

Effect of Primary Spectrum Availability on Outage Probability of Cognitive Relay Systems in Secondary Spectrum Usage

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Abstract—In this paper, we investigate the outage probability of cognitive relaying networks where the relays use primary spectrums in a secondary purpose. The outage probability has been studied and useful conditions of achieving the full diversity have been given based on the assumption that the primary spectrum is always available for secondary usage. However, the available spectrum for cognitive nodes does not always exist practically. The performance then depends on the probability that the primary spectrum is available for the secondary usage. In this paper, we prove that if the probability of spectrum availability for cognitive nodes and the number of primary nodes become large, then the outage probability decreases and the full diversity can be achieved. We show that for a given average number of primary nodes having the spectrum available for cognitive nodes, the outage performance becomes better with a few primary nodes having a high probability of the spectrum availability for cognitive nodes than with many primary nodes having a low probability of the spectrum availability. We also show that if the primary spectrum is not always available for secondary usage, then there is no impact of cooperative detection of available spectrum with multiple relay nodes on the outage performance.

Keywords—Cognitive relaying, energy detection, outage probability, full diversity order, beacon signal

I. INTRODUCTION

Cognitive radios are increasingly regarded as an efficient solution to enhance utilization of the limited spectral resource in wireless communications [1]-[3]. Wireless network systems have been operated in private frequency bands. According to the FCC (Federal Communications Commission) report on spectrum utilization, measurement data shows that licensed frequency bands are not utilized severely [4]. Many searchers and legislators have argued in favor of a more efficient use of the spectrum, leading to the possible coexistence of several network operators in a shared frequency band, and have been studying cognitive radios and devices that can adapt their operating characteristics to the channel condition, as a candidate for secondary spectrum access.

A cognitive radio allows a unlicensed user to access a spectrum hole unoccupied by a primary (licensed) user. By doing so, spectrum utilization can be improved significantly while reducing the white spaces in the spectrum [1]. In cooperative relay networks, one or more intermediate nodes

are used to support signal transmission when the direct path from source to destination is in a deep fade. Relays can provide transmit power savings due to path loss reduction and spatial diversity. Recently, the combination of cognitive radio and cooperative relaying techniques has been investigated [5], [6].

Inspired by cognitive radios and relay networks, [5] investigated a cognitive wireless relay network which is defined by a source node, a destination node, and several clusters each of which consists of a number of cognitive relay nodes and only one primary node. The cognitive relay nodes are grouped as clusters based on their geographical proximity. The aim of [5] is to understand the impact of spectrum acquisition performance of the cognitive relay nodes on the outage probability.

While [5] assumes that an unoccupied spectrum by the primary node always exists, this assumption is not realistic. Therefore, in this paper, we consider the probability of presence of an unoccupied spectrum by the primary node (i.e. available spectrum for cognitive nodes). This probability is identical with the probability of sending a beacon signal from the primary node p_{bc} because when an unoccupied spectrum by the primary node exists, the primary node broadcasts a beacon signal to inform cognitive nodes of the availability of a spectrum. We consider multiple primary nodes in each cluster in order to increase the probability of presence of available spectrum for cognitive nodes. We investigate effect of primary spectrum availability on the outage probability and show that the outage probability decreases and the full diversity can be achieved if the number of primary nodes is large and/or p_{bc} is close to 1. We also show that the probability of sending a beacon signal as well as the probability of sensing a beacon signal impacts to the outage performance. For a given average number of primary nodes having the spectrum available for cognitive nodes, the outage performance becomes better with a few primary nodes having a large p_{bc} than with many primary nodes having a small p_{bc} . And if $p_{bc} \neq 1$, then there is no impact of cooperative detection of available spectrum with multiple relay nodes on the outage performance.

