

Travel Time Prediction for Float Car System Based on Time Series

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Abstract— Recently, the Float Car technology is playing a more and more important role in real-time traffic service systems because it can collect real-time traffic information with low cost, high coverage and high efficiency. Meanwhile, the ability to accurately predict travel times in transportation networks is becoming a critical component for many Intelligent Transportation Systems. This paper focuses on the research of travel time prediction method based on Float Car Data. To gain the inherent characteristic of traffic information, a mechanism of dynamically extracting traffic periodic trends through the statistical analysis of historical data is present. On the basis of it, a series of improvements based on time series are proposed to predict the travel time information. The Float Car Data in Beijing are used as experiment data to verify the methods.

Keywords— Intelligent Transportation System (ITS); travel time prediction; trends extraction; time series

I. INTRODUCTION

With the development of Intelligent Transportation System[1] and the increasing need of dynamic navigation services, how to obtain real-time, accurate traffic flow forecasting information[2] has become a very active research topic. An extensive amount of research has been conducted in traffic prediction and many methods and ideas have been proposed. Many different studies in the past focused on travel time forecasting.. Time series [3,4] is one of the most popular methods of prediction. However, the high degree of complexity and randomness and uncertainty in the traffic flow makes a single prediction model prediction difficult to achieve good results. In addition, prediction of travel time is potentially more challenging for arterial roads than for freeways because vehicles traveling on arterials are subject not only to queuing delay but also to signal delays[5]. Therefore, the combination prediction model is the further research direction of short-term prediction of travel time[6,7].

This paper focuses on the research of travel time prediction method on the basis of the float car data. A prediction model has been advanced which is based on the

time series analysis. The highlighted ideas and contributions are described as follows:

1) The statistics for the historical mean and variance of the traffic information of the main roads in Beijing is made, and the inherent cyclical trends in the traffic flow has been found successfully by analyzing historical data of float cars which makes the foundations for short-term traffic flow prediction.

2) The time series prediction method is improved. A time series difference method is presented based on the statistical analysis of historical data, which better meets the steady input requirement of the time series model. A real-time information repair method is proposed based on the statistical theory, which meets the requirement of continues data in the time series model. A modeling method of time series is proposed which can update periodically and make the time series model can better meet the uncertainties of dynamic traffic flow prediction.

In this paper, we present a mechanism of dynamically extracting traffic periodic trends based on the statistical analysis of historical data accumulated by float car traffic information processing system. On the basis of time series method, a series of improvements are proposed according to the inherent properties of traffic information, and a travel time prediction method is present using float car data. Finally, experiments are performed based on traffic information data generated by float car system in order to verify the accuracy of the prediction method.

The rest of this paper is organized as follows: the second section describes the mechanism of extracting traffic periodic trends and its outlier eliminating algorithm; the third section presents the process of float car travel time prediction modeling and its core algorithm; the forth section is about experiments and analysis and the last section is the conclusion.

II. STATISTICAL ANALYSIS OF HISTORICAL DATA

Finding laws from the transportation system through observation and analysis is a general principle of scientific traffic prediction. Extracting the periodic trend via statistical analysis of historical data and grasping the development rules of the predicting object can provide a valuable reference for the prediction model building. This section firstly describes the basic process of dynamically extracting historical trend, and

then the historical data outliers processing algorithms are present.

A. Extraction of historical traffic trend

Firstly, historical data is divided by dimensions according to the principle of multidimensional data analysis. After that, the analysis of traffic patterns shows that traffic flow has a periodic and tidal change. For example, during weekday traffic rush hours, the average speed on part of the roadways, especially the main trunks significantly decreases. Moreover, on the same day each week, the variation curve of the average speed on the same roadway is similar. As the organization form of historical data should be consistent with traffic patterns' features, historical data can be divided into three dimensions, that is, links, days of the week and intervals. The value of links contains the whole collection of links. Take Beijing as an example, Xizhimen and Tiananmen are included. The value of days of the week consists of Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday, and the value of intervals includes every five minutes intervals from 00:00 to 23:59, for instance, from 08:00 to 08:05 and from 08:06 to 08:10. The content of each data block unit indicates the parameter to analyze, that is, the travel time. The division of historical data showed in Fig.1.

