

Inter-channel Interference correlation matrix in OFDM system based on GCE-BEM

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Abstract—Orthogonal Frequency Division Multiplexing (OFDM) is regarded as a feasible alternative for high data rate communication systems, mainly because of its inherent equalization simplicity. Based on the channel model described by generalized complex exponential basis expansion model (GCE-BEM) in OFDM systems, which is more accurate than the conventional complex exponential basis expansion model (CE-BEM), we calculate the correlation matrix of inter-channel interference (ICI) in the channel estimation method described in [1] and [3]. The correlation matrix is verified to be a scaled identity matrix under the same assumption as in [2], the property of which is a premise of optimal pilot insertion scheme proposed in [2].

Keywords—OFDM, GCE-BEM, ICI, correlation matrix

I. INTRODUCTION

The demand for high data rate transmissions calls for broadband communication systems. When the bandwidth gets large, the sampling period can become smaller than the delay spread of the channel, especially in multipath scenarios, which gives rise to frequency-selective channels. Carrier frequency-offset due to the oscillators' mismatch, together with high relative mobility between the transmitter and receiver causes the transmission channel to change rapidly in time, which is referred to as the time-selectivity of the channel. The frequency-selective channel conjunction with the time-selective channel which is called doubly-selective channel in wireless communications can introduce intersymbol interference (ISI) and intercarrier interference (ICI) in Orthogonal Frequency Division Multiplexing (OFDM) systems. In an OFDM system, the bandwidth of each subcarrier is small enough to be considered to experience a flat fading in a frequency-selective channel. The narrowband nature of subcarriers makes the signal robust against frequency selectivity. OFDM can transform the time-variant frequency-selective channel into a set of time-variant frequency-flat subcarriers. However OFDM is relatively sensitive to time selectivity of a mobile channel. Time variations of the channel within an OFDM symbol duration result in ICI and lead to an irreducible error floor in conventional receivers. The doubly selective (both time- and frequency-selective) channel estimation is essential for

channel compensation and data detection. Based on the basis expansion model (BEM), several channel estimation algorithms have been proposed for doubly selective channel [1-3].

Recently, the BEM is used to represent the time-varying channel for its parsimony of coefficients to be estimated and its algebraic ease. In mobile communication systems with OFDM, the CE-BEM is often used to represent the time-variant channel for its parsimony of coefficients to be estimated and its algebraic ease. The CE-BEM channel model, however, can bring about high modeling error because the channel estimation period of CE-BEM is limited to the length of one OFDM symbol. An improved channel model so called generalized CE-BEM (GCE-BEM) can provide lower modeling error than CE-BEM. The frequency resolution of GCE-BEM also can be increased arbitrarily at the expense of higher computational complexity.

Based on the GCE-BEM channel models, pilot-assisted time-varying channel estimators are proposed for OFDM systems in [1] and [3]. The pilot insertion scheme used in [3] is depicted in Fig. 1. If the pilot clusters are inserted equidistantly in the frequency domain in OFDM systems with the CE-BEM channel model adopted, the pilot insertion scheme in [1] can be verified to be an optimal one in the sense of minimum mean square error under the premise of the correlation matrix of the inter-channel interference being a scaled identity matrix. But can it be still a scaled identity matrix if the more accurate GCE-BEM channel model is adopted? In this paper, we calculate the ICI correlation matrix based on GCE-BEM and verify that it can be a scaled identity matrix under the assumptions normally used in the literatures. The property of the ICI correlation matrix being a scaled identity matrix is very useful for the analysis and design of optimal pilot insertion schemes in OFDM systems [2].

