

An Evolutionary Dynamics Game in Cooperative Spectrum Sensing

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Abstract—Cooperative spectrum sensing has greatly improved the performance of the cognitive radio. If the cognitive users belong to different groups, they seem to be less insensitive to achieve a higher throughput. In this paper , an evolutionary game framework is proposed to study the interactions between selfish users in the cooperative sensing .We compare the dynamics and the stationary strategy of the secondary users, and propose a distributed learning algorithm that helps the secondary users approach the Nash equilibrium with local payoff observation only. Simulation results show that, in the cooperative sensing game, the average throughput with more than two secondary users is higher than that when the secondary users sense the primary user individually without cooperation.

Keywords— cognitive radios, cooperative spectrum sensing, distributed learning algorithm , dynamics game

I. INTRODUCTION

As the demand for high data rate communications increase, traditional Fixed Spectrum Allocation(FSA) methods can not satisfy such services providing. That is why many spectrum regulatory bodies, such as the USFCC, are focusing on opening the licensing procedures using more flexible approaches such as secondary spectrum access based on deploying Cognitive Radios(CR)[1].

Spectrum sensing has become an essential function of cognitive radio devices[2], in order to protect the primary users from interference caused by secondary users' operation. Cooperative spectrum sensing with relay nodes's help and multiuser collaborative sensing has been shown to greatly improve the sensing performance[3]-[5]. In[6], the authors proposed that cooperation can reduce the detection time of the primary user and increase the overall agility in sensing. The author of [7]studied the spatial diversity in multiuser networks to improve spectrum sensing capabilities of centralized cognitive radio networks.

In most existing cooperative spectrum sensing schemes [3]-[7], the secondary users are generally assumed to belong to the same authority. They will fuse their sensing outcomes voluntarily to a centralized controller, such as the secondary base station. However, the the secondary users may be

selfish and do not serve a common goal, with the emerging applications of mobile adhoc networks envisioned in civilian usage. Since sensing the primary user frequency band also consumes a certain amount of energy and time, if multiple secondary users occupy different sub-bands and can over hear the others' sensing outcomes, they will have more time for their own data transmission.

In this paper we will study the interactions between the selfish users and their stationary strategy, and model the cooperative spectrum sensing as an evolutionary game.

If the secondary users agree to sense cooperately , the cost can be shared among them. while all of the secondary users will be punished by a very low payoff if no one senses the primary user. By using replicator dynamics in [8], we obtain the equations that show the users' behavior dynamics, and derive the equilibrium strategy when all secondary users are supposed homogeneous in their individual data rates and the received SNRs of the primary user. Further more, we develop a distributed learning algorithm that can help the secondary users only with their own payoff to approach their optimal strategy. The results of simulation show that the users tend to have less incentive to contribute to the cooperative sensing, as the number of secondary users and the cost of sensing increases. Meanwhile, they can still achieve a higher average throughput in the spectrum sensing game than that of the single-user sensing in the cognitive radio network.

The rest of this paper is organized as follows. The system model is presented in Section II. In Section III, we analyze the behavior dynamics of the secondary users in the cooperative spectrum sensing as an evolutionary game, and develop a distributed learning algorithm that approaches equilibrium. The results of simulation are shown in Section IV. Finally, Section V concludes the paper.

II. THE MODEL OF SYSTEM

When a secondary user is sensing the spectrum channel of the primary user in a cognitive radio network, there are two hypotheses for the received signal $r(t)$: absent or present ,denoted by H_0 and H_1 respectively. Then,

$$r(t) \begin{cases} hs(t) + w(t) & \text{if } H_1 \\ w(t) & \text{if } H_0 \end{cases} \quad (1)$$

In (1), h is the gain of the channel, $w(t)$ is an additive white Gaussian noise (AWGN) with mean zero and variance σ_w^2 ; $s(t)$ is the signal of the primary user, which is assumed to be a random process with mean zero and variance σ_s^2 ; and $w(t)$ and $s(t)$ are assumed to be mutually independent.