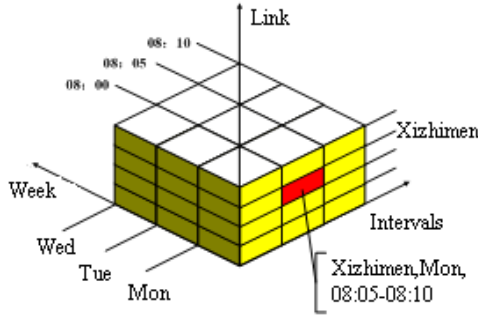


Figure 1. Multidimensional division cube of historical data

On the basis of multidimensional division of historical data collections, the parameter travel time is analyzed. With mathematical statistics method, using the sample mean and sample variance of each data block showed in Fig.1 as the population mean and variance estimation, a collections of statistical characteristics is obtained and taken as measurable representation of periodic trends.

In short, the extraction of historical traffic periodic trends is a process of getting the statistical characteristics of historical data. However, historical data not only reflects the inherent trends of information but also includes outliers such as acquisition system error. In order to ensure the accuracy of the extraction of periodic trends, outliers should be found and eliminated before calculating the statistical characteristics. Next section describes the detailed algorithm of this process.

B. Eliminating outlier algorithm

In this section, two kinds of eliminating outlier algorithms are proposed.

1) Horizontally eliminating outlier algorithms

The basic idea of horizontally eliminating outlier algorithm is, according to statistical theory, excluding emergencies, the same link during the same interval (usually 5minutes) on the same day of the week, that is, each block showed in Fig.1, has the average speed (the quotient of link length and the travel time) that approximately obeys normal distribution. Therefore, the normal test can be used to eliminate the historical data outliers of each block.

Suppose the average speed variable of link in each data block obeys normal distribution, then

$$U = \frac{x_i - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \quad (1 \leq i \leq n) \quad (1)$$

where, x_i is the average speed variable of each data block, μ is the average speed mean of each data block, and σ is the average speed variance of each data block, both of μ and σ can be approximately replaced by sample statistics.

According to the principle of normal distribution hypothesis test, using U as the test statistic, bilateral hypothesis test is processed:

$$P\{|U| > \mu_{1-\alpha}\} = \alpha \quad (2)$$

for a given test level α , the rejection region of the test is

$$W = \{|U| > \mu_{1-\alpha}\} \quad (3)$$

that is,

$$W = \{U < -\mu_{1-\alpha} \text{ or } U > \mu_{1-\alpha}\} \quad (4)$$

Thus, the average speed observed value x_i falling into the rejection region can be seen as outliers, and should be removed before calculating historical data statistics. As this eliminating outlier algorithm is carried out by horizontally compare the data in the same interval, it is named horizontally eliminating outlier algorithm.

2) Vertically eliminating outlier algorithms

In accordance with international norms, the information interval of real-time traffic information processing system is based on a 5-minute standard. However, the change frequency of traffic jam status is usually not so high that the daily actual traffic jam status can be used to divide the time zone. Suppose the average speed in the same time zone obeys normal distribution, then outliers can be detected and eliminated by hypothesis test.

The division of time zone begins with half an hour. Suppose the average speed in each time zone respectively obeys normal distribution, construct the statistic between two adjacent time zones in order to test whether their means and variances are approximately equal, and then determine whether the two time zones can be combined.

Suppose the average speed variables x and y of the two adjacent time zones respectively obey normal distribution, then

$$U_1 = \frac{x_i - \mu_1}{\sigma_1/\sqrt{n_1}} \sim N(0,1) \quad (1 \leq i \leq n_1) \quad (5)$$

$$U_2 = \frac{y_i - \mu_2}{\sigma_2/\sqrt{n_2}} \sim N(0,1) \quad (1 \leq i \leq n_2) \quad (6)$$

where, x_i and y_i respectively obey normal distribution, are equal hypothesis test. y_i represent the average speed of the two adjacent time zones, μ_1 and μ_2 represent the mean of the two time zones' average speed, σ_1 and σ_2 represent the variance of the two time zones' average speed.

The necessary condition for the combination of two adjacent time zones is $\sigma_1 = \sigma_2$ and $\mu_1 = \mu_2$.

On the basis of statistical theory, hypothesis test is firstly carried out on $H_0: \sigma_1 = \sigma_2$, $H_1: \sigma_1 \neq \sigma_2$. With the means μ_1 and μ_2 unknown, construct F test statistic:

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1 - 1, n_2 - 1) \quad (7)$$

where,

$$S_1^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (x_i - \bar{x})^2 \quad (8)$$

$$S_2^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (y_i - \bar{y})^2 \quad (9)$$

under the condition of H_0 ,

$$F = \frac{S_1^2}{S_2^2} \sim F(n_1 - 1, n_2 - 1) \quad (10)$$

the rejection region is:

$$W = \left\{ \begin{array}{l} F < F_{\frac{\alpha}{2}}(n_1 - 1, n_2 - 1) \text{ or} \\ F > F_{1-\frac{\alpha}{2}}(n_1 - 1, n_2 - 1) \end{array} \right\} \quad (11)$$

while under the condition of H_1 , the two time zones can not be combined.