II. DATA MODEL

We assume the typical wide-sense stationary uncorrelated scattering (WSSUS) model such that

$$E\{h(n,l)h^*(n+\Delta n, l+\Delta l)\} = r(\Delta n)\sigma_l^2\delta(\Delta l) \quad (1)$$

Where $r(\Delta n)$ denotes the normalized tap autocorrelation, $\delta(\cdot)$ is Dirac's delta function and σ_l^2 denotes the variance of the l th tap. We denote T_s as sampling period, $t_{\max}$ and $f_{\max}$ as maximum delay spread and maximum Doppler frequency,

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respectively, which are assumed to be known at the receiver beforehand, so DFT of $h(n,l)$, i.e., $H(k,l) = 0$ for $k\Delta f > f_{\max}$ or $l\Delta t > t_{\max}$, where Δf and Δt are Doppler frequency resolution and delay spread resolution, respectively. Now we adopt the parsimonious GCE-BEM to approximate the channel's time variation in a discrete form.

We consider an OFDM system with N subcarriers and denote the channel order by L . The discrete-time impulse response of the time-varying channel can be expressed by GCE-BEM as

$$h(n,l) = \sum_{q=-Q}^Q h_q(l) e^{j\omega_q n} \quad (2)$$

where $h_q(l)$ is the BEM coefficient of the l th channel path, $l \in \{0, \dots, L-1\}$, and n is the time index. The order of BEM is $2Q+1$ and $\omega_q = 2\pi q/N_s$ where $N_s = G(N+L)$ is the total number of symbols in G OFDM blocks with N symbols for data and L symbols for the cyclic prefix in one OFDM block, which defines the size of the observation window. If $G = 1$, the GCE-BEM of (1) will be degraded to CE-BEM.

The pilot insertion scheme proposed in [3] is shown in Fig. 1.

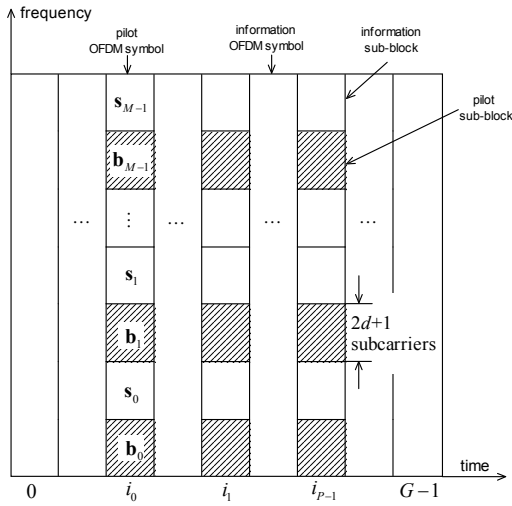


Figure 1. The pilot insertion scheme for channel estimation

It can be proved that the pilot insertion scheme in Fig. 1 is an optimal one in the sense of minimum mean square error under the premise of the correlation matrix of the ICI being a scaled identity matrix.

The optimal condition 1) the pilot tones must be equipowered, 2) the pilot tones must be equispaced, 3) the number of pilot sub-blocks in any pilot symbol must be the same, 4) the pilot symbols must be equispaced in the G OFDM blocks.

III. CALCULATION OF THE CORRELATION MATRIX

Before calculating the ICI correlation matrix, we make following two assumptions, the rationale of which is explained in [2].

A1) Data symbols are of zero mean and unit variance, and are uncorrelated with each other, i.e. $E\{\mathbf{s}_d \mathbf{s}_d^H\} = \mathbf{I}_D$, where $\mathbf{I}_D$ is a $D \times D$ identity matrix.

A2) The BEM coefficients $h_q(l)$ are complex Gaussian random variables with zero mean and variance $\sigma_{q,l}^2$ and are uncorrelated with each other for different q and l .

