

# A Unified Framework for the Joint Optimal Allocation of Subchannel and Power in Multi-hop Relay Network.

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**Abstract**—This paper provides a unified framework for the joint optimal subchannel and resource allocation in multi-hop relay network. Specifically, we use a simple rearrangement inequality and show that seemingly difficult problems can be efficiently solved. This includes several existing solution methods as special cases. The proposed framework can be applied to the MIMO relaying and multi-hop OFDM.

## I. INTRODUCTION

Multi-hop relay network is a promising technology that can provide conventional wireless networks with various performance gains such as coverage extension and power efficiency. In several communication systems assisted by multi-hop relaying, parallel subchannels naturally arise, e.g. multi-hop OFDM and MIMO relaying [2], [8]. Under these circumstances, joint optimal allocation of subchannel and power further improve the system performance.

Herdin [2] have shown that pairing subchannels of the first hop and the second hop in the same order can increase the capacity of dual-hop OFDM without theoretical justification. Since then, several attempts have been made in an effort to prove the optimality of pairing the subchannels of both hops in the same order. Previous works in this direction have used the theory of majorization [1], [4]. Other works consider the joint design of subchannel pairing and power allocation, but its applicability is restricted to specific performance criteria, mainly capacity [3], [5].

In this paper, we provide a unified framework for the joint optimal allocation of subchannel and power. The existing design approaches in this area are rather ad-hoc in nature. The main contribution of this paper is that it unifies the existing design methods.

The paper is organized as follows. In Section II, we introduce the system model and the problem formulation of interest. Then, we present the main result of this paper in section III, which shows the problem posed in section II can be solved efficiently. In order to show the applicability and generality of our solution method, we give several design examples in section IV. Finally, we conclude this paper in section V. The following notations are used throughout this paper.  $\sigma(\cdot)$  and  $|\cdot|$  denote the permutation and the magnitude, respectively.

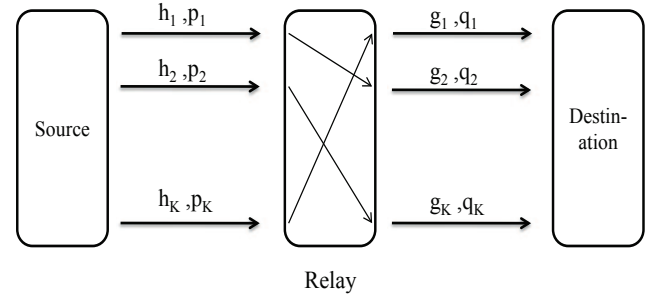


Figure 1. Two hop relay network

## II. SYSTEM MODEL AND REVIEW

We consider the two hop relay network consisting of source, relay and destination and each node has  $K$  parallel subchannels as shown in Fig. 1. For the sake of concreteness, one can think of these subchannels as OFDM or MIMO subchannel. In this model, source first transmits the information signal  $x_k$  on  $k$ th subchannel, and the received signal at relay is then given by

$$r_k = \sqrt{\frac{P_s}{K}} h_k x_k + n_k \quad (1)$$

where  $n_k$ ,  $h_k$ , and  $P_s$  denote additive white Gaussian noise with variance 1, the channel gain between source and relay on  $k$ th subchannel, and the maximum power available at source, respectively. After receiving the signal, the relay allocates  $q_k$  amount of power into  $k$ th subchannel and then forwards it on the  $\sigma(k)$ th subchannel toward destination. The received signal at the destination is finally given by

$$y_{\sigma(k)} = \sqrt{q_k} g_{\sigma(k)} s_{\sigma(k)} + z_{\sigma(k)} \quad (2)$$

where  $z_k$ ,  $g_k$ , and  $s_k$  denotes additive white Gaussian noise with variance 1, the channel gain between relay and destination on  $k$ th subchannel, and the transmit signal at the relay, respectively.

