

Anisotropic Diffusion for Preserving Boundary-edge

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Abstract— In this paper, a new approach for anisotropic diffusion based on the direction of a boundary-edge is proposed to better preserve edges along with the removal of noises. Although the anisotropic diffusion is effective in removing noises in the image, it often destructs the structural information of the object in the image because of a sequence of inaccurate diffusions. In the proposed method, prewitt mask is performed for the directions of boundary-edge. As a result, when the magnitude of boundary-edge is large enough and the number of pixels whose values are similar in the 3×3 pixel structure are one figure among 5,6 and 7, the center pixel is considered as the boundary-edge with accurate edge direction. In the case of boundary-edge, the weight of diffusion is selected adaptively according to the directions of boundary-edge. The proposed diffusion is performed at 8-directions(north, south, east, west, north_left, north_right, south_left, south_right) for preservation of boundary-edge and remove of noise. Experiments results show that the proposed method can eliminate noise and can remain contour of boundary-edge in images with noises.

Keywords — anisotropic diffusion, prewitt mask, boundary edge

I. INTRODUCTION

Anisotropic diffusion is one of the widely used techniques in the field of image process for denosing. The denosing filter based on anisotropic diffusion was introduced by Perona and Malik[2] and has constituted since then a powerful tool for signal and image enhancement. Although the existing denosing filters are termed as “edge preserving” and “feature preserving” there exist major problems of the filtering approach.

First, the anisotropic diffusion is sensitive to the shape and size of the filter kernel. If the cross-kernel is used as a filter, edges are concentrated in horizontal and vertical direction.

Second, the existing filters of anisotropic diffusion do not enhance edges. In case any portion of the filter kernel contains an edge, smoothing will be inhibited.

In this paper, the following method is proposed to solve the above problems.

First, to solve the problem that edges are concentrated in horizontal and vertical direction, the 3×3 window filter including the diagonal direction is used.

Second, presitt mask is employed to determine the boundary-edge. In the case of boundary-edge, the edge directions of the pixel which is decided as a boundary-edge is predicted. And then diffusion is performed by applying weight to the predicted directions to enhance edges.

We first analyzed the anisotropic diffusion of Perona and Malik [2] and GHAD(Gradient Histogram-Based Anisotropic Diffusion)[8] in Section 2. Section 3 presents the proposed anisotropic diffusion method based on directions of boundary-edge. Section 4 shows the experimental results and conclusions are given in Section 5.

II. ANISOTROPIC DIFFUSION

Anisotropic diffusion algorithms remove noise from an image by modifying the image via a partial differential equation. Perona and Malik [2] replaced the classical isotropic diffusion equation with equation (1) based on cross-kernel.

$$I_s^{t+1} = I^t + \frac{\lambda}{|\eta_s|} \sum_{p \in \eta_s} g(\nabla I_{s,p}) \nabla I_{s,p}, \quad (1)$$

$$\eta_s = \{north, south, east, west\}$$

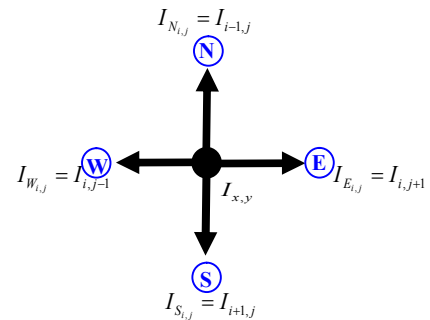


Figure 1. The structure of cross-kernel

The gradient $\|\nabla I\|$ indicates nearest-neighbor differences :

$$\begin{aligned}\nabla_N I_{(i,j)} &\equiv I_{i-1,j} - I_{i,j} & \nabla_S I_{(i,j)} &\equiv I_{i+1,j} - I_{i,j} \\ \nabla_E I_{(i,j)} &\equiv I_{i,j-1} - I_{i,j} & \nabla_W I_{(i,j)} &\equiv I_{i,j+1} - I_{i,j}\end{aligned}$$

