

Performance Analysis of LDPC codes Over WS-EWC coded Optical CDMA Networks

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Abstract—One extended Welch-Costas (EWC) code family for the wavelength-division-multiplexing/spectral-amplitude coding (WDM/SAC; WS) optical code-division multiple-access (OCDMA) networks is proposed. This system has a superior performance as compared to the previous modified quadratic congruence (MQC) coded OCDMA networks. However, since the performance of such a network is unsatisfactory when the data bit rate is higher, one class of quasi-cyclic low-density parity-check (QC-LDPC) code is adopted to improve that. Simulation results show that the performance of the WS-EWC coded OCDMA network can be greatly improved by using the LDPC codes when data rate is 2.5 Gb/s.

Keywords—wavelength-division-multiplexing (WDM), spectral-amplitude-coding (SAC), optical code-division multiple-access (OCDMA), quasi-cyclic low-density parity-check (QC-LDPC) codes

I. INTRODUCTION

In local area networks, optical code-division multiple-access (OCDMA) techniques are proposed to provide flexible solutions for asynchronous and high-speed communication. Due to the unipolar characteristic of optical signals, the effect of multiple access interference (MAI) influences the performance of OCDMA network seriously, especially when there are a large number of active users [1]. Therefore, several spectral-amplitude-coding (SAC) OCDMA networks are proposed to eliminate the MAI from other users [2-7].

A (N, w, λ) code family that has fixed cross correlation λ between codewords is suitable for SAC-OCDMA network, where N is code length and w is the code weight. In such a network, the principal limitation lies in the phase-induced intensity noise (PIIN) when the signal power is relatively high. It has been shown that the effect of PIIN is proportional to the square of photocurrent and the electrical bandwidth of the receiver [4]. The system performance can not be improved by increasing the received optical power when PIIN is the major noise.

To suppress the PIIN in the photodiodes of the decoders, several codes with ideal λ ($\lambda = 1$) are proposed [4-6]. In [4], the modified quadratic congruence (MQC) code is generated by stuffing extra chips at the original code sequences. It has been shown that the effect of PIIN can be efficiently suppressed by using codes with ideal λ , and the system performance can be significantly improved. In [5], one SAC scheme combined with wavelength-division-multiplexing (WDM) is proposed for OCDMA networks. This WDM/SAC (WS) OCDMA network not only preserves the ability for MAI elimination, but also reduces the influence of PIIN. Moreover, the WS-OCDMA network has a superior performance than the conventional SAC-OCDMA networks.

In this paper, one new $(p^2+p+1, p+1, 1)$ code family extended from the Welch-Costas (WC) sequences [8] for each prime number p (where $p > 2$) is proposed. This EWC code family can be directly used in SAC-OCDMA networks. The performance of EWC code is nearly the same as that of MQC code for similar code lengths. Furthermore, by using mapping method in [5], one WS-EWC code family for WS-OCDMA networks can be easily constructed from the proposed EWC code family. Numerical results show that the WS-EWC coded OCDMA network outperforms than the EWC coded OCDMA network. Thus, in the rest of this paper, we will focus only on analysis of WS-EWC coded OCDMA network.

The main noises considered in this paper are PIIN and thermal noise. Since both these two noises are proportional to the noise-equivalent bandwidth of the photodetector, the performance of such a network is unsatisfactory when the data bit rate is higher. One way to alleviate this problem is to use the forward-error correction (FEC) schemes. Among the FEC schemes, low-density parity-check (LDPC) codes [9-13] have recently received more attention due to the superior error correction ability. Vasic *et al.* [11] show that the LDPC codes outperform other FEC schemes for the WDM systems, and we have demonstrated the performance of LDPC codes over SAC-OCDMA network in [14]. Therefore, we adopt the LDPC code in [13] to improve the performance of the proposed network when data rate is high. Simulation results show that

Table I. Some EWC and WS-EWC codewords.

l	k	EWC codeword X_k	WS-EWC codeword $A_{k,l}$
0	0	0010101000100	0010101000100 00000000000000
0	1	0100011001000	0100011001000 00000000000000
0	2	1001001000010	1001001000010 00000000000000
0	3	1000010100100	1000010100100 00000000000000
0	4	0011000101000	0011000101000 00000000000000
0	5	0100100100010	0100100100010 00000000000000
0	6	0101000010100	0101000010100 00000000000000
0	7	1000100011000	1000100011000 00000000000000
0	8	0010010010010	0010010010010 00000000000000
0	9	1110000000001	1110000000001 00000000000000
0	10	0001110000001	0001110000001 00000000000000
0	11	0000001110001	0000001110001 00000000000000
0	12	0000000001111	0000000001111 00000000000000

the performance of WS-EWC coded OCDMA network at 2.5 Gb/s can be significantly improved by the use of LDPC codes.

