

The Comparison between the TE₂₁ Mode and the Four-Horn Monopulse Technique for LEO Satellite Tracking

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Abstract— One of the most well-known techniques, which are used in the ground stations in order to track the LEO satellites, is monopulse. Two most popular of them are four-horn and TE₂₁ mode monopulse techniques. In this paper the monopulse tracking technique using TE₂₁ mode is analyzed and compared with the four-horn monopulse tracking technique. The key parameters of TE₂₁ mode monopulse technique including the linearity and slope of the error signal are calculated. It is demonstrated that the linearity of the error signal is improved in comparison with the four-horn monopulse technique. Although the slope of the error signal decreases in the TE₂₁ mode tracking technique, the estimation of the satellite angle is more accurate in most of the cases compared to the four-horn tracking technique.

Keywords— LEO satellites, four-horn monopulse, multimode monopulse technique

I. INTRODUCTION

The advances in the satellite communication systems and efforts made to solve the system's problems led the researchers to take the LEO satellites into consideration. These satellites have a smaller size and lower weight than the satellites in higher earth orbits; as a result the cost of satellite production and also their launching systems are less than other satellites but angular velocity of these satellites is very high. So the most significant problem of these satellites is the requirement of tracking and consequently more complex data reception stations are needed. Major complexity of tracking systems is related to antenna tracking subsystems. The first developed systems for antenna tracking was similar to radar tracking technique like four-horn monopulse. But due to some problems in the satellite systems and also in order to have more efficiency, the tracking techniques of satellites were proposed [1]. One of these techniques was multimode monopulse tracking technique. At first it was tried to produce difference signal by using generation of different modes in a conical corrugated horn, which in the monopulse technique was generated by several horns. In this horn different modes are produced which are dependent on the dimensions of the horn. By more investigation and considering to different aspects of the problem the designers of satellite tracking

stations realized that the best mode for tracking is TE₂₁ (along with TE₁₁ as dominant mode) [2]. In this paper different aspect of tracking technique of the four-horn monopulse technique as the classic monopulse and TE₂₁ multimode monopulse is compared.

The paper is organized as follows. The antenna far field pattern of TE₁₁ and TE₂₁ modes are investigated in section 2. The fundamental of the multimode monopulse tracking technique is described and its parameters are compared to counterpart parameters of the four-horn monopulse in section 3. By using simulation results, different aspects of two techniques are investigated in section 4. Finally, conclusion is presented.

II. THE RADIATION PATTERNS OF TE₁₁ AND TE₂₁ MODES

In the introduction of this paper, the mentioned modes of TE₁₁ and TE₂₁ are used as sum and difference signals in classic monopulse tracking technique, respectively. In this section for more understanding of this subject the pattern of these two modes are studied. Suppose the horn producing two mentioned modes is placed at the focal point of the parabolic antenna. Its radius is “ a ” and operating at the frequency of “ f_c ”. The far field of the mentioned modes is described in [3] and by some algebraic computations it can be shown that far field components of these modes in the spherical coordinate are obtained as:

$$E_{\theta_{11}} = \frac{J_1(u)}{u} \frac{\sin \phi}{\tau'_{11} \sqrt{1 - \left(\frac{1}{\tau'_{11}}\right)^2}} \quad (1)$$

$$E_{\phi_{11}} = \frac{J'_1(u)}{1 - \left(\frac{u}{\tau'_{11}}\right)^2} \frac{\cos \phi}{\tau'_{11} \sqrt{1 - \left(\frac{1}{\tau'_{11}}\right)^2}} \quad (2)$$

$$E_{\theta_{21}} = \frac{J_2(u)}{u} \frac{2 \sin 2\phi}{\tau'_{21} \sqrt{1 - \left(\frac{2}{\tau'_{21}}\right)^2}} \quad (3)$$

$$E_{\phi_{21}} = \frac{J'_2(u)}{1 - \left(\frac{u}{\tau'_{21}}\right)^2} \frac{\cos 2\phi}{\tau'_{21} \sqrt{1 - \left(\frac{2}{\tau'_{21}}\right)^2}} \quad (4)$$