The edge-stopping function $g\|\nabla I\|$ is updated at every iteration as a function of the brightness gradient :

$$\begin{aligned}C'_{N_{i,j}} &= g(\|\nabla_N I'_{i,j}\|), & C'_{S_{i,j}} &= g(\|\nabla_S I'_{i,j}\|), \\ C'_{E_{i,j}} &= g(\|\nabla_E I'_{i,j}\|), & C'_{W_{i,j}} &= g(\|\nabla_W I'_{i,j}\|).\end{aligned}$$

Perona and Malik[2] suggested two different edge stopping $g(\cdot)$ functions in their anisotropic diffusion equation (2) and (3).

$$g(\nabla I) = \frac{1}{1 + \left(\frac{\|\nabla I\|}{K}\right)^2} \quad (2)$$

$$g(\nabla I) = e^{-(\|\nabla I\|/K)^2} \quad (3)$$

The inverse proportion edge stopping function in equation (2) can remove the noise in the large area efficiently, because its diffusion is in inverse proportion to gradient. On the other hand, this function can't preserve contour information. The exponent edge stopping function in Equation (3) can remain contour. And the more the number of the diffusion, the more salient noises with high gradient are in equation (3) function.

The GHAD(Gradient Histogram-Based Anisotropic Diffusion)[8] improved the edge stopping function of RAD(Robust Anisotropic Diffusion)[3] with equation (4).

$$g(\nabla I) = \begin{cases} \left[1 - \frac{(\|\nabla I\|)^2}{5K^2}\right]^2, & \frac{\|\nabla I\|^2}{5} \leq K^2 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

If $\frac{\|\nabla I\|^2}{5} \leq K^2$, the diffusion is carried out. Otherwise, the diffusing is not carried out, where K is a gradient magnitude parameter. In the anisotropic diffusion method, the gradient magnitude is used to detect an image edge or boundary as a step discontinuity in intensity.

The features of PM-method(edge stopping function of Perona and Malik[2]) and GHAD-method(edge stopping function of Hae Yong Kim[8]) are shown respectively in Figure 2.

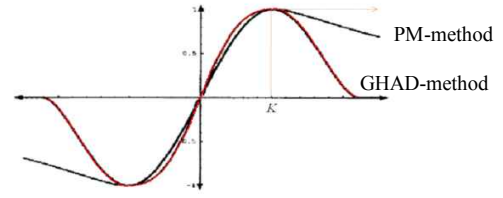


Figure 2. Comparison between PM-method and GHAD-method

III. THE PROPOSED METHOD

To solve the problem that edges are concentrated in horizontal and vertical direction and enhance edges, the proposed method is organized as followings.

A. Detection of boundary-edge's directions

The directions of a boundary-edge is detected by using prewitt mask operations. Prewitt mask is organized as follows:

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad G_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \quad (5)$$

The magnitude of an edge is obtained by equation (6).

$$M = \sqrt{G_x^2 + G_y^2} \quad (6)$$

If the magnitude of an edge obtained by equation (6) is more than the threshold of edge magnitude, this edge is decided as a boundary-edge. The direction of the detected boundary-edge is calculated by using equation (7).

$$D = \arctan\left(\frac{G_y}{G_x}\right) \times \frac{180}{\pi} \quad (7)$$

Directions of boundary-edge are shown in Figure 3. The direction of the detected boundary-edge is classified as four kinds of categories.

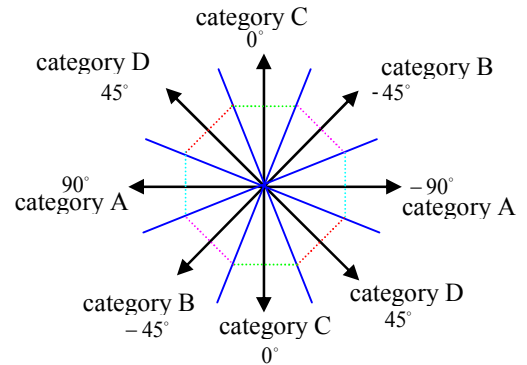


Figure 3. Directions of boundary-edge using Prewitt mask operations

Category D) The difference of the pixel values in the *North_West* direction and the *South_East* direction are very small.