II. SYSTEM MODEL

We consider a cognitive wireless relay network where a source node is assisted by several cognitive relay nodes as

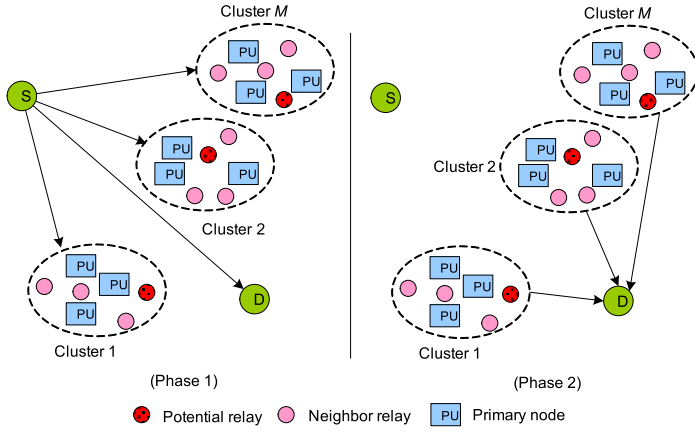


Figure 1. System model.

shown in Figure 1. There are M clusters assigned with different frequency band. Each cluster consists of B primary (licensed) nodes and $N + 1$ cognitive (unlicensed) relay nodes including one potential relay node and N neighbor relay nodes. Each primary user is assigned different spectrum and broadcasts a beacon signal over a side channel with a probability p_{bc} to inform a relay node of the availability of a spectrum. If the potential relay node detects the beacon signal with a probability P_d and acquires the spectrum hole unoccupied by a primary user successfully, then it decodes the message from the source. The potential relays that can acquire the spectrum hole and decode the message successfully, are included in the set of successful potential relays $R(s)$.

The communication of this system is composed of two phases. In the first phase, the source broadcasts its message, and both the relays and the destination receive the message. In the second phase, the successful potential relays decode and transmit to the destination over the acquired spectrum.

In the first phase, the received signal at the potential relay in cluster i is

$$y_{sr_i} = h_{sr_i}x_s + n_{sr_i}, \quad (1)$$

where x_s is the transmitted signal from the source, and h_{sr_i} and n_{sr_i} are the channel coefficient and the noise, respectively, from the source to the potential relay. The received signal at the destination is

$$y_{sd} = h_{sd}x_s + n_{sd}, \quad (2)$$

where h_{sd} and n_{sd} are the channel coefficient and the noise, respectively, from the source to the destination. In the second phase, the received signal at the destination from the successful potential relay in cluster i , i.e. $r_i \in R(s)$ is

$$y_{r_id} = h_{r_id}x_{r_i} + n_{r_id}, \quad (3)$$

where x_{r_i} is the transmitted signal from the successful potential relay in cluster i , and h_{r_id} and n_{r_id} are the channel

coefficient and the noise, respectively, from the relay to the destination. h_{sr_i} , h_{sd} , and h_{r_id} are zero-mean circularly symmetric complex Gaussian random variables with variances $1/\lambda_{sr_i}$, $1/\lambda_{sd}$, and $1/\lambda_{r_id}$, respectively. n_{sr_i} , n_{sd} , and n_{r_id} are also zero-mean circularly symmetric complex Gaussian random variables with variance N_o . We assume the transmit power of the source and the relays is P .

III. OUTAGE PROBABILITY

The outage probability is one of the most commonly used performance measures in wireless systems and is defined as $P_{out} = Pr[I < R]$, where I is the mutual information as

$$I = \frac{1}{M+1} \log \left(1 + \text{SNR}|h_{sd}|^2 + \text{SNR} \sum_{r_i \in R(s)} |h_{r_id}|^2 \right) \quad (4)$$

where $\text{SNR} = P/(N_oW)$, and R is a predefined threshold rate. To obtain the outage probability, we derive the followings and show the numerical results for two cases: (i) with multiple primary users and a potential relay node, and (ii) with a primary user and a potential relay node cooperated by neighbor relay nodes.

A. With Multiple Primary Users and One Potential Relay Node in Each Cluster

The probability that b primary nodes among total B primary nodes send the beacon signal is

$$Pr[\hat{B} = b] = \binom{B}{b} p_{bc}^b (1 - p_{bc})^{B-b}, \quad (5)$$

where $\hat{B}$ is the number of the primary nodes sending the beacon signal and p_{bc} is the probability that each primary node sends the beacon signal.