Where

$J_n(u)$ are Bessel functions of the first kind, order n ;

" c " is the speed of light in free space;

$k_{c_{n,1}} a = \tau'_{n,1}$ is the 1th zero of the derivative of the Bessel function of the first kind, order n , which their values for $n = 1, 2$ equal to:

$$k_{c_{1,1}} a = \tau'_{1,1} = 1.841 \quad (5)$$

$$k_{c_{2,1}} a = \tau'_{2,1} = 3.054 \quad (6)$$

Also,

$$u = ka \sin \theta; \quad k = \frac{2\pi}{\lambda} \text{ and } \lambda = \frac{c}{f_c}.$$

Pattern of each electrical field can be obtained by following equation:

$$P(\theta, \phi) = E_\theta^2(\theta, \phi) + E_\phi^2(\theta, \phi) \quad (7)$$

Thus, pattern of TE_{11} and TE_{21} modes could be drawn by using electrical fields of these modes for different values θ and ϕ . According to above equations, two mentioned modes (unlike pattern of the circular uniform aperture antenna) have different patterns respect to ϕ .

The equations (1)-(4) are related to spherical coordinate whereas the pattern of the antenna must be calculated in antenna pedestal coordinate, because the purpose of this paper is investigation of monopulse tracking technique in the ground station antenna pedestal. In this paper the pattern of the aperture antenna in the spherical coordinate is converted to Elevation over Azimuth coordinate [5].

According to above conversion three-dimensional pattern of two modes TE_{11} and TE_{21} for typical values Azimuth=78 deg and Elevation=52 deg at $f_c = 8.25$ GHz and $a = 5.4$ m, are shown in Fig. 1 and Fig. 2 respectively. Remind that two previous values are only typical values and mentioned patterns are same for all of the Azimuth and Elevation values.

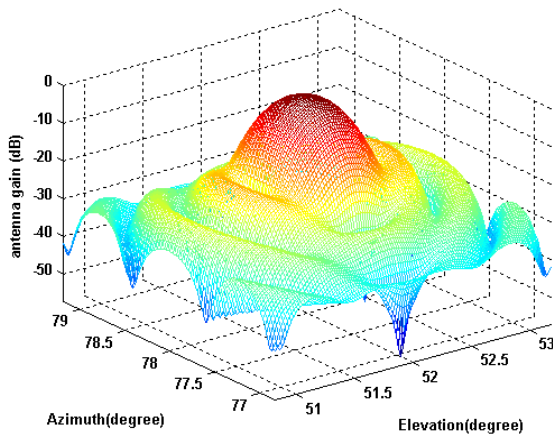


Figure 1. The Three dimensional radiation pattern of the TE_{11} mode

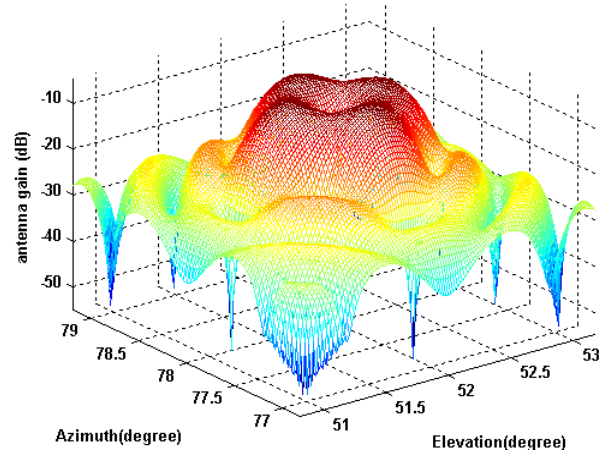


Figure 2. The Three dimensional radiation pattern of the TE_{21} mode

III. ANALYSIS OF FOUR-HORN AND TE_{21} MODE MONOPULSE TECHNIQUE

monopulse tracking techniques which are used by any structure have a common property which is linearity of error signal over convergence region around antenna axis. In other words, they obtained error signal from sum and difference pattern has a linear slope. Therefore the slope of this error signal and value of it can be estimated by angle offset of satellite signal from antenna axis and thus move antenna axis to desired point. The monopulse technique is explained in detail in [4]. Hence the error signal in monopulse techniques can be presented as:

$$E_e = \frac{E_d}{E_s} \quad (8)$$

$$E_e = K_m \theta \quad (9)$$

$$\theta = \sqrt{\theta_{az}^2 + \theta_{el}^2} \quad (10)$$

Where E_d is the difference signal, E_s is the sum signal, E_e is the error signal, θ is satellite angle offset from antenna axis and K_m is the slope of error signal in monopulse techniques. Note that in four-horn monopulse E_d and E_s can be obtained as defined in [6] and in multimode monopulse E_d and E_s are the signals which are obtained from TE_{11} and TE_{21} modes, respectively.

Based on eq. (9) it can be shown that slope of error signal is calculated by the following equation:

$$K_m = \left. \frac{\partial E_e}{\partial \theta} \right|_{\theta_i=0} \quad (11)$$

In above equation $\theta_i = 0$ is the time in which the ground station antenna axis is toward the satellite's signal.

Considering the most important property in monopulse tracking techniques that is linearity of error signal, it is better to study the deviation of error signal in monopulse techniques (four-horn monopulse for different horn's offset from antenna axis and multimode) from the linear signal matched to them. By doing this, Fig 3 shows these deviations for two techniques of the four-horn and multimode monopulse. As described in [6],

the best angular offset of feed horns from antenna axis in four-horn monopulse technique is $\theta_0 = \theta_b / \sqrt{2}$ where θ_b is half power beam width of circular uniform aperture antenna. Note that unlike four-horn monopulse technique, the TE₂₁ multimode monopulse uses one horn so deviation from linear signal is single value. It can be observed that deviation of error signal from matched linear signal in the multimode monopulse technique is less than four-horn monopulse for the horn's entire offset except for $\theta_0 = \theta_b / \sqrt{2}$.

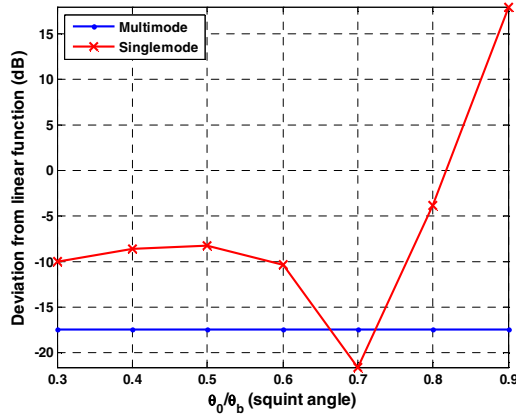


Figure 3. The comparison between deviation of the error signal in four-horn and multimode monopulse from linear function

Also the slope of linear signal that is matched to error signal can be compared to slope values obtained by real error signal. It was done for four-horn monopulse in [6] for different horn's offset related to antenna axis. Fig 4 shows slope value of error signal by using eq. (11) and slope of linear function that is matched to it. It is obvious that the slope value of error signal in multimode monopulse is less than the slope of error signal in four-horn monopulse for $\theta_0 > 0.4\theta_b$ (and also the least difference is obtained for $\theta_0 = \theta_b / \sqrt{2}$).