We use the energy detector to sense the licensed spectrum, then the test statistics $T(r)$ can be written as

$$T(r) = \frac{1}{N} \sum_{t=1}^N |r(t)|^2 \quad (2)$$

where N is the number of collected samples.

The sensing performance can be expressed by the probability of detection p_d and the probability of false alarm p_f respectively. In [9], the probability of false alarm p_f is given as

$$p_f(\lambda) = Q\left(\left(\frac{\lambda}{\sigma_w^2} - 1\right)\sqrt{N}\right) \quad (3)$$

where λ is the threshold of the energy detector, and $Q(\cdot)$ is the complementary distribution function of the standard Gaussian. If we assume the primary signal is a complex PSK signal, the probability of detection p_d is also given in [9] as

$$p_d(\lambda) = Q\left(\left(\frac{\lambda}{\sigma_w^2} - \gamma - 1\right)\sqrt{\frac{N}{2\gamma + 1}}\right) \quad (4)$$

where γ denotes the received signal-to-noise ratio (SNR) of the primary user when it is present.

While sensing the activity of primary user, the secondary users cannot transmit data at the same time. If f_s denotes the sampling frequency and T denotes the frame duration, then the time duration for data transmission is $T - \delta(N)$, where $\delta(N) = N/f_s$. When the licensed spectrum user is absent and there is no false alarm, the secondary user's average throughput is

$$R_{H_0}(N) = \frac{T - \delta(N)}{T} (1 - p_f) C_{H_0} \quad (5)$$

where C_{H_0} represents the data rate of the secondary user while the primary user is absent. When the licensed spectrum user is present and not detected, the average throughput of the secondary user is

$$R_{H_1}(N) = \frac{T - \delta(N)}{T} (1 - p_d) C_{H_1} \quad (6)$$

where C_{H_1} represents the data rate of the secondary user while the primary user is present.

Suppose p_{H_0} is the probability that the primary user is absent, then the total throughput of the secondary user can be written as

$$R(N) = p_{H_0} R_{H_0}(N) + (1 - p_{H_0}) R_{H_1}(N) \quad (7)$$

The goal usually is to maximize the total throughput (7). In practice the target detection probability $\bar{P}_d$ is required to be close to 1; moreover, we usually have $C_{H_1} < C_{H_0}$. Therefore, (7) can be approximated by

$$\tilde{R}(N) = p_{H_0} R_{H_0}(N) = p_{H_0} \frac{T - \delta(N)}{T} (1 - p_f) C_{H_0} \quad (8)$$

From (8), there is a tradeoff for a secondary user to choose an optimal N that maximizes the throughput $\tilde{R}(N)$. In order to keep a low p_f with a small N , a good choice is using the other secondary users in the same licensed band in the cooperative spectrum sensing.

III. SPECTRUM SENSING GAME PAGE

A. Problem Formulation

A snapshot of the cooperative spectrum sensing is shown in Fig.1. The entire licensed band is divided into K sub-bands, and each secondary user occupies exclusively in one of the K sub-bands while the primary user is absent. The data transmission is slotted in the intervals of length T . Before data transmission, the secondary users need to sense the primary user's activity. The secondary users can jointly sense the primary user's presence jointly, and exchange their sensing results through a narrow-band signalling channel, as shown in Fig.1. In this means, each secondary user can spend less time detecting while enjoying a low false alarm probability p_f via some decision fusion rule [10], and the spectrum sensing cost (N , or $\delta(N)$) can be shared by whoever is willing to contribute (C).