In this paper, we consider two popular choices of relaying schemes which are decode and forward (DF) relaying and amplifying and forward (AF) relaying. Depending on which

relaying mode is considered,  $s_k$  can be chosen as follows

$$s_k = \begin{cases} x_k & \text{for DF} \\ r_k & \text{for AF} \end{cases} \quad (3)$$

The above model can be applied in the MIMO relaying where canonical transmission turns out to be a parallel subchannel and Multi-hop OFDM.

Let  $S_K$  be the set of all such permutations on  $\{1, \dots, K\}$ . The problem we are interested in is then given by

$$\begin{aligned} \max_{\sigma \in S_N, \mathbf{q}} \quad & \sum_{k=1}^K f_k \left( |g_{\sigma(k)}|^2 q_k \right) \\ \text{s.t.} \quad & q_k \geq 0, \sum_k q_k \leq P_r \end{aligned} \quad (4)$$

where  $f_k$  and  $P_r$  denote an arbitrary objective function that we are interested in and the maximum power available at relay, respectively. The above problem turns out to be a difficult mixed integer nonlinear programming. In principle, the above problem can be solved by calculating optimal power for all possible choices of permutations, which is, however, prohibitive due to its exponential complexity.

The next section shows that the above problem can be simply solved if the objective function satisfies certain properties. As will be shown in section III, many interesting examples satisfy these properties.

### III. JOINT OPTIMAL DESIGN OF SUBCHANNEL MATCHING AND POWER CONTROL

In this section, we provide a general result on the joint optimal subchannel and power allocation. The following lemma, which is a generalized version of the well-known rearrangement inequality in mathematics, is useful for the subsequent analysis.

*Lemma 1:* Let  $f_1, \dots, f_K$  be real valued functions defined on an interval  $I$ . Then,

$$\sum_{k=1}^K f_k(x_{N-k+1}) \leq \sum_{k=1}^K f_k(x_{\sigma(k)}) \leq \sum_{k=1}^K f_k(x_k) \quad (5)$$

for any sequences  $x_1 \leq \dots \leq x_K$  and permutations  $\sigma \in S_N$  if and only if

$$f_{k+1} - f_k \text{ is nondecreasing on } I, 1 \leq k < K \quad (6)$$

*Proof:* The proof is simple, and interested readers may refer to [6]. ■

With the help of lemma 1, we can now present the main result of this paper.

*Theorem 1:* If  $f_{k+1}(x) - f_k(x)$  is nondecreasing, the optimal permutation  $\sigma^*$  is given by

$$|g_{\sigma^*(1)}|^2 \leq \dots \leq |g_{\sigma^*(K)}|^2 \quad (7)$$

and the problem (4) can be simplified into

$$\begin{aligned} \max_{\mathbf{q}} \quad & \sum_{k=1}^K f_k \left( |g_{\sigma^*(k)}|^2 q_k \right) \\ \text{s.t.} \quad & q_k \geq 0, \sum_k q_k \leq P_r \end{aligned} \quad (8)$$

*Proof:* The proof is omitted. ■

It is worthwhile noting that several previous works have used the majorization theory to prove the optimality of pairing subchannels in the same order. The above theorem is more general than those results and includes the results in [1], [4] as special cases.

### IV. DESIGN EXAMPLES

In this section, we show how theorem 1 helps us find the optimal solution to (4) through specific examples.

*Example 1:* (AF capacity maximization)

Let  $\pi$  be a permutation such that

$$|h_{\pi(1)}|^2 \leq \dots \leq |h_{\pi(K)}|^2 \quad (9)$$

and set  $f_k(x) = \log \left( 1 + \frac{P_s |h_{\pi(k)}|^2}{K} \frac{x}{\alpha_{\pi(k)} + x} \right)$ , where  $\alpha_k = 1 + \frac{P_s |h_k|^2}{K}$ . Then, AF capacity maximization problem can be expressed as [5]

$$\begin{aligned} \max_{\sigma \in S_N, \mathbf{q}} \quad & \sum_{k=1}^K \log \left( 1 + \frac{P_s/K |h_{\pi(k)}|^2 |g_{\sigma(k)}|^2 q_k}{\alpha_{\pi(k)} + |g_{\sigma(k)}|^2 q_k} \right) \\ \text{s.t.} \quad & q_k \geq 0, \sum_k q_k \leq P_r \end{aligned} \quad (10)$$