Each potential relay performs energy detection and estimates the presence of the beacon signal from the primary nodes. Energy detection is a simple technique for detecting an unknown signal. The average probability of detection, P_d , for each potential relay in Rayleigh fading is given by [7]

$$P_d = e^{-\frac{u}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda}{2} \right)^n + \left(\frac{1 + \lambda_{pr}}{\lambda_{pr}} \right)^{u-1} \times \left\{ e^{-\frac{\lambda}{2(1+\lambda_{pr})}} - e^{-\frac{u}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda \lambda_{pr}}{2(1+\lambda_{pr})} \right)^n \right\}, \quad (6)$$

where $u = TW$ is time-bandwidth product and we assume that T and W are chosen to restrict u to integer values, and λ is the decision threshold. Then, the probability that the potential relay detects at least one beacon signal among b beacon signals, A_d , is

$$A_d = 1 - (1 - P_d)^b. \quad (7)$$

Using (7), the probability that k potential relays are successful potential node among total M potential relays is

$$Pr[K = k | \hat{B} = b] = \binom{M}{k} A_d^k (1 - A_d)^{M-k}, \quad (8)$$

where K is the number of successful potential relay nodes.

The probability of each potential relay being in the successful potential relay set is

$$Pr[r_i \in R(s)] = \exp\left(-\lambda_{sr_i} \frac{2^{(M+1)R} - 1}{\text{SNR}}\right) A_d, \quad (9)$$

using the approximation technique in [8] as $\text{SNR} \rightarrow \infty$, then

$$Pr[R(s) | K = k, \hat{B} = b] \sim \left(\frac{2^{(M+1)R} - 1}{\text{SNR}}\right)^{k-|R(s)|} \times \prod_{r_i \in R(s)} A_d \prod_{r_i \notin R(s)} \lambda_{sr_i}. \quad (10)$$

And the outage probability conditioned on $R(s)$, K , and $\hat{B}$ is given by [8]

$$Pr[I < R | R(s), K = k, \hat{B} = b] \sim \left(\frac{2^{(M+1)R} - 1}{\text{SNR}}\right)^{|R(s)|+1} \frac{\lambda_{sd}}{(|R(s)| + 1)!} \prod_{r_i \in R(s)} \lambda_{r_i d}. \quad (11)$$

So, using (5), (8), (10), and (11), the outage probability can be obtained as

$$P_{out} = \sum_{b=0}^B \sum_{k=0}^M \sum_{|R(s)|=0}^k Pr[I < R | R(s), K = k, \hat{B} = b] \times Pr[R(s) | K = k, \hat{B} = b] Pr[K = k | \hat{B} = b] \times Pr[\hat{B} = b] \quad (12)$$

$$\sim \sum_{b=0}^B \sum_{k=0}^M \left(\frac{2^{(M+1)R} - 1}{\text{SNR}}\right)^{k+1} \binom{B}{b} p_{bc}^b (1 - p_{bc})^{B-b} \times \binom{M}{k} A_d^k (1 - A_d)^{M-k} \Gamma, \quad (13)$$

where

$$\Gamma = \lambda_{sd} \sum_{|R(s)|=0}^k \prod_{r_i \notin R(s)} \lambda_{sr_i} \prod_{r_i \in R(s)} A_d \lambda_{r_i d} \frac{1}{(|R(s)| + 1)!} \quad (14)$$

For $\lambda_{r_i d} = \lambda_{rd}$ and $\lambda_{sr_i} = \lambda_{sr}$,

$$\Gamma = \lambda_{sd} \sum_{|R(s)|=0}^k \binom{k}{|R(s)|} \lambda_{rd}^{|R(s)|} \lambda_{sr}^{k-|R(s)|} \frac{A_d^{|R(s)|}}{(|R(s)| + 1)!} \quad (15)$$