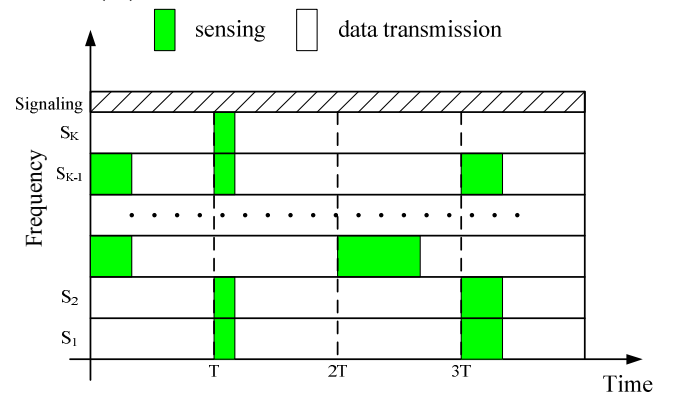


Figure 1. Cooperative spectrum sensing

When the secondary users donot serve for a common rule, they will obviously work selfishly by pursuing the highest throughput. If the secondary user is able to overhear the detection results from the other users, it will take use of that by denying D to take part in spectrum sensing. We assume that the users who deny to joint in cooperative sensing will have more time for their own data transmission, if each secondary user in the cognitive radio network achieves the same false alarm probability p_f . The secondary users, meanwhile, are punished by a very low throughput if no one cooperates in sensing, with the hope that someone else may sense the spectrum.

So, we can view the spectrum sensing as a noncooperative game. The the secondary users are the players of the game, denoted by $S = \{s_1, s_2, \dots, s_K\}$. Every player s_k follows the same strategy, denoted by $A = \{C, D\}$. Assume that the secondary users contributing in cooperative sensing forms a set $\xi_C = \{\xi_1, \xi_2, \dots, \xi_J\}$. So for a contributor $s_j \in \xi_C$, the payoff fuction can be defined as the throughput of the secondary user:

$$\tilde{U}_{C,s_j} = p_{H_0} \left(1 - \frac{\delta(N)}{|\xi_C|T} \right) (1 - p_f^{\xi_C}) C_{s_j}, \text{ if } \xi_c \in [1, K] \quad (9)$$

where $|\xi_c|$ represents the number of contributors, and C_{s_j} is the data rate for user s_j when the primary user is absent. We also assume that the sensing cost, $\delta(N)$ is equally shared by all contributors. So the p_f will be low for the group of contributors with a large number N . So the payoff for a denier s_i can be given as

$$\tilde{U}_{D,s_i} = p_{H_0} (1 - p_f^{\xi_C}) C_{s_i}, \text{ if } |\xi_c| \in [1, K-1] \quad (10)$$

From (5), we can consider $|\xi_c| = 0$ as the situation that no secondary user contributes to spectrum sensing and waits for the others to sense. In this case, the payoff for a denier becomes

$$\tilde{U}_{D,s_i} = 0, \text{ if } |\xi_c| = 0 \quad (11)$$

In this paper, we mainly focus on the logical-OR rule for the decision fusion rule to derive the $p_f^{\xi_C}$. Denote the detection and false alarm probability for a contributor $s_j \in \xi_C$ by p_{d,s_j} and p_{f,s_j} , then we have the following

$$p_d = 1 = \prod_{s_j \in \xi_C} (1 - p_{d,s_j}) \quad (12)$$

$$\text{and} \quad p_f = 1 = \prod_{s_j \in \xi_C} (1 - p_{f,s_j}) \quad (13)$$

Therefore, each individual user's targe detection probability can be expressed with $\bar{p}_d$ for a set ξ_C as

$$\bar{p}_{d,s_j} = 1 - (1 - \bar{p}_d)^{1/|\xi_C|} \quad (14)$$

Hence, from (5) we can write p_{f,s_j} as

$$p_{f,s_j} = Q \left(\sqrt{2\gamma_{s_j} + 1} Q^{-1}(\bar{p}_{d,s_j}) + \sqrt{N/|\xi_C|} \gamma_{s_j} \right) \quad (15)$$

where $Q^{-1}(\cdot)$ denotes the inverse function of $Q(\cdot)$.