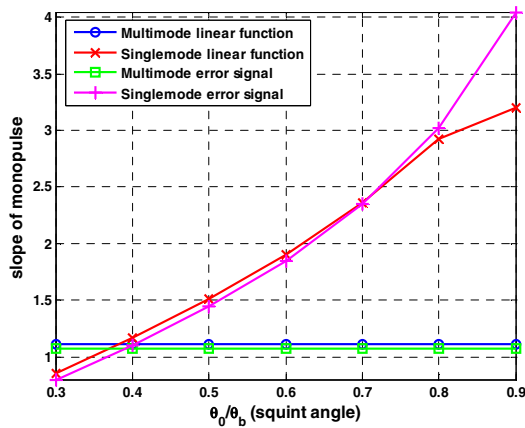


Figure 4. The comparison between the slope of the error signal in four-horn and multimode monopulse techniques

IV. COMPARISON BETWEEN ESTIMATION ERROR OF THE FOUR-HORN AND MULTIMODE MONOPULSE TECHNIQUES

Bearing in mind what is explained in analysis of the multimode monopulse technique and utilization of results obtained from analysis of four-horn monopulse in [6], the estimation accuracy of these two techniques could be compared. In this section different parameters of estimation including bias, variance (var) and root mean square error (RMSE) are analyzed. Assuming that real value of angle is θ and estimated value is $\hat{\theta}$ the mentioned parameters can be calculated by the following equations:

$$\text{bias}(\theta) = E(\theta - \hat{\theta}) \quad (12)$$

$$\text{var}(\hat{\theta}) = E(\hat{\theta}^2) - E(\hat{\theta})^2 = E((\theta - \hat{\theta})^2) - E((\theta - \hat{\theta})^2) \quad (13)$$

$$\begin{aligned} \text{RMSE}(\theta - \hat{\theta}) &= \sqrt{E((\theta - \hat{\theta})^2) + E((\theta - \hat{\theta})^2)} \\ &= \sqrt{\text{var}(\theta) + \text{bias}(\theta)^2} \end{aligned} \quad (14)$$

It should be noted that time average values are used instead of statistical average (expectation). Fig 5 shows the location of the satellite respect to antenna axis. In this figure θ_b is 3dB beam width of circular uniform aperture antenna.

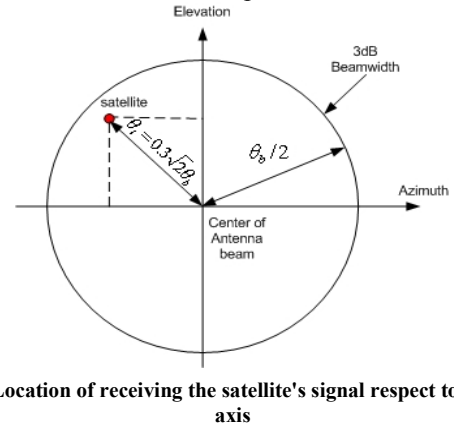


Figure 5. Location of receiving the satellite's signal respect to the antenna axis

Several independent simulation results should be averaged in order to obtain the bias, variance and RMSE parameters in some specific SNRs. Assume that satellite transmitted signal is x and additive noise is n , thus received signal is:

$$y = x + n \quad (15)$$

Where n is the white Gaussian noise with zero mean and variance σ_n^2 . The maximum levels of received power by sum and difference patterns (TE₁₁ and TE₂₁ modes respectively) are obtained as bellow:

$$(TE_{11})_{\max} = 0.3235 \text{ volt} \quad (16)$$

$$(TE_{21})_{\max} = 0.1862 \text{ volt} \quad (17)$$

In addition it is assumed that the additive noises in two modes are independent thus variance of additive noises are calculated by following equations:

$$\sigma_{n_{TE_{11}}}^2 = TE_{11_{\max}} \times \frac{1}{2} 10^{\frac{\text{SNR}}{10}} \quad (18)$$

$$\sigma_{r_{TE_{21}}}^2 = TE_{21_{\max}} \times \frac{1}{2} 10^{-\frac{SNR}{10}} \quad (19)$$