In (13), by simply taking $E[\hat{B}] = B p_{bc}$ and $[B p_{bc}] \triangleq \bar{B}$ as the number of the primary nodes sending the beacon signal, and $E[K | \hat{B} = \bar{B}] = M \left\{1 - (1 - P_d)^{\bar{B}}\right\}$ and $[M \left\{1 - (1 - P_d)^{\bar{B}}\right\}] \triangleq \bar{K}$ as the number of the potential relay nodes relaying to the destination, a simple approximated outage probability can be obtained as

$$\bar{P}_{out} \sim \left(\frac{2^{(M+1)R} - 1}{\text{SNR}}\right)^{\bar{K}+1} \bar{\Gamma}, \quad (16)$$

where

$$\bar{\Gamma} = \lambda_{sd} \sum_{|R(s)|=0}^{\bar{K}} \prod_{r_i \notin R(s)} \lambda_{sr_i} \times \prod_{r_i \in R(s)} \left\{1 - (1 - P_d)^{\bar{B}}\right\} \lambda_{r_i d} \frac{1}{(|R(s)| + 1)!} \quad (17)$$

For $\lambda_{r_i d} = \lambda_{rd}$ and $\lambda_{sr_i} = \lambda_{sr}$,

$$\bar{\Gamma} = \lambda_{sd} \sum_{|R(s)|=0}^{\bar{K}} \binom{\bar{K}}{|R(s)|} \lambda_{rd}^{|R(s)|} \lambda_{sr}^{\bar{K}-|R(s)|} \times \left\{1 - (1 - P_d)^{\bar{B}}\right\}^{|R(s)|} \frac{1}{(|R(s)| + 1)!}. \quad (18)$$

The outage probability with $P_d = 1$ and $p_{bc} = 1$ obtained in [5] is

$$P_{out,p} \sim \left(\frac{2^{(M+1)R} - 1}{\text{SNR}}\right)^{M+1} \Gamma_p, \quad (19)$$

where

$$\Gamma_p = \lambda_{sd} \sum_{|R(s)|=0}^M \prod_{r_i \notin R(s)} \lambda_{sr_i} \prod_{r_i \in R(s)} \lambda_{r_i d} \frac{1}{(|R(s)| + 1)!} \quad (20)$$

and for $\lambda_{r_i d} = \lambda_{rd}$ and $\lambda_{sr_i} = \lambda_{sr}$,

$$\Gamma_p = \lambda_{sd} \sum_{|R(s)|=0}^M \binom{M}{|R(s)|} \lambda_{rd}^{|R(s)|} \lambda_{sr}^{M-|R(s)|} \frac{1}{(|R(s)| + 1)!}. \quad (21)$$

In this case, full diversity can be achieved as observed in (19). So, we can achieve full diversity if in each cluster the following is satisfied.

$$\Omega \triangleq \frac{1}{\Gamma_p} \sum_{b=0}^B \sum_{k=0}^M \left(\frac{2^{(M+1)R} - 1}{\text{SNR}}\right)^{k-M} \binom{B}{b} p_{bc}^b (1 - p_{bc})^{B-b} \times \binom{M}{k} \left\{1 - (1 - P_d)^b\right\}^k (1 - P_d)^{b(M-k)} \Gamma \approx 1. \quad (22)$$

1) Numerical Results: Figure 2 shows the outage probabilities at large SNR. (19) by $P_d = 1$ and $p_{bc} = 1$ can be achieved the full diversity of order $M+1$. With the same B , if p_{bc} becomes small then P_{out} is increased. And with the same p_{bc} , if B becomes large then the full diversity can be achieved. Also, in Figure 4(a), we can be confident of achieving the full diversity at large B because of $\Omega \rightarrow 1$ with a large B . In Figure 3, though B is large, P_{out} is increased with a small p_{bc} comparing the case of $B = 1$, $p_{bc} = 1$ with the case of $B = 10$, $p_{bc} = 0.3$. With the same B , P_{out} becomes low at increasing p_{bc} and the full diversity is achieved. However, it is not always realized shown as Figure 4(b). If B is small, though $p_{bc} \rightarrow 1$, then the full diversity can not be achieved.