B. Analysis of the Game

Since the data transmission for each secondary user is continuous, the spectrum sensing game will evolve over time. Therefore, we can use game theory to analyze the dynamics of the players and derive the equilibrium [8].

1) DynamicsoftheSensingGame

The key concept of evolutionary game is *replicator dynamics*, which describes the evolution of strategies in time. Specically, consider a large population of homogeneous individuals who are programmed to the same set of pure strategies A in a symmetric game with payoff function U . At time τ , let $p_{a_i}(t) \geq 0$ be the number of individuals who are currently programmed to pure strategy $a_i \notin \vartheta$, and let $p(t) = \sum p_{a_i}(t) > 0$ be the total population. Then the associated population state is defined as the vector $x(t) = \{x_{a_1}(t), \dots, x_{|A|}(t)\}$, where $x_{a_i}(t)$ idefined as the population share $x_{a_i}(t) = p_{a_i}(t)/p(t)$. By replicator dynamics, the evolution dynamics of $x_{a_i}(t)$ in given as the following differential equation

$$\dot{x}_{a_i} = \epsilon [\bar{U}(a_i, x_{-a_i}) - \bar{U}(x)] x_{a_i} \quad (16)$$

where $\bar{U}(a_i, x_{-a_i})$ is the instantaneous average payoff of the individuals using a_i , $\bar{U}(x)$ is the instantaneous average the whole population, and ϵ is some positive number representing the time scale. From (16), we can know that when the strategy a_i has a higher payoff than the average level, the population share using the same strategy will also grow. By analogy, we can view $x_{a_i}(t)$ as the probability that one player in a symmetric game adopts pure strategy a_i , and $x(t)$ can be equivalently viewed as a mixed strategy a for that player.

Then, we can generalize (16) to the spectrum sensing game with heterogeneous players, as C_{s_i} may vary among different users. Denote the probability that user s_j adopts strategy $h \in A$ at the time t by $x_{h,s_j}(t)$, then the time evolution of $x_{h,s_j}(t)$ is governed by the following differential equation:

$$\dot{x}_{h,s_j} = \frac{1}{\bar{U}_{s_j}(x)} [\bar{U}_{s_j}(h, x_{-s_j}) - \bar{U}_{s_j}(x)] x_{h,s_j} \quad (17)$$

where $\bar{U}_{s_j}(h_i, x_{-s_j})$ is the average payoff for player s_j using pure strategy h , and $\bar{U}_{s_j}(x)$ is s_j 's the average payoff using mixed strategy x_{s_j} .

2) Equilibrium Analysis

If each user s_j maximizes its total payoff by choosing the optimal probability of being a contributor x_{h,s_j} , where $h = C(OR D)$, the outcome of the game can be characterized by the Nash Equilibrium[8]. In Nash equilibria (NE), given that the other players adopt their equilibrium strategies, no player can gain a higher payoff value by unilaterally deviating from the equilibrium strategy.

In the cognitive radio network, most of the secondary users are assumed to be located faraway from the primary base station. So as we discussed, in order to guarantee low p_f , the number of sampled signals N should be large. Further assume that for all $s_i \in \xi$, all users have the same data rate, then the payoff functions in (10)-(12) become

$$U_C(J) = U_0 \left(1 - \frac{\tau}{J}\right), \quad \text{if } J \in [1, K] \quad (18)$$

$$\text{and } U_D(J) = \begin{cases} U_0, & \text{if } J \in [1, K-1] \\ 0, & \text{if } J = 0 \end{cases} \quad (19)$$

where $U_0 = p_{H_0}(1 - \hat{p}_f)C$, $J = |\xi_c|$.