II. CONSTRUCTION AND ANALYSIS OF WS-EWC CODED OCDMA NETWORK

In this section, we first introduce the methods for constructing the EWC and WS-EWC code families, respectively. Next, the configuration of WS-EWC coded OCDMA network is demonstrated. Finally, performance analysis of this WS-EWC coded OCDMA network is provided.

A. EWC code family

The proposed EWC code family can be constructed by using the following two steps:

Step 1: Let $GF(p)$ denote a finite field over an odd prime p ($p > 2$) and β is a primitive element of $GF(p)$. We can construct a multilevel sequence $y_{a,b}$ with elements of $GF(q)$ by using

$$y_{a,b}(c) = \begin{cases} d\beta^{(b+c)} + a, & c = 1, 2, \dots, p-1 \\ a, & c = p \\ p-b-2, & c = p+1 \end{cases} \quad (1)$$

where $d \in \{1, 2, \dots, p-1\}$, $a \in \{0, 1, \dots, p-1\}$ and $b \in \{0, 1, \dots, p-2\}$. Each sequence $y_{a,b}$ is generated by fixing parameters a and b , and total p^2-p different sequences can be obtained. Then, we can add another $2p+1$ sequences without affecting the final code property as follows

$$y_{a,b}(c) = \begin{cases} a, & c = 1, 2, \dots, p \\ p-1, & c = p+1 \end{cases}, \text{ for } b = p-1 \quad (2)$$

and

$$y_{a,e}(c) = \begin{cases} c-1, & c = 1, 2, \dots, p \\ p-e, & c = p+1 \end{cases}, \text{ for } a = p, e = 0, 1, \dots, p \quad (3)$$

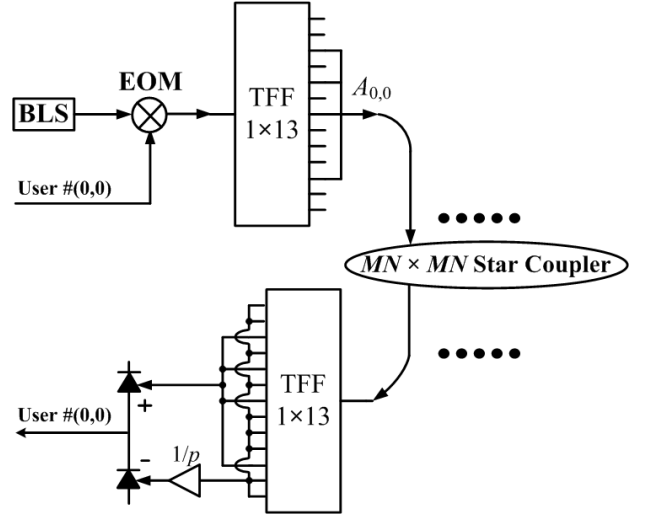


Fig. 1. WS-EWC coded OCDMA networks.

Therefore, total p^2+p+1 multilevel sequences can be obtained.

Step 2: Next, the proposed EWC codewords X_k 's can be constructed from the generated sequences $y_{a,b}$ and $y_{a,e}$ by the following mapping method:

(i). For $0 \leq a \leq p-1$, $0 \leq b \leq p-1$, and $0 \leq k \leq p^2-1$

$$X_k(i) = \begin{cases} 1, & \text{if } i = (c-1) * p + y_{a,b}(c), c = 1, 2, \dots, p+1 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

(ii) For $a = p$, $0 \leq e \leq p$, and $p^2 \leq k \leq p^2+p$

$$X_k(i) = \begin{cases} 1, & \text{if } i = e * p + y_{a,e}(c), c = 1, 2, \dots, p \\ 1, & \text{if } i = (e + y_{a,e}(c)) * (p+1), c = p+1 \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where $i = 0, 1, \dots, p^2+p$. Therefore, the proposed EWC code family has the code length $N = p^2+p+1$, code weight $w = p+1$, cross correlation $\lambda = 1$, and the cardinality p^2+p+1 . Table I shows the EWC codewords with parameters $p = 3$, $d = 1$, and $\beta = 2$.