The test image consists of ten kinds of boundary-edges shown in Figure 4. prewitt's mask operations apply to the test image. As a result, the magnitudes of edges are obtained in Figure 6. And then the directions of edges are obtained in Figure 7.

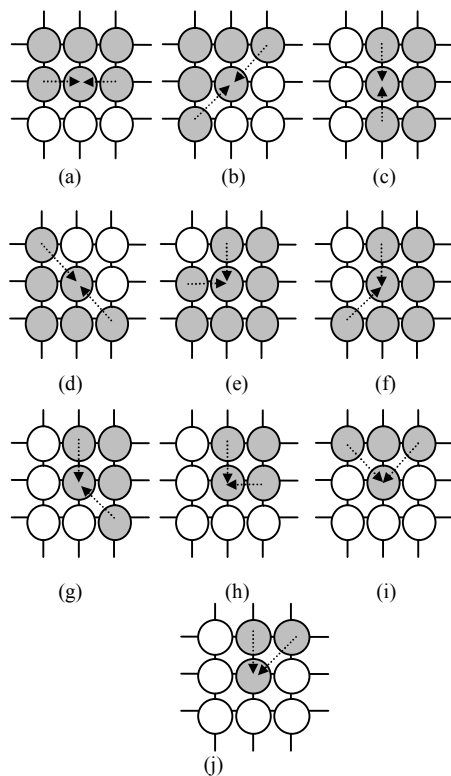


Figure 4. Patterns of boundary-edge

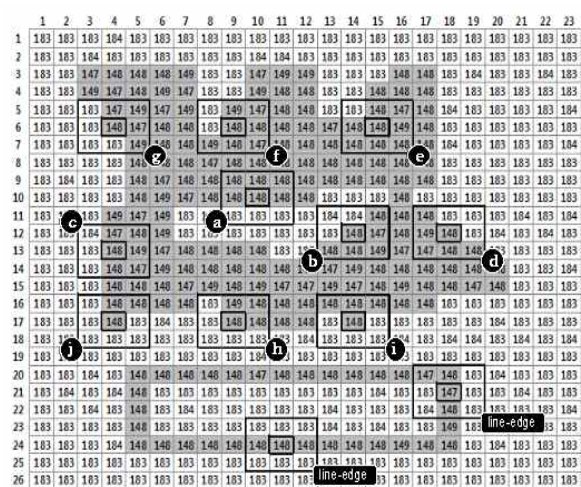


Figure 5. Pixel values of test image

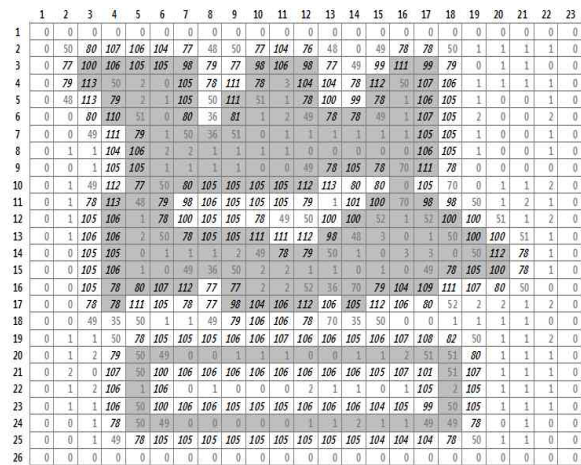


Figure 6. Magnitudes of edges

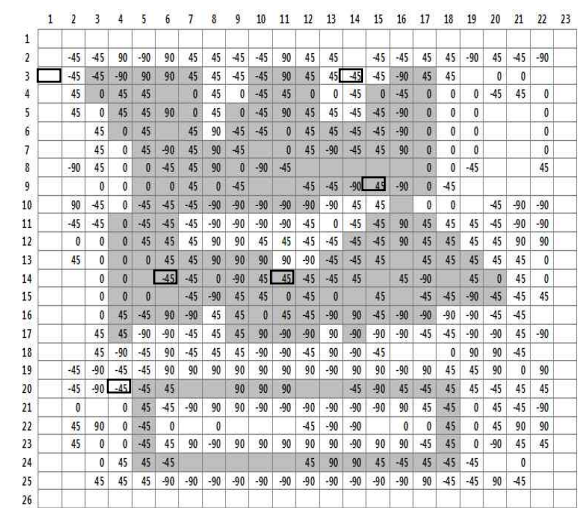


Figure 7. Directions of edge

All the directions of edges with large magnitude shown in Figure 6 and Figure 7 are not accurate. For an example, the direction of an edge is not accurate at thick rectangles in Figure 7.