Now we calculate the autocorrelation matrix of $\mathbf{Ici}$, i.e. $E\{\mathbf{Ici}(\mathbf{Ici})^H\}$, where $\mathbf{Ici} = [\mathbf{Ici}(i_0), \dots, \mathbf{Ici}(i_{p-1})]^T$. We omit the i_p in $\mathbf{F}_L^d(i_p)$ below.

$$\begin{aligned} \mathbf{Ici}(i_p) &= \sum_{q=-Q}^Q \Psi_q^d(i_p) \text{diag}\{\mathbf{s}(i_p)\} \mathbf{F}_L^d \mathbf{h}_q \\ &= \sum_{q=-Q}^Q \tilde{\mathbf{F}}(i_p) \Gamma_q \bar{\mathbf{F}}(i_p) \text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \mathbf{s}(i_p) \end{aligned} \quad (3)$$

Where $\mathbf{F}$ denotes $N \times N$ unitary discrete Fourier transform matrix and the N by $(L+1)$ matrix $\mathbf{F}_L$ is $\sqrt{N}$ times the first $L+1$ columns of $\mathbf{F}$, the $M_{ip} \times N$ matrix $\tilde{\mathbf{F}}(i_p)$ is the corresponding M_{ip} rows of $\mathbf{F}$ according to the position of pilots in the i_p th OFDM block, and the $N \times d_D$ matrix $\bar{\mathbf{F}}(i_p)$ is the corresponding d_D columns of $\mathbf{F}^H$ according to the position of data in the i_p th OFDM block. The N by N matrix $\Psi_q^{(i)} := \mathbf{F}^H \Gamma_q^{(i)} \mathbf{F}$ the element of $\Psi_q^{(i)}$ can be shown as $[\Psi_q^{(i)}]_{m,n} = \varphi_q^{(i)}(m-n)$,

where $\varphi_q^{(i)}(k) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi q(i(N+L)+n)/N_s} e^{-j2\pi kn/N}$. Note that

$\varphi_q^{(i)}(0)$ appears on the main diagonal of $\Psi_q^{(i)}$, $\varphi_q^{(i)}(1)$ on the first sub-diagonal, $\varphi_q^{(i)}(-1)$ on the first super-diagonal, and so on. $\Psi_q^{(i)}$ is the ICI matrix induced by the time variations of the channel. If the channel is time invariant, then $Q = 0$ and $\Psi_q^{(i)}$ will reduce to an identity matrix. The detail explanation of (3) can be found in [3].

According to the optimal pilot insertion condition, the number of pilot sub-blocks in any pilot symbol is the same and we omit the subscript ip of M_{ip} below.

We assume that the BEM coefficients $h_q(l)$ are zero mean, complex Gaussian random variables with variance $\sigma_{q,l}^2$.

The autocorrelation matrix of $\mathbf{Ici}$ is

$$E\{\mathbf{Ici}(\mathbf{Ici})^H\} = \begin{bmatrix} \mathbf{A}_{0,0} & \dots & \mathbf{A}_{0,P-1} \\ \vdots & \dots & \vdots \\ \mathbf{A}_{P-1,0} & \dots & \mathbf{A}_{P-1,P-1} \end{bmatrix} \quad (4)$$

Where $\mathbf{A}_{p1,p2}$ ($p1, p2 \in \{0, \dots, P-1\}$) is the correlation matrix of $\mathbf{Ici}(i_{p1})$ and $\mathbf{Ici}(i_{p2})$, i.e. $\mathbf{A}_{p1,p2} = E\{\mathbf{Ici}(i_{p1})(\mathbf{Ici}(i_{p2}))^H\}$.

$\forall p1, p2 \in \{0, \dots, P-1\}$ and $p1 \neq p2$, $\mathbf{A}_{p1,p2} = \mathbf{0}$ for $E\{\mathbf{s}(i_{p1})(\mathbf{s}(i_{p2}))^H\} = \mathbf{0}$.

