (lower figure). Again, it is found that fitted Log-Normal distribution gives better regression coefficient (0.9903) compared to fitted Pareto distribution (0.9627). As distribution analysis has favoured more, logarithm of inter-arrival data is considered for time-series analysis next. This is done with an expectation that, temporal behaviour of the logarithm data will exhibit randomness and lack of temporal correlation. This same method has been adopted for the data collected from academic institute LAN as well.

B. Time-Series Analysis

Time series analysis of data is mostly concerned about the correlation structure present in the data. However, to contrast with temporal dependency, Lag plot is often employed to examine randomness among data, which plots values of the series

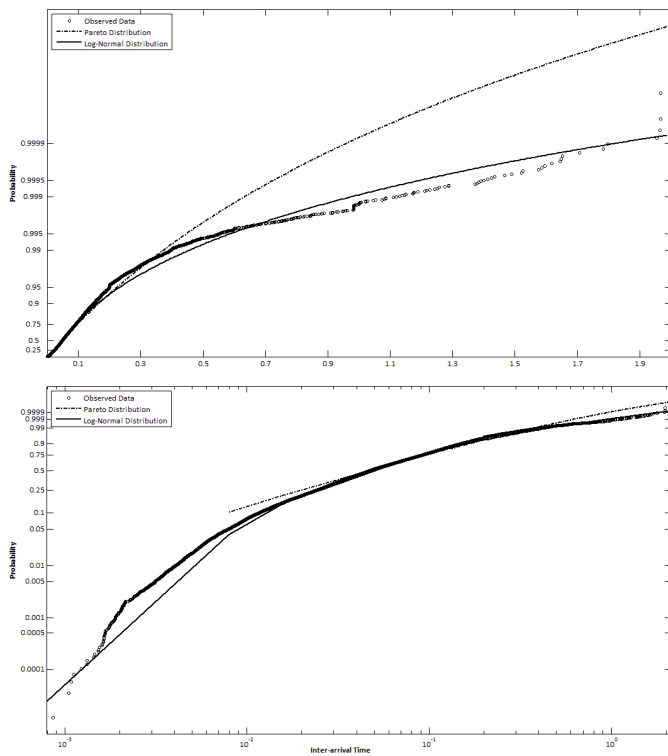


Fig. 4. Probability plot of Inter-arrival Time

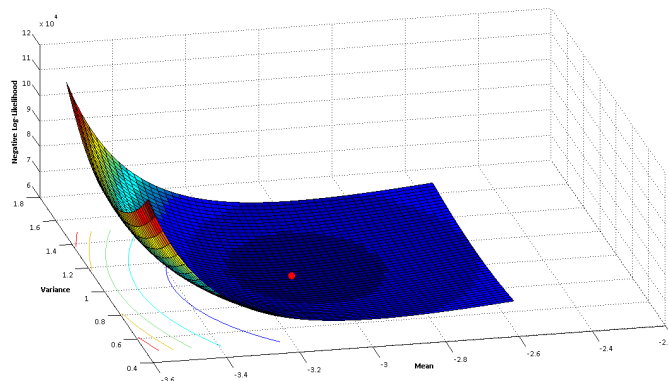


Fig. 5. Negative Log-Likelihood Surface for Fitted Log-Normal Distribution Parameters

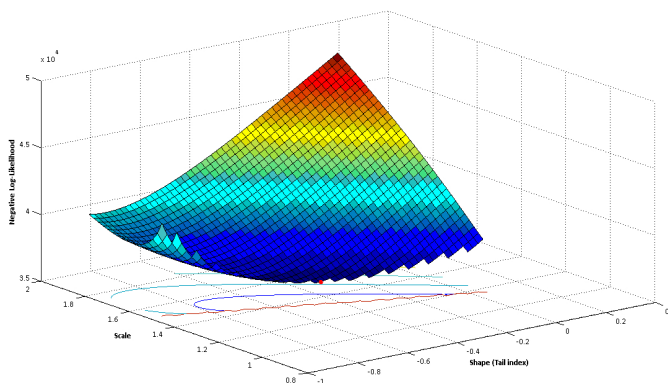


Fig. 6. Negative Log-Likelihood Surface for Fitted Pareto Distribution Parameters