According to the symmetry setting, (16) can be applied to the special case in which all players have the same evolution dynamics and equilibrium strategy. If we consider x as the probability of one secondary user contributes to spectrum sensing, the average payoff for pure strategy C can be obtained as

$$\bar{U}_C = \sum_{j=0}^{K-1} \binom{K-1}{j} x^j (1-x)^{K-1-j} U_C(j+1) \quad (20)$$

where $\binom{K-1}{j} x^j (1-x)^{K-1-j}$ is the probability of

$J+1$ users contribute to cooperative sensing. Similarly, the average payoff for pure strategy D can be given by

$$\bar{U}_D = \sum_{j=0}^{K-1} \binom{K-1}{j} x^j (1-x)^{K-1-j} U_D(j) \quad (22)$$

Therefore, the average payoff $\bar{U} = x\bar{U}_C + (1-x)\bar{U}_D$, so (16) becomes

$$\dot{x} = \epsilon x(1-x)(\bar{U}_C - \bar{U}_D) \quad (23)$$

C. Learning Algorithm for Nash Equilibrium

We can derive the strategy adjustment for the secondary user from (17). Denote the pure strategy taken by user s_j at time t by $A_{s_j}(t)$. Define an indicator function $1_{s_j}^h(t)$ as

$$1_{s_j}^h(t) = \begin{cases} 1, & \text{if } A_{s_j}(t) = h \\ 0, & \text{if } A_{s_j}(t) \neq h \end{cases} \quad (24)$$

At some interval mT , we can approximate $\bar{U}_{s_j}(h, x_{-s_j})$ by

$$\bar{U}_{s_j}(x) = \frac{\sum_{t \leq mT} \tilde{U}_{s_j}(A_{s_j}(t), A_{-s_j}(t)) 1_{s_j}^h(t)}{\sum_{t \leq mT} 1_{s_j}^h(t)} \quad (25)$$

where $\tilde{U}_{s_j}(A_{s_j}(t), A_{-s_j}(t))$ is the payoff value for s_j as determined by (9)-(11). Similarly, $\bar{U}_{s_j}(t)$ is approximated by

$$\tilde{U}_{s_j} = \frac{1}{m} \sum_{t \leq mT} \tilde{U}_{s_j}(A_{s_j}(t), A_{-s_j}(t)) \quad (26)$$

Then, the derivative $\dot{x}_{h,s_j}(mT)$ can be approximated by substituting (25) and (26) into (17). Therefore, the probability of user s_j taking action h can be adjusted by

$$\bar{U}_{s_j}(x) = \frac{1}{m} \sum_{t \leq mT} \tilde{U}_{s_j}(A_{s_j}(t), A_{-s_j}(t)) \quad (27)$$

with being the step size of adjustment chosen by s_j . We will demonstrate the convergence of the learning algorithm in the next section.

IV. SIMULATION RESULTS AND ANALYSIS

The parameters used in the simulation are as follows. We assume that the primary signal is a baseband QPSK modulated signal, the sampling frequency is $f_s = 4\text{MHz}$, and the frame duration is $T = 5\text{ms}$. The probability that the primary user is inactive is set as $p_{H_0} = 0.9$, and the required target detection probability is 0.95. The received γ_{s_j} 's are in the low SNR regime, with an average value of -12 dB.

We first illustrate the optimal equilibrium strategy for the secondary users assuming a homogeneous setting as in Section III-B2, where the data rate is $C = 1\text{Mbps}$. In Fig.2, we figure the optimal probability of being a contributor x^* with different number of secondary users. The x-axis represents the ratio of sensing time duration over the frame duration $\tau = \delta(N)/T$.

From Fig.2, we can see that x^* decreases as τ as increases. For the same τ , x^* decreases as the number of secondary users increases. This indicates that the increasing sensing cost and more users existing in the network are the incentives, which contribute to the drop in cooperative sensing. Because when players are faced with a high sensing cost and more counterpart players, they will tend to wait for others to sense the spectrum and can then enjoy a free ride.