In the case of the same pilot OFDM block, the autocorrelation matrix of $\mathbf{Ici}(i_p)$ is

$$\mathbf{A}_{p,p} = E\left\{\left(\sum_{q=-Q}^Q \Psi_q^d(i_p) \text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \mathbf{s}(i_p)\right) \left(\sum_{q=-Q}^Q \Psi_q^d(i_p) \text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \mathbf{s}(i_p)\right)^H\right\} \quad (5)$$

Because of the independence assumption of $h_q(l)$, (A3) can be expressed as

$$\mathbf{A}_{p,p} = E \left\{ \sum_{q=-Q}^Q \left(\mathbf{\Psi}_q^d(i_p) \text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \mathbf{s}(i_p) \left(\mathbf{s}(i_p) \right)^H \left(\text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \right)^H \left(\mathbf{\Psi}_q^d(i_p) \right)^H \right) \right\} \quad (6)$$

We calculate (A4) from inner to outside.

$$E \left\{ \mathbf{s}(i_p) \left(\mathbf{s}(i_p) \right)^H \right\} = \mathbf{I}_{d_D} \quad (7)$$

The m th diagonal element of

$$E \left\{ \text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \left(\text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \right)^H \right\} \text{ is } E \left\{ \left(\sum_{l=0}^L e^{jlm/N} h_q(l) \right) \left(\sum_{l=0}^L e^{-jlm/N} h_q(l) \right)^H \right\} = \sum_{l=0}^L E \left\{ |h_q(l)|^2 \right\} = \sum_{l=0}^L \sigma_{q,l}^2$$

which is independent of m , so

$$E \left\{ \text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \left(\text{diag}\{\mathbf{F}_L^d \mathbf{h}_q\} \right)^H \right\} = \left(\sum_{l=0}^L \sigma_{q,l}^2 \right) \mathbf{I}_{d_D} \quad (8)$$

Finally, we calculate

$$\mathbf{\Phi}_q(i_p) := E \left\{ \mathbf{\Psi}_q^d(i_p) \left(\mathbf{\Psi}_q^d(i_p) \right)^H \right\} = \mathbf{\Psi}_q^d(i_p) \left(\mathbf{\Psi}_q^d(i_p) \right)^H.$$

$$\mathbf{\Phi}_q(i_p) = \tilde{\mathbf{F}}(i_p) \mathbf{\Gamma}_q \bar{\mathbf{F}}(i_p) \left(\bar{\mathbf{F}}(i_p) \right)^H \mathbf{\Gamma}_q^H \left(\tilde{\mathbf{F}}(i_p) \right)^H.$$

Where $\bar{\mathbf{F}}(i_p)$ is the corresponding d_D columns part of IFFT matrix $\mathbf{F}^H$ according to the position of data in the i_p th OFDM block. The i th row the j th column element of $\mathbf{\Phi}_q(i_p)$ can be written as

$$(\mathbf{\Phi}_q(i_p))_{(i,j)} = \frac{1}{N^2} \sum_{n=n_0}^{n_{dD-1}} \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \exp \left[-j2\pi \left(\frac{mk_1 - m_k k_2 - n(k_1 - k_2)}{N} + \frac{q(k_1 - k_2)}{N_s} \right) \right] \quad (9)$$

Where n_a ($\forall a \in \{0, \dots, d_D - 1\}$) is the index of data symbol subcarrier in the p th OFDM block, m_b ($\forall b \in \{0, \dots, M - 1\}$) is the index of non zero pilot symbol subcarrier in the p th OFDM block. We note that $(\mathbf{\Phi}_q(i_p))_{(i,j)}$ is independent of i_p , so the diagonal sub-matrices in (4) are equal, that is $\mathbf{A}_{p,p} = \mathbf{A}_{q,q}$, $\forall p, q \in \{0, \dots, P - 1\}$.