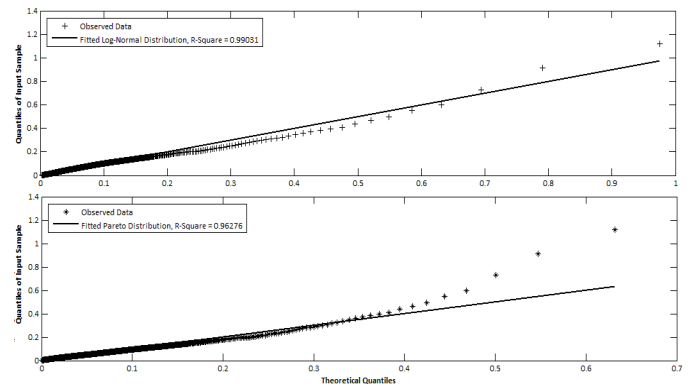


Fig. 7. QQ-plot of Fitted Distributions

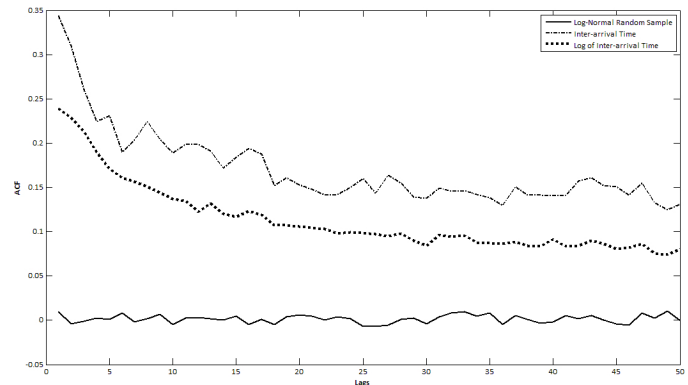


Fig. 8. ACF comparison

at different lags against each other. Considering the discussion of the previous subsection, which favours more towards Log-Normal distribution, figure 8 shows Auto-Correlation Function (ACF) of a log-normally generated random numbers using fitted parameters, inter-arrival and logarithm of inter-arrival data. The plot clearly shows that inter-arrival time exhibits very slow decay in ACF, similar to LRD assumption whereas log-normally generated random samples shows no correlation at all. Further, contrary to the expected behaviour, logarithm of inter-arrival also shows temporal correlation by virtue of slow decay in ACF. Confirmation of the slow decay in ACF and manifestation of self similarity with LRD can be observed in figure 9, which estimates $H = 0.82$ using Aggregated Variance method for the logarithm of inter-arrival.

On the other hand, randomness of the data is also important, which can be seen from lag plot shown in figure 10. Lag plot shows that data is randomly distributed on either side of the mean value with approximately 25% of the population lying in each of the quadrants. Thus, slow decay of ACF can only be possible if large number of contiguous samples lie on either side of the mean value with values very near to each other. To investigate into this phenomena, we resort to the spectral analysis of the data and compare the same with synthetically generated fGN dataset corresponding to the estimated $H = 0.82$ value and variance of the data. Result of the analysis is