B. WS-EWC code family

For WS-OCDMA networks, the users are divided into M ($M \geq 1$) groups [5]. Each user is indexed as user $\#(k, l)$ and assigned a codeword $A_{k,l}$, $k = 0, 1, \dots, N-1$, and $l = 0, 1, \dots, M-1$. Let X_k 's ($k = 0, 1, \dots, N-1$) denote the EWC codewords and $E_l = (e_0, e_1, \dots, e_{M-1})$ is a vector whose l th element is one and others are zero. The WS-EWC codeword of the k th user in the l th group can be obtained as

$$A_{k,l} = [a_{k,l}(m)] = X_k \otimes E_l = [e_0 X_k \ e_1 X_k \ \dots \ e_{M-1} X_k], \ m = 0, 1, \dots, MN-1 \quad (6)$$

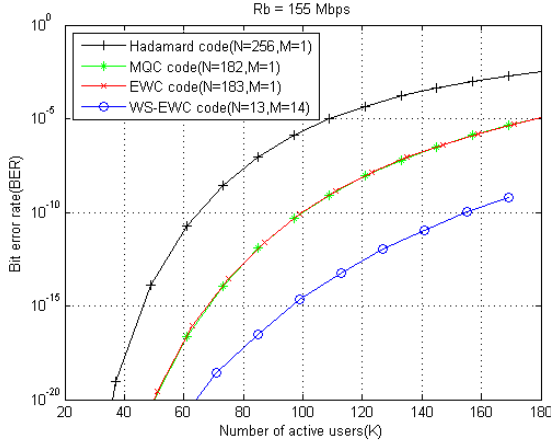


Fig. 2. BER versus number of active users.

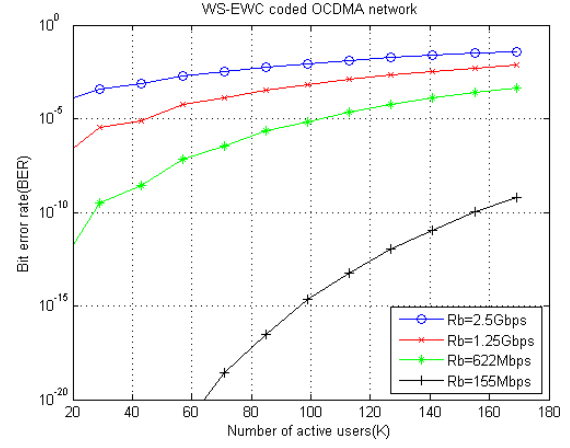


Fig. 3. BER versus number of active users.

where $a_{k,l}(m)$ is the m th element of $A_{k,l}$ and $\otimes$ is Kronecker product. Thus, the code length of the WS-EWC code is MN . For clarity of exposition, some WS-EWC codewords corresponding to the EWC codewords with $M = 2$, $l = 0$ are shown in Table I.

C. Construction of WS-EWC coded OCDMA networks

Take the case in Table I as an example. The structure of encoder for user $\#(0, 0)$ using WS-EWC codeword $A_{0,0}$ is shown in Figure 1. The information bits of user $\#(0, 0)$ are on-off keying the broadband incoherent light source, and the resulting optical signals are directed to the thin film filter (TFF). The connections between the TFF and the star coupler are determined by user $\#(0, 0)$'s codeword $A_{0,0}$. Thus, the output ports #2, #4, #6, #10 of the TFF are connected to generate user $\#(0, 0)$'s optical codeword $\{\lambda_2, \lambda_4, \lambda_6, \lambda_{10}\}$. After the encoding process, these coded spectral signals are combined in a star coupler and broadcast to the links connected to the receiver.

The decoding process is similar to conventional SAC systems and can be accomplished by passing the received signal through one TFF and detecting the results differentially. Assume user $\#(k, l)$ is the desired user, the connections between the TFF and the two photodiodes are respectively determined by the following two vectors:

$A_{k,l}^{(0)} = X_k \otimes E_l$ and $A_{k,l}^{(1)} = \bar{X}_k \otimes E_l$, where $\bar{X}_k = (1 - x_{k,0}, 1 - x_{k,1}, \dots, 1 - x_{k,N-1})$ is the complement of X_k . If $A_{k,l}^{(0)}(i)$ (or $A_{k,l}^{(1)}(i)$) is "1", the i th output port of the TFF is connected to the upper (or lower) photodiode. Thus, the user $\#(k,l)$'s decoder can eliminate the MAI from other interfering users by using the following correlation subtraction scheme:

$$A_{q,s} \odot A_{k,l}^{(0)} - \frac{1}{p} A_{q,s} \odot A_{k,l}^{(1)} = \begin{cases} p+1, & q=k, s=l \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

where $\odot$ is the dot-product of two vectors [7].