In case of time invariant channel, $Q=0$, (A7) becomes

$$(\mathbf{\Phi}_q(i_p))_{(i,j)} = \frac{1}{N^2} \sum_{n=n_0}^{n_{dD-1}} \sum_{k_1=0}^{N-1} \exp \left[-j2\pi \frac{(m_i - n)k_1}{N} \right] \sum_{k_2=0}^{N-1} \exp \left[-j2\pi \frac{(n - m_j)k_2}{N} \right] \quad (10)$$

The $(n - m_j)$ in (10) is an integer, so the last summation in (10) is zero, and $(\mathbf{\Phi}_q(i_p))_{(i,j)} = 0$, $\forall i \neq j$, that means the case of no ICI.

In case of $G=1$, $N_s=N$, (A7) becomes

$$(\mathbf{\Phi}_q(i_p))_{(i,j)} = \frac{1}{N^2} \sum_{n=n_0}^{n_{dD-1}} \sum_{k_1=0}^{N-1} \exp \left[-j2\pi \frac{(m_i - n + q)k_1}{N} \right] \sum_{k_2=0}^{N-1} \exp \left[-j2\pi \frac{(n - m_j - q)k_2}{N} \right] \quad (11)$$

The $(n - m_j)$ in (11) is an integer, and the last summation in (11) is also zero, and $(\mathbf{\Phi}_q(i_p))_{(i,j)} = 0$, $\forall i \neq j$.

Now we calculate the diagonal element of $\mathbf{\Phi}_q(i_p)$, let $m_i = m_j = m$, then (9) becomes

$$(\mathbf{\Phi}_q(i_p))_{(i,i)} = \frac{1}{N^2} \sum_{n=n_0}^{n_{dD-1}} \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \exp \left[-j2\pi \left(\frac{-(n-m)(k_1 - k_2)}{N} + \frac{q(k_1 - k_2)}{N_s} \right) \right] \quad (12)$$

We note the item $(n-m)$ in (12), the m only affects the beginning position of data sub-block in the pilot OFDM block and the position of data sub-block is chosen uniformly, so it does not affect the result of summation, we can omit the m , and (12) is independent of m , which means the diagonal element of $\mathbf{\Phi}_q(i_p)$ is the same. The non-diagonal elements of $\mathbf{\Phi}_q(i_p)$ are not zero in the case of $Q \neq 0$ and $G \neq 1$ but they are rather small compared with the diagonal ones. For example, when $N_s = G(N+L) = 64(64+7)$, $q=5$, the ratio of non-diagonal element over diagonal element is smaller than 10^{-5} . So $\mathbf{\Phi}_q(i_p)$ can be expressed as

$$\mathbf{\Phi}_q(i_p) \approx c_2 \mathbf{I}_M \quad (13)$$

Where

$$c_2 = \frac{1}{N^2} \sum_{n=n_0}^{n_{dD-1}} \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \exp \left[-j2\pi \left(\frac{-n(k_1 - k_2)}{N} + \frac{q(k_1 - k_2)}{N_s} \right) \right]$$

Substitute (7), (8) and (13) in (6), we have $\mathbf{A}_{p,p} \approx \sigma_l^2 \mathbf{I}_M$, where

$$\sigma_l^2 = \frac{d_D}{N^2} \sum_{q=-Q}^Q \sum_{n=n_0}^{n_{dD-1}} \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \exp \left[-j2\pi \left(\frac{-n(k_1 - k_2)}{N} + \frac{q(k_1 - k_2)}{N_s} \right) \right] \left(\sum_{q=-Q}^Q \sum_{l=0}^L \sigma_{q,l}^2 \right).$$

Then the autocorrelation matrix of $\mathbf{Ici}$ is

$$E \left\{ \mathbf{Ici}(\mathbf{Ici})^H \right\} \approx \sigma_l^2 \mathbf{I}_{MP} \quad (14)$$

IV. CONCLUSIONS

We have calculated the correlation matrix of ICI vector introduced by time variant channel modeled by GCE-BEM in the channel estimation method described in [1] and [3]. The scaled identity property of the ICI matrix is a premise of optimal pilot insertion scheme in some literatures. We verified that the correlation matrix is a scaled identity matrix approximately.

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