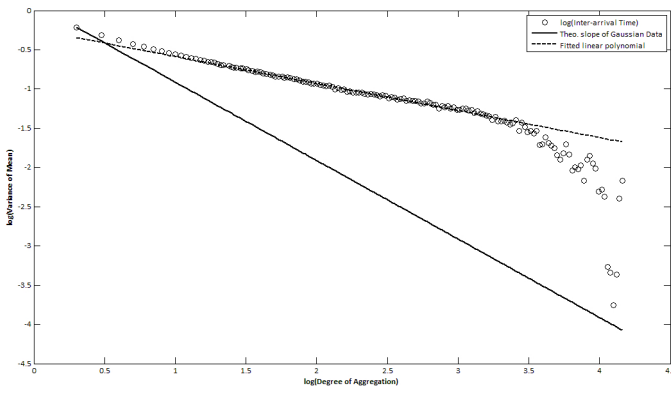


Fig. 9. Aggregated Variance Plot

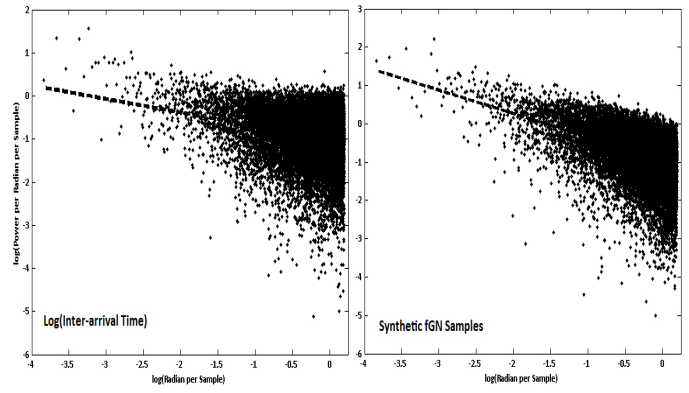


Fig. 11. Periodogram Analysis

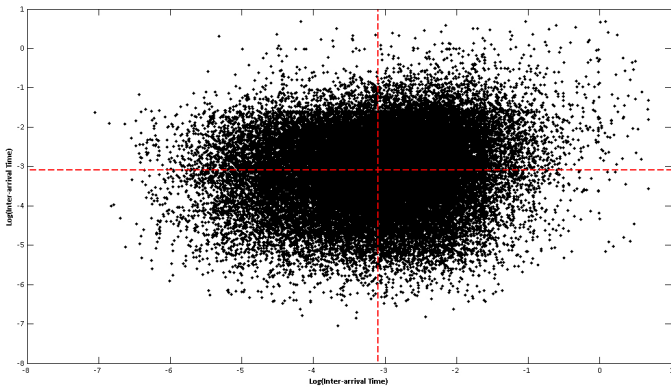


Fig. 10. Lag Plot

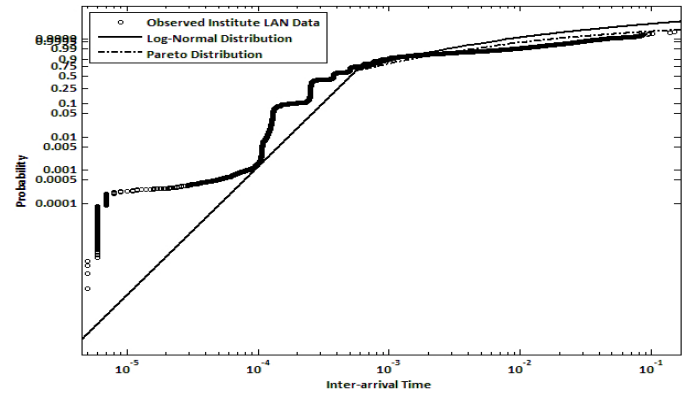


Fig. 12. Probability plot of Institute LAN Data

shown in figure 11.