B. Selection of the accurate boundary-edge's direction

To select the area with an accurate boundary's direction, the number of pixels whose values are similar is counted based on the 3×3 pixel structure

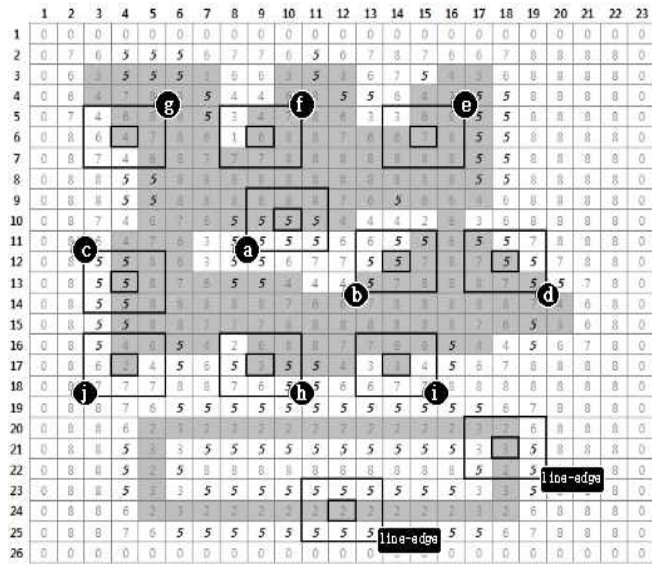


Figure 8. The number of homogeneous pixels

In case the difference between the center pixel value and a neighboring pixel value in the 3×3 pixel structure is less than ± 0.1 , count the number of the neighboring pixel. The result of counting is shown in Figure 8.

The boundary of a straight line pattern having 5 homogeneous pixels such as (a), (b), (c), (d) shown in Figure 8 and the boundary of the corner pattern having 6 and 7 homogeneous pixels such as (e), (f) shown in Figure 8 have accurate edge directions. But the boundary of the corner pattern having 3, 4 and 5 homogeneous pixels such as (g), (h), (i) shown in Figure 8 includes inaccurate edge directions.

The block diagram of the proposed method is depicted in Figure 9. The boundary edge direction is calculated for pattern C having an accurate edge direction. But the boundary edge direction is not calculated for pattern A and pattern B having an inaccurate edge direction.

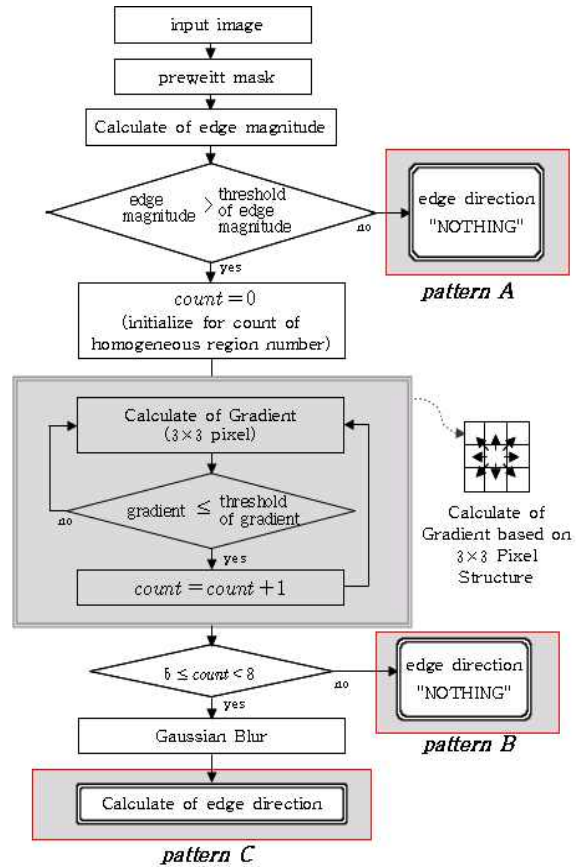


Figure 9. Process for detection of correct boundary-edge's directions

C. Weighted anisotropic diffusion based on boundary-edge's directions

The proposed diffusion exploiting an equation (8) is performed at 3x3 window filter including 8-directions (north, south, east, west, north_left, north_right, south_left, south_right) for preservation of boundary-edge and remove of noise .