It is easy to check  $\frac{d}{dx} (f_{k+1}(x) - f_k(x)) \geq 0$  and hence  $f_{k+1}(x) - f_k(x)$  is nondecreasing. Indeed, taking derivative and then disregarding the denominator, we have

$$\begin{aligned} & \frac{P_s |h_{\pi(k+1)}|^2}{K} \alpha_{\pi(k+1)} \left\{ (\alpha_{\pi(k)} + x)^2 + \frac{P_s |h_{\pi(k)}|^2 x^2}{K} \right\} \\ & - \frac{P_s |h_{\pi(k)}|^2}{K} \alpha_{\pi(k)} \left\{ (\alpha_{\pi(k+1)} + x)^2 + \frac{P_s |h_{\pi(k+1)}|^2 x^2}{K} \right\} \\ & \geq 0 \end{aligned}$$

Therefore, theorem 1 tells us that we can always choose  $\sigma^*$  such that (7) holds and the we only have to solve the following simple one.

$$\begin{aligned} \max_{\mathbf{q}} \quad & \sum_{k=1}^K \log \left( 1 + \frac{P_s/K |h_{\pi(k)}|^2 |g_{\sigma^*(k)}|^2 q_k}{\alpha_{\pi(k)} + |g_{\sigma^*(k)}|^2 q_k} \right) \\ \text{s.t.} \quad & q_k \geq 0, \sum_k q_k \leq P_r \end{aligned} \quad (11)$$

which is a convex problem that can be solved efficiently. It is interesting to note that [8] adopts the brute force search to solve (10). However, our result says that brute force search is not necessary, thereby greatly reducing the search complexity.

*Example 2: (MSE minimization)*

Let  $\pi$  be as in (9) and set  $f_k(x) = -\frac{\alpha_{\pi(k)} + x}{\alpha_{\pi(k)} + \alpha_{\pi(k)}x}$ , where  $\alpha_k = 1 + \frac{P_s |h_k|^2}{K}$ . The MSE minimization problem for AF relaying can be posed as [7]

$$\begin{aligned} \max_{\sigma \in S_N, \mathbf{q}} & - \sum_{k=1}^K \frac{\alpha_{\pi(k)} + |g_{\sigma(k)}|^2 q_k}{\alpha_{\pi(k)} + \alpha_{\pi(k)} |g_{\sigma(k)}|^2 q_k} \\ \text{s.t. } & q_k \geq 0, \sum_k q_k \leq P_r \end{aligned} \quad (12)$$

By checking the derivative,  $f_{k+1}(x) - f_k(x)$  is nondecreasing.

$$\begin{aligned} & \frac{d}{dx} (f_{k+1}(x) - f_k(x)) \\ &= \frac{x(\alpha_{\pi(k+1)} - \alpha_{\pi(k)})}{\alpha_{\pi(k)} \alpha_{\pi(k+1)} (1+x)} \geq 0 \end{aligned}$$

Therefore, we can use theorem 2 and problem (12) simplifies to

$$\begin{aligned} \max_{\mathbf{q}} & - \sum_{k=1}^K \frac{\alpha_{\pi(k)} + |g_{\sigma^*(k)}|^2 q_k}{\alpha_{\pi(k)} + \alpha_{\pi(k)} |g_{\sigma^*(k)}|^2 q_k} \\ \text{s.t. } & q_k \geq 0, \sum_k q_k \leq P_r \end{aligned} \quad (13)$$

where  $\sigma^*$  is a permutation that satisfies (7).