Left figure shows periodogram for the data with estimated $H = 0.615$ and right figure shows periodogram of the synthetic fGN data with estimated $H = 0.81$. Comparing these two figures, one clearly notes that, synthetic fGN data shows a steady declining slope not only near the origin but also towards right-end. On the other hand, logarithm of inter-arrival time shows declining slope which diminishes as one moves towards right-end. According to the alternative definition of LRD based on periodogram method, logarithm of inter-arrival time data exhibits LRD as there is a negative slope near origin. However, it is also important to note physical significance of this requirement for LRD. Power-law like decay in periodogram of a LRD sequence near origin signifies that maximum signal power contained in the time-series lies with the portion of the signal where it does not vary or varies with extremely small frequency; this complies to the persistence of self-similarity in LRD. Further, for any LRD data, signal power contained in smaller frequency shall be more than the signal power contained in little higher frequency. This in effect means that, for any LRD data, a continuous non-zero negative slope is expected as one moves towards right-end of the periodogram. This is confirmed by the synthetic fGN sequence in figure 11. However, logarithm of inter-arrival time do not satisfy this and it appears to become pretty

flat as one moves towards right-end. This observation implies that, for the logarithm of inter-arrival time, samples vary with equiprobability at smaller as well as higher frequency with possible exception near origin; this in turn, justify observation made in [28]. Further, samples lying near origin are the one who possibly arrive during a burst as a result of capture effect. Also, substantial difference is seen in the H -value estimated by time-domain analysis and frequency-domain analysis for logarithm of inter-arrival time due to flatness towards right-end of the periodogram.

V. ANALYZING DATA COLLECTED FROM INSTITUTE LAN

It was mentioned earlier that, applicability of analysis of Bellcore data in the current Internet context has been established through analysis of traffic trace of an academic institute LAN. We, however, present only a few figures related this data analysis here due to space limitation. Probability plot analysis of the data has been done, which is shown in figure 12.

Similar to the observations made in case of Bellcore data, fitted Pareto distribution is unable to predict about inter-arrival time present in the lower tail, even though it is similar to the observed data towards upper tail. Number of samples falling in the lower tail constitutes approx. 74% of the population. On the other hand, Log-Normal distribution deviates from the observed samples both in the extreme upper and lower tail but fails in prediction for approximately 0.035% samples

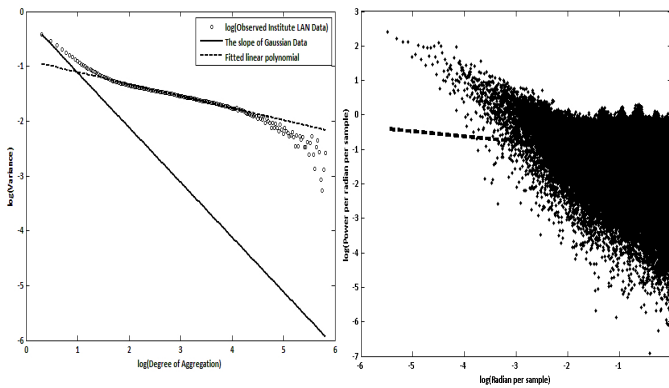


Fig. 13. Hurst parameter of Institute LAN Data (Aggregated Variance and Periodogram method)

in the lower tail and 4% samples in the upper tail; this is very minimal compared to the deviation in case of Pareto distribution. One can therefore proceed with Log-Normal assumption as before. Hurst parameter of the logarithm of inter-arrival time, both aggregated variance (left figure) and periodogram method (right figure), is shown in figure 13. Self-similarity with LRD of the data is visible in aggregated variance method and is found to be $H = 0.89$, however, periodogram does not reflect such high degree of LRD with $H = 0.61$ for first 10% of points - this is similar to the observation made for Bellcore data.

VI. CONCLUSION

In this paper we have shown that, Log-Normal distribution models network traffic inter-arrival time better than Pareto distribution, specially, in the lower tail region. Primary reason attributed for this observation is, exponential back-off approach in the Ethernet layer and greediness inherent into the protocol resulting into a competition amongst senders and resulting into capture effect. It has also been shown that, even though network data exhibits Self-similarity with Long Range Dependence (LRD) in Time-domain analysis, same is not prominent in case frequency domain analysis. All this observations, point towards better characterization requirement of network traffic and design of techniques to ascertain if LRD is a natural phenomena in network traffic.

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