In Figure 4, $\Omega \approx 1$ means obtaining the full diversity. The approximation by ignoring Γ and Γ_p is close to the exactness. If the average number of primary nodes sending a beacon signal $B_{avg} = B \times p_{bc}$ is fixed, then P_{out} becomes low with a large p_{bc} , that is, the performance of P_{out} is higher with a few

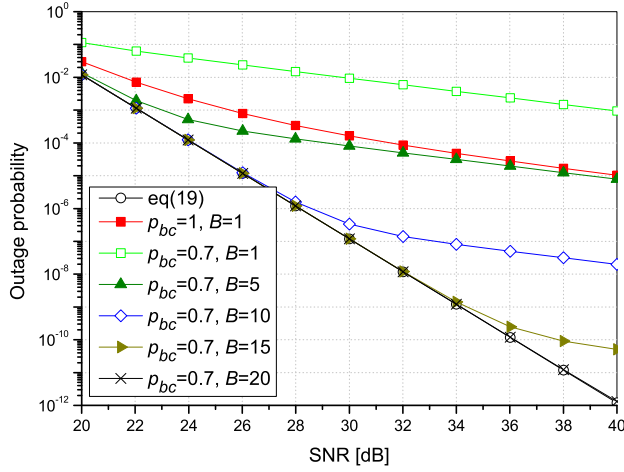


Figure 2. Comparison of the outage probability for different B : $R = 1$, $M = 4$, $\lambda = 13.98$, $u = 5$, $\lambda_{pr} = 10$, and $\lambda_{sd} = \lambda_{sr} = \lambda_{rd} = 1$.

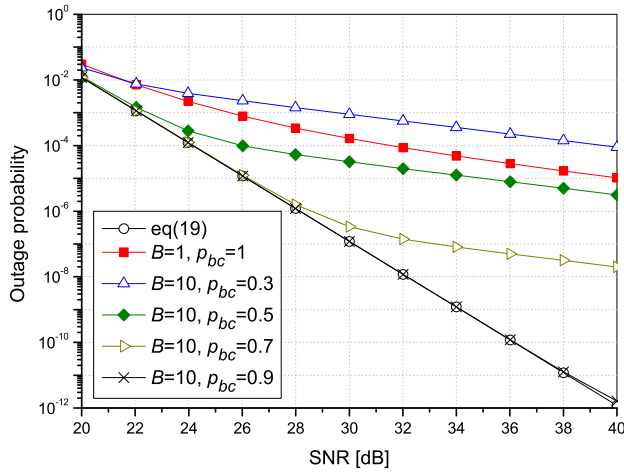


Figure 3. Comparison of the outage probability for different p_{bc} : $R = 1$, $M = 4$, $\lambda = 13.98$, $u = 5$, $\lambda_{pr} = 10$, and $\lambda_{sd} = \lambda_{sr} = \lambda_{rd} = 1$.

primary nodes that scarcely use their spectrum than with many primary nodes that tightly use their spectrum. For example, in Figure 4(b), when $B_{avg} = 3$, the case of $B = 10$ and $p_{bc} = 0.3$ achieves larger P_{out} than that of $B = 3$ and $p_{bc} = 1$, and also, when $B_{avg} = 1$, the case of $B = 10$ and $p_{bc} = 0.1$ achieves larger P_{out} than that of $B = 3$ and $p_{bc} = 0.33$.

B. With One Primary User and One Potential Relay Node Cooperated by Neighbor Relay Nodes in Each Cluster

In this section, we consider $B = 1$ and assume that in each cluster, all neighbor nodes cooperate in energy detection same as operating at the potential relay node. N neighbor relay nodes independently detect the presence of the beacon signal from the primary user, and if at least one neighbor relay detects