To calculate the bias, variance, and RMSE, the noise with the calculated variance is added to the sum and difference signals. Assume that a antenna with diameter $a = 5.4$ m at $f_c = 8.25$ GHz and its axis is align to Azimuth=78 deg and Elevation=52 deg and satellite's angle offset respect to antenna axis is similar to Fig 5. The values of bias, variance, and RMSE are calculated by over 500 independent simulations. Figs 6-8 show bias, variance, and RMSE parameters of multimode monopulse versus counterpart parameters of the four-horn monopulse, respectively (for different horn's offset). By comparing these values to their counterpart values, obtained by four-horn monopulse tracking technique, it can be concluded that RMSE of multimode monopulse technique is better than the RMSE of four-horn monopulse except for $\theta_0 = \theta_b / \sqrt{2}$. This result agrees with linearity of error signal that explained previously. By paying more attention it can be seen that bias of the multimode is less than the bias of the four-horn monopulse (except for $\theta_0 = \theta_b / \sqrt{2}$), but its variance is more than the variance of four-horn monopulse.

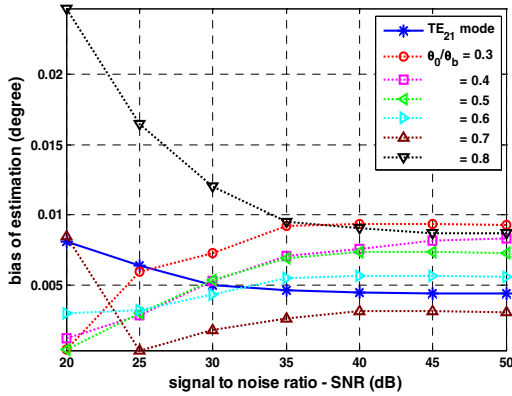


Figure 6. The bias of estimation error in four-horn and multimode monopulse

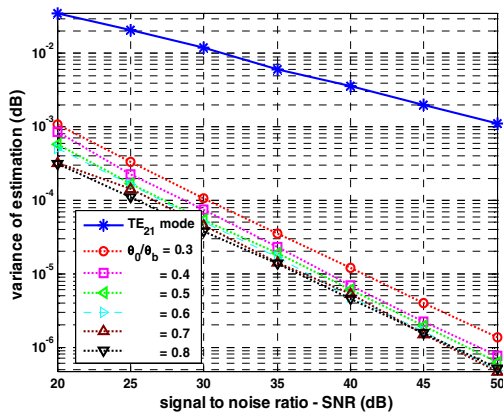


Figure 7. The variance of estimation error in four-horn and multimode monopulse

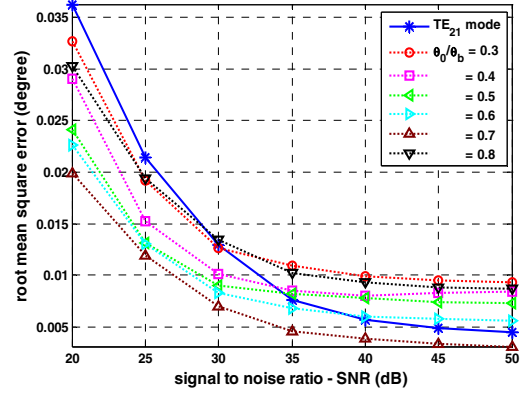


Figure 8. RMSE of estimation error in four-horn and multimode monopulse

V. CONCLUSION

In this paper four-horn and multimode monopulse tracking techniques have been analyzed. It has been illustrated that the slope of error signal in TE₂₁ multimode feed is less than the slope of error signal in four-horn with the offset feed of $\theta_0 > 0.4\theta_b$. Although estimation bias of this technique is less than the four-horn monopulse technique, their variance is more. However, it has been shown that RMSE estimation of TE₂₁ multimode monopulse technique in all of the cases are better than RMSE of four-horn monopulse techniques except for the case that the offset of horns is $\theta_0 = \theta_b / \sqrt{2}$.

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