$$I_s^{t+1} = I^t + \frac{\lambda}{|\eta_s|} \sum_{p \in \eta_s} g(\nabla I_{s,p}) \nabla I_{s,p}, \quad (8)$$

$$\eta_s = \begin{cases} \text{east, west, south, north} \\ \text{north_west, north_east,} \\ \text{south_west, south_east} \end{cases}$$

Weighting based on the predicted boundary-edge's direction is applied to the proposed edge-stopping function $g\|\nabla I\|$ at every iteration the proposed edge-stopping function $g\|\nabla I\|$ is as follows.

$$\begin{aligned} \text{If } \frac{\|\nabla I\|^2}{4} > K^2 \quad & \text{then } \frac{1}{1 + \left(\frac{\|\nabla I\|}{K}\right)^2} \\ \text{elseif } \text{pattern } C \quad & \text{then } 1 \\ \text{else } \text{pattern } A, B \quad & \text{then } \left[1 - \frac{\|\nabla I\|^2}{4K^2}\right]^2. \end{aligned}$$

K of edge stopping function is calculated by using GHAD[8]. In GHAD, the user specifies the desired number ν of edge-elements in the final filtered signal/image. Alternatively, the user can specify the proportion $\varpi = \nu/n$, where n is the number of possible edge-elements of the image. In each diffusion iteration, GHAD computes the absolute gradient values throughout the image. These gradient values are sorted in increasing order, generating the ordered sequence $\vec{v} = (v_0, \dots, v_{n-1})$. In order to make the output image to have ν edge elements, GHAD set $K = v_{n-1-\nu}$ in every iteration.

With this setting, the diffusion considers ν frontiers between neighboring pixels as edge-elements to be preserved, and $n - \nu$ frontiers as homogeneous regions where the noise must be suppressed.

All above criteria try to find a parameter K to appropriately separate edges from noises. However, we argue that the classification of a pair of neighboring pixels in “edge” or “noise” depends not only on the image to be filtered, but also on the desired scale of the filtered image. If the user wants to obtain a coarse-scale image with only the key edges, a large K should be specified. And if the user wants to obtain a fine-scale image with all the detailed edges, a small K should be chosen.

IV. EXPERIMENTS RESULT

Figure 10 depicts the restoration of a piecewise constant image corrupted by Gaussian noise. The original image Figure 10(a) has background gray level 183~184, with edge with intensities 147~149. This image was converted to floating-point range $[0, 1]$, and the zero-mean Gaussian noise with standard deviation 0.03 was added, resulting in Figure 10(b).

Diffusion is iterated until convergence and during diffusion the edge stopping functions of PM-inverse proportion method, PM-exponent method and GHAD method smudge many important edges (Figure 10 (c),(d),(e)). This happens because the grayscale intensity leaks continuously through smooth edges.

The proposed method is the best one in remaining contour of edge and removing noise between anisotropic diffusions methods (Figure 10 (f)).