*Example 3: (DF capacity maximization)*

For DF relaying, the objective is given by

$$f_k(x) = \log \left( 1 + \min \left\{ \frac{P_s |h_{\pi(k)}|^2}{K}, x \right\} \right) \quad (14)$$

and the corresponding problem is posed as

$$\begin{aligned} \max_{\sigma \in S_N, \mathbf{q}} & \sum_{k=1}^K \log \left( 1 + \min \left\{ P_s |h_{\pi(k)}|^2 / K, \right. \right. \\ & \left. \left. |g_{\sigma(k)}|^2 q_k \right\} \right) \\ \text{s.t. } & q_k \geq 0, \sum_k q_k \leq P_r \end{aligned} \quad (15)$$

where  $\pi$  is a permutation satisfying (9). By definition of  $f_k(x)$ , it is easy to observe that  $f_k(x)$  is nondecreasing. To verify that  $f_{k+1}(x) - f_k(x)$  is also nondecreasing, let  $x \geq y$ . There are six cases we have to consider depending on the relations on  $x, y, \frac{P_s |h_k|^2}{K}$ , and  $\frac{P_s |h_{k+1}|^2}{K}$ . For example, for the case when

$$\begin{aligned} y \leq \frac{P_s |h_{\pi(k)}|^2}{K} \leq x \leq \frac{P_s |h_{\pi(k+1)}|^2}{K} \text{ we have} \\ \{f_{k+1}(x) - f_k(x)\} - \{f_{k+1}(y) - f_k(y)\} \\ = \log \left( \frac{1 + \min \left\{ \frac{P_s |h_{\pi(k+1)}|^2}{K}, x \right\}}{1 + \min \left\{ \frac{P_s |h_{\pi(k+1)}|^2}{K}, y \right\}} \right) \\ + \log \left( \frac{1 + \min \left\{ \frac{P_s |h_{\pi(k)}|^2}{K}, y \right\}}{1 + \min \left\{ \frac{P_s |h_{\pi(k)}|^2}{K}, x \right\}} \right) \\ = \log \left( \frac{(1+x)}{\left(1 + \frac{P_s |h_{\pi(k)}|^2}{K}\right)} \right) \geq 0 \end{aligned}$$

Using the similar method for the other five cases, we can conclude that  $f_{k+1}(x) - f_k(x)$  is nondecreasing. So, the problem (15) is simplified into

$$\begin{aligned} \max_{\mathbf{q}} & \sum_{k=1}^K \log \left( 1 + \min \left\{ P_s |h_{\pi(k)}|^2 / K, \right. \right. \\ & \left. \left. |g_{\sigma^*(k)}|^2 q_k \right\} \right) \\ \text{s.t. } & q_k \geq 0, \sum_k q_k \leq P_r \end{aligned}$$

Since the objective has nondifferentiable points, we transform the above problem to the one with differentiable objective function. To this end, we notice that optimum is attained when  $|g_{\pi(k)}|^2 q_k \leq \frac{P_s |h_{\pi(k)}|^2}{K}$ . So, the problem is equivalent to solve the following one

$$\begin{aligned} \max_{\mathbf{q}} & \sum_{k=1}^K \log \left( 1 + |g_{\sigma^*(k)}|^2 q_k \right) \\ \text{s.t. } & q_k \geq 0, \sum_k q_k \leq P_r \\ & |g_{\sigma^*(k)}|^2 q_k \leq \frac{P_s |h_{\pi(k)}|^2}{K} \end{aligned} \quad (16)$$

which is a convex problem whose solution can be obtained efficiently. Again,  $\sigma^*$  is a permutation that satisfies (7).

In addition to the examples listed above, the proposed method can be used to find optimal or suboptimal solution in many other problems such as BER minimization, etc.

## V. CONCLUSIONS

We have proposed a unified framework for joint optimal allocation of subchannel and power, the proposed one includes and generalizes the existing previous works. We also showed that many interesting problems fall into the proposed framework, thereby yielding simple solution methods despite of the difficult nature of the original problem.

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