D. Performance analysis of the WS-EWC OCDMA networks

In the following analysis, we have considered the effects of PIIN and thermal noise simultaneously, since they are the dominant sources of noise that exist in the SAC-OCDMA networks. Assume that the light sources are unpolarized with equal magnitude $P_{sr}/\Delta\nu$ over bandwidth $\Delta\nu$ Hz, where P_{sr} is the effective source power at the receiver. The average optical power for one balanced detector when the desired user transmits bit "1" is $P_{sig} = (RP_{sr}w/MN)$, where R is the responsivity of the photodiodes. Assume that $K-1$ interfering users are active and K' users among them are sending "1", the variances of PIIN for desired user bit "1" and "0" are [5]

$$\langle I_1^2 \rangle = \frac{BR^2 P_{sr}^2}{MN\Delta\nu} \left\{ w + \left\lfloor \frac{K'}{M} \right\rfloor \times \left[3\lambda + \alpha\lambda + \frac{w\lambda}{N} (1 + \alpha) \left(\left\lfloor \frac{K'}{M} \right\rfloor - 1 \right) \right] \right\} \quad (8)$$

and

$$\langle I_0^2 \rangle = \frac{BR^2 P_{sr}^2}{MN\Delta\nu} \left\{ \left\lfloor \frac{K'}{M} \right\rfloor \times \left[\lambda + \alpha\lambda + \frac{w\lambda}{N} (1 + \alpha) \left(\left\lfloor \frac{K'}{M} \right\rfloor - 1 \right) \right] \right\} \quad (9)$$

respectively. Here $\lfloor x \rfloor$ denotes the largest integer less than or equal to x , α is $\lambda/(w-\lambda)$, and B is the noise-equivalent bandwidth of the photodetectors. If the probabilities of sending bit "1" and "0" are equal for each user, the bit error rate (BER) is [5]

$$P_E = \sum_{K'=0}^{K-1} \binom{K-1}{K'} \times \left(\frac{1}{2} \right)^{K'} \times \left(\frac{1}{2} \right)^{K-1-K'} \times \frac{1}{4} \left\{ \operatorname{erfc} \left(\sqrt{\frac{P_{th}^2}{2(\langle I_0^2 \rangle + \langle I_{th}^2 \rangle)}} \right) + \operatorname{erfc} \left(\sqrt{\frac{(P_{sig} - P_{th})^2}{2(\langle I_1^2 \rangle + \langle I_{th}^2 \rangle)}} \right) \right\} \quad (10)$$

where $\langle I_{th}^2 \rangle$ is the variance of thermal noise and P_{th} is the threshold of the decision device. By using the above equations with $P_{th} = P_{sig}/2$ and $\lambda = 1$, we can obtain the relationship between BER and the number of active users in Figure 2. The parameters used here are the same as the ones in [5]. The BER of SAC-OCMA networks adopting Hadamard code, MQC code, and EWC code are also shown in Fig. 2 for comparison. Since the cross-correlation of the Hadamard code is $N/4$, the corresponding BER is always worse than the ones for other codes when P_{sr} is -10 dBm. The BER performances of MQC code and EWC code with similar code length and code weights are almost equal and lower than Hadamard code. The WS-EWC code has a superior performance than other codes. This is because that the OCDMA network adopting WS-EWC code suffers less PIIN than that adopting other codes when there are a large number of active users.

Figure 3 shows the relationship between BER and the number of active users K for the WS-EWC coded OCDMA network when the data rates R_b are 155 Mb/s, 622 Mb/s, 1.25 Gb/s and 2.5 Gb/s, respectively. Since both PIIN and thermal noise are proportional to the noise-equivalent bandwidth of the photodetector, the BER is unsatisfactory when the data bit rate is higher. Therefore, we utilized the LDPC code in [13] to improve the BER performance of the 2.5 Gb/s WS-EWC coded OCDMA network in the next section.

III. PERFORMANCE ANALYSIS OF WS-EWC CODED OCDMA NETWORK WITH AND WITHOUT LDPC CODES

A binary $[N_{LDPC}, K_{LDPC}]$ regular LDPC code is defined as the null space of a sparse parity-check matrix $\mathbf{H}$ over GF(2), where N_{LDPC} and K_{LDPC} represent the block length and the number of message bits of the LDPC code, respectively. In this section, we construct several LDPC codes with similar code lengths but different code rates r ($r = K_{LDPC}/N_{LDPC}$) by using the method we proposed in [13]. The sum-product algorithm (SPA) in log-domain [11], [14] is used for decoding the LDPC codes, and the maximum number of decoding iterations is 50.