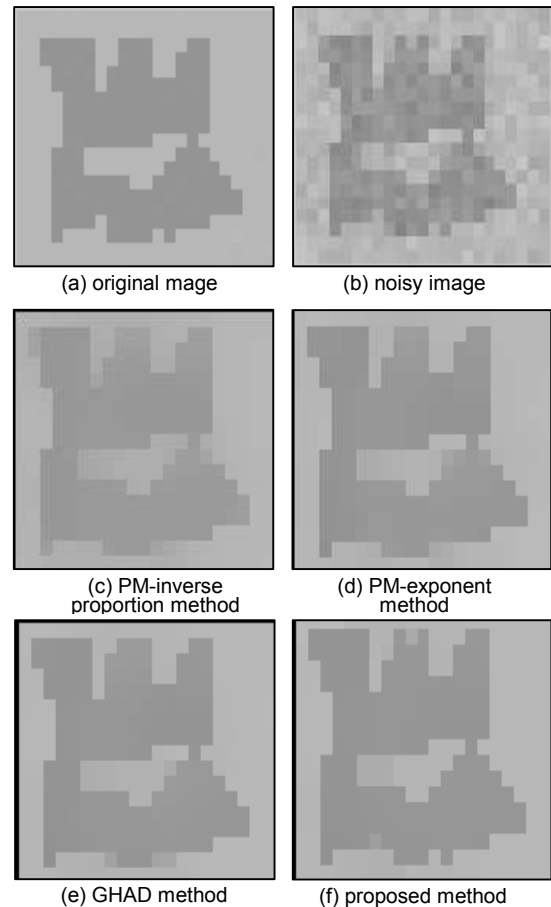


Figure 10. Restoration of a piecewise constant image corrupted by Gaussian noise

TABLE 1. COMPARISON OF PSNR ($K(\varpi) = 0.25\%$, $t = 10$)

Types of diffusions	Process time (sec)	PSNR (dB)
PM-inverse proportion method	0.1140	33.9285
PM-exponent method	0.1040	35.3033
GHAD method	0.0930	35.9793
proposed method	0.2500	37.5594

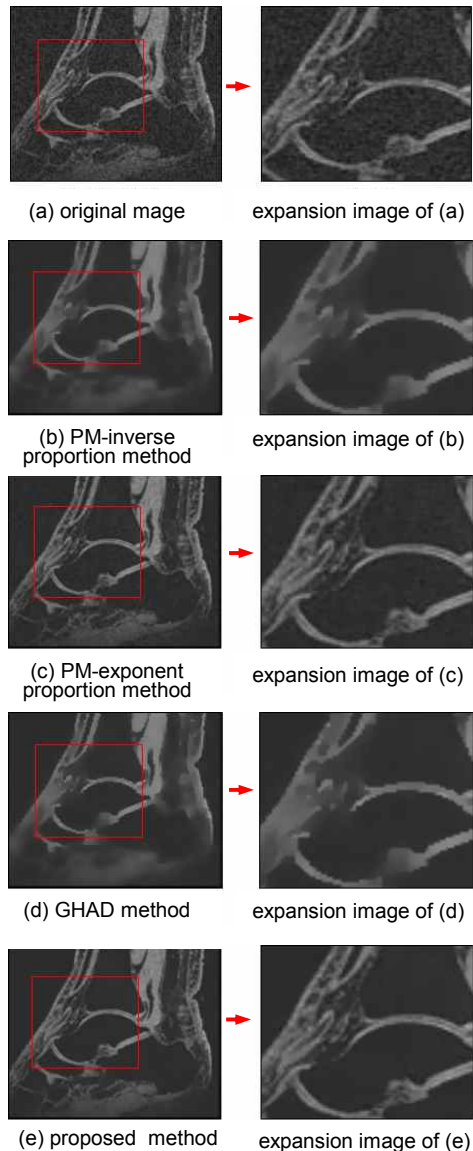


Figure 11. Visual comparison of experiments result in the magnetic resonance image

Our method has been tested by using magnetic resonance image with noise. In the filtered image of Figure 11, we can appreciate the noise reduction while most structures are still present in the image.

V. CONCLUSION

The anisotropic diffusion filters do not enhance edges. When any portion of the filter window contains an edge, the coefficient of variation will be high and smoothing is inhibited. Therefore, noise in the neighborhood of an edge is remained after filtering.

To overcome the shortcoming of these conventional anisotropic diffusion methods, a new anisotropic diffusion method based on directions of boundary-edge is proposed.

In this paper, Prewitt mask is performed for directions of boundary-edge. According to the directions of boundary-edge, diffusion is performed adaptively using weight to preserve edges and remove noises.

Experiments results show that the conventional inverse proportion method[2] and GHAD[8] cannot remain contour of edge. However, the proposed method is performed adaptively to preserve edges and remove noises.

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