Fig.3 shows the average throughput per user when all users adopt the equilibrium strategy. It is obviously that there is a tradeoff between the throughput and the sensing cost. The optimal value of τ is around 0.15, and will slightly increase as the number of user increases. This is because the false alarm probability p_f in (13) tends to increase as the number of user increases. So the users need to collect

more samples for better detection in order to have a low p_f . As more users share the increasing sensing cost, the optimal

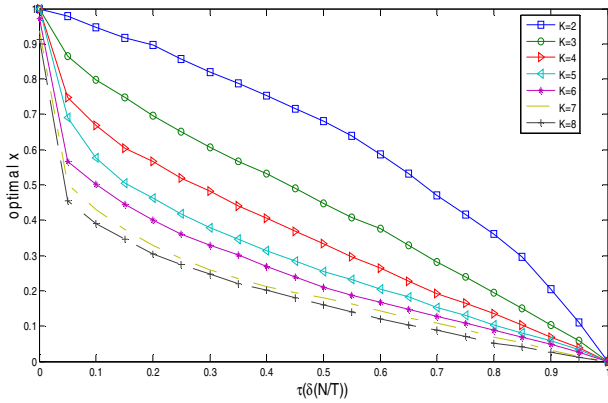


Figure 2. Probability of being a contributor

average throughput per user still has a bit of increase. We also plot the optimal throughput for the single-user sensing for comparison. It is interesting that the average throughput values for games more than 2 users are all higher than that of the single-user sensing, while the throughput for the 2-user game is not. The reason is that when there are more than 2 users in the game, the chance that no user contributes to sensing is smaller; it is more likely that neither user senses the spectrum in the 2-user game.

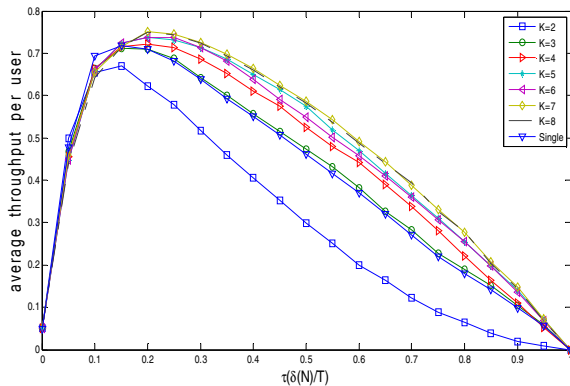


Figure 3. Average throughput per user

We finally show the learning curve for the probability of being a contributor in a 3-player game in Fig.4, with $\tau = 0.5$, the step-size of learning $\eta_{s_i} = 0.002$, $\gamma_1 = -13dB$, $\gamma_2 = -12dB$, and $\gamma_3 = -11dB$. We see that in the long run, all three users can gradually reach the equilibrium strategy, which is about 0.44.

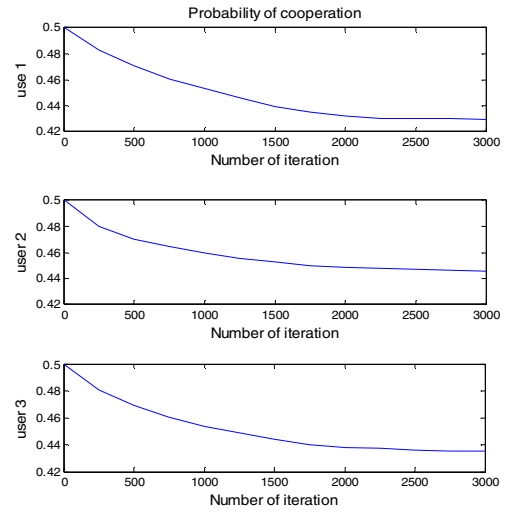


Figure 4. Learning curve for a 3-user sensing game

V. CONCLUSION

In this paper, we propose an evolutionary game-theoretical framework for distributed cooperative sensing in cognitive radio networks. By employing the theory of replicator dynamics, we study the behavior dynamics of secondary users, and further propose a distributed learning algorithm that gradually converges to the Nash equilibrium. From the simulation results, the average throughput per user in a K -user sensing game ($K > 2$) is still higher than that in the single-user sensing.

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