Let u_i be the i th bit in a LDPC codeword u and s_i is the corresponding received sample. Assume the channel error probability is calculated by using Gaussian approximation, thus, the initial log-likelihood ratio of bits can be calculated as

$$L(u_i) = \ln \frac{P(u_i = 0 | s_i)}{P(u_i = 1 | s_i)} = \ln \left(\frac{\sigma_1}{\sigma_0} \right) + \frac{(s_i - 1)^2}{2\sigma_1^2} - \frac{s_i^2}{2\sigma_0^2} \quad (11)$$

where σ_0^2 and σ_1^2 denote the variances of the noises when the desired bits are "0" and "1", respectively. Thus, by considering the code rate r of LDPC codes, these variances can be represented as $\sigma_0^2 = (r \times \text{SNR}_0)^{-0.5}$ and $\sigma_1^2 = (r \times \text{SNR}_1)^{-0.5}$, respectively. Notice that $\text{SNR}_1 = P_{sig}^2 / (\langle I_1^2 \rangle + \langle I_{th}^2 \rangle)$ (or $\text{SNR}_0 = P_{sig}^2 / (\langle I_0^2 \rangle + \langle I_{th}^2 \rangle)$) represents the signal-to-noise ratio (SNR) of the WS-EWC coded OCDMA

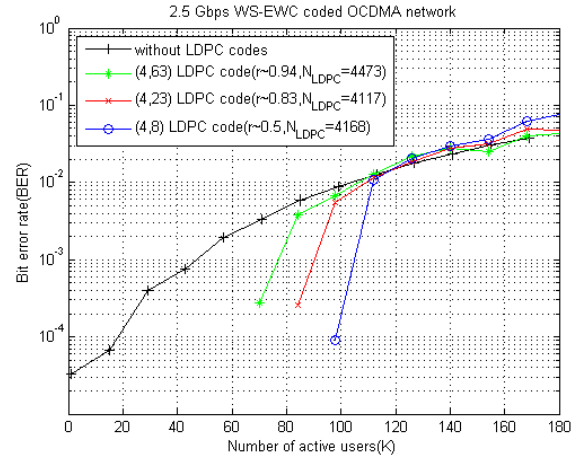


Fig. 4. BER performance of the 2.5Gb/s WS-EWC coded OCDMA network with and without LDPC codes.

networks without LDPC codes when the desired user transmits bit "1" (or "0").

From the above discussion, we know that the values of σ_0 and σ_1 are highly related to the number of active users K in the networks. To simulate the real situation of the networks, the number of active interfering users K' ($0 \leq K' \leq K-1$) is randomly selected. Figure 4 shows the relation between the bit error rates and the active users of the 2.5 Gb/s WS-EWC coded OCDMA networks with and without LDPC codes. In this figure, we observe that for the 2.5 Gb/s WS-EWC coded OCDMA networks without LDPC codes, zero active user can be supported when BER is 10^{-9} . Notice that the lowest BERs in Fig. 4 are obtained from the simulations. After that point, we spent much time for simulating at least gigabits, but no error bits happen. Thus, it is reasonable to say that 69 active user can transmit with BER better than 10^{-9} through the use of $N_{LDPC} = 4473$, $r \sim 0.94$ LDPC code. Naturally, more redundancy will result in better BER performance. For a [4168, 2084] LDPC code with a code rate $r \sim 0.5$, transmission of 97 active users with BER better than 10^{-9} can be supported. Form these results, it has been clearly shown that the capacity of the WS-EWC coded OCDMA networks can be significantly improved by the use of LDPC codes.

IV. CONCLUSIONS

One new WS-EWC code family for the WS-OCMA networks is proposed. Numerical results show that the WS-EWC coded OCDMA network has a superior performance as compared to the conventional SAC-OCMA networks. While the data rate is 155 Mb/s, there is no need to apply LDPC codes to this OCDMA network due to the excellent performance. However, while the data rate is increasing to 2.5 Gb/s, no one can transmit with BER better than 10^{-9} . To improve that, the LDPC codes we proposed in [13] is adopted. Simulation results show that the performance of the WS-EWC coded OCDMA network can be greatly improved by using the LDPC codes when data rate is 2.5 Gb/s.

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