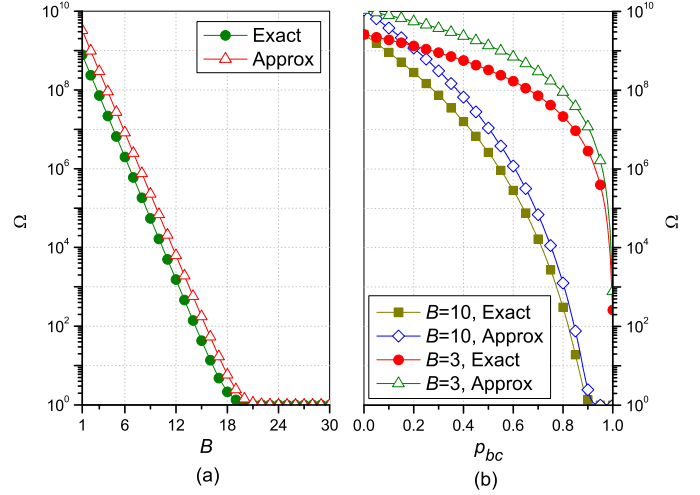


Figure 4. (a) The required number of primary nodes B with $p_{bc} = 0.7$ and (b) the required p_{bc} for achieving full diversity.

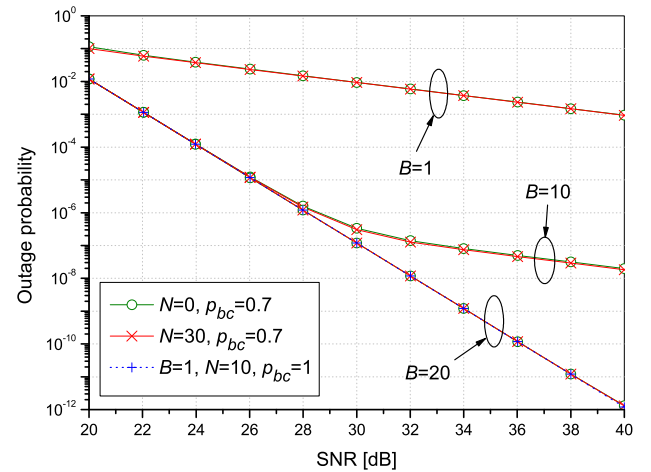


Figure 5. Comparison of the outage probability for different N and B : $R = 1$, $M = 4$, $\lambda = 13.98$, $u = 5$, $\lambda_{pr} = 10$, and $\lambda_{sd} = \lambda_{sr} = \lambda_{rd} = 1$.

it, then the potential relay decides that there is the spectrum hole. The probability of detection, C_d , in this scenario is

$$C_d = 1 - (1 - A_d)^{N+1} = 1 - (1 - P_d)^{b(N+1)}, \quad (23)$$

where P_d and A_d are given in (6) and (7), respectively. The approximated outage probability as $\text{SNR} \rightarrow \infty$ can be obtained by (13) with $B = 1$ and C_d instead of A_d .

This scenario can be extended easily to another scenario with multiple primary user by $B > 1$.

1) Numerical Results: Figure 5 shows that with $B = 1$, if $p_{bc} \neq 1$ then there is no effect of cooperating relays on P_{out} , but if $p_{bc} = 1$ then the full diversity is achieved by using cooperating nodes. This results also occur with $B > 1$

and there is only effect from the presence of multiple primary users. Though many relays cooperate to the potential relay, if each primary user uses its spectrum tightly ($p_{bc} \neq 1$) then the cooperating relays do not improve P_{out} . However, if the number of primary users is increased then P_{out} is improved even if $p_{bc} \neq 1$, because the relays can get many opportunities for acquiring the spectrum hole by multiple primary users.

IV. CONCLUSION

In this paper, we investigate the outage probability of cognitive relaying networks where the relays use primary spectrums in a secondary purpose. We consider that the probability of presence of an unoccupied spectrum by the primary node (i.e. available spectrum for cognitive nodes) is not always zero. In order to increasing the probability of presence of available spectrum for cognitive nodes, multiple primary nodes are in each cluster. We investigate effect of primary spectrum availability on the outage probability and show that the full diversity can be achieved if the number of primary nodes is large and/or p_{bc} is close to 1. We also show that the probability of sending a beacon signal as well as the probability of sensing a beacon signal impacts to the outage performance. P_{out} is lower with a few primary nodes that scarcely use their spectrum than with many primary nodes that tightly use their spectrum. And if $p_{bc} \neq 1$, then there is no impact of cooperative sensing a beacon signal with multiple relay nodes on the outage performance.

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