On the condition of H_0 , the further test should be processed on $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$. T test statistic is constructed with $\sigma_1 = \sigma_2$:

$$T = \frac{\bar{x} - \bar{y}}{S_w \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2) \quad (12)$$

under the condition of H_0 , the rejection region is

$$W = \left\{ |T| > t_{1-\frac{\alpha}{2}}(n_1 + n_2 - 2) \right\} \quad (13)$$

After the above two hypothesis tests processing, the combination of time zones can be determined. The average speed variable x of links in each combined time zone approximately obeys normal distribution, so the normal test can be used to detect and eliminate the other kind of outliers, which is defined as vertical outliers.

III. TRAVEL TIME PREDICTION

Travel time prediction is based on the travel time of link i during the past p moments $TT(t - k\Delta t)$ ($k = 1, 2, \dots, p$) and the related historical statistic data $h(t + k\Delta t)$ ($k = 0, \pm 1, \pm 2, \dots$). It aims at calculating the travel time of link i in the k future periods $TT(t + k\Delta t)$, where, Δt is sampling interval and generally not more than 15 minutes in short-term traffic prediction.

This section firstly gives a brief introduction to the basic time series model and its features, and then proposes the improvement of time series method in travel time prediction based on float car data.

A. The basic time series model

The series consisting of a series of random variables $x_1, x_2, \dots, x_n$ is called random series. It can be represented by a collection of random variables $\{x_i\}$, $i = 1, 2, \dots, n$, also, it can be defined as a random vector X with a component x_i in multidimensional random space. If the random series is sorted by time, the subscript of x_i is an integer variable t representing increment of equal interval. This random series

is called time series and represented as $\{x_i\}$, $i = 1, 2, \dots, n$, where t is a moment or a period of time[8].

Time series models mainly include the autoregressive (AR) models, the moving average (MA) models and the autoregressive moving average (ARMA) models and so on.

Because of their different structures of linear difference equation, they have completely different statistical properties. There are two aspects of the most important differences: one is the properties of autocorrelation and partial correlation, and the other is power spectrum. The different properties of autocorrelation and partial correlation are often used to identify the type of models.

The AR model has the autocorrelation property of tailing. Its autocorrelation coefficients have the distribution of unlimited exponential decay or damped oscillation. The MA model has the autocorrelation property of censoring. It has finite autocorrelation coefficients, the number of which is determined by the order of the model. However, in terms of partial correlation properties, the AR model has the partial correlation properties of censoring while the MA model has the partial correlation properties of tailing.

B. Real-time information filling

The methods of time series prediction require the input data have continuity of time. In the past, the studies on time series were often based on simulation system. Simulation system can provide continual input data, however, when time series is applied into real-time operating system which collects data through float cars, there must be discontinuous collecting data. Therefore, it is difficult to fill discontinuous real-time traffic information when time series is applied into actual system.

An information filling algorithm combining historical data and real-time data is proposed in this paper.

The historical mean extracted by dynamic historical data statistical analysis module can reflect the basic status of the average speed of road. On the same day of the week, without special incidents, the developing trend of the link average speed should be basically the same as the statistical results. The slight difference is in the arrival time of daily morning and evening rush hours and the duration of the peak. This is just showed in the horizontal variance among historical data statistical results. However, the deviation of the speed curve of that day from historical mean must have continuity, that is, when data in current period is missing, the deviation of real-time data of the past few periods from historical mean can be used to estimate the deviation of current data from the mean, so as to achieve the purpose of filling missing data.

Therefore, the following statistic is constructed:

$$Z_b = \frac{x_{t-1} - \mu_{t-1}}{\sigma_{t-1}}, Z_c = \frac{x_t - \mu_t}{\sigma_t} \quad (14)$$

where, x_t is current missing real-time information, x_{t-1} is the information collected or filled in the past few periods of time, μ_{t-1} and μ_t are the historical mean of two periods of time respectively, and σ_{t-1} and σ_t are the historical variance of two periods of time respectively.

Suppose the deviations of the historical means of the two periods of time from historical mean are respectively equal,

that is, $Z_b = Z_c$, and then the current estimated value can be calculated:

$$x_t = \frac{x_{t-1} - \mu_{t-1}}{\sigma_{t-1}} \sigma_t + \mu_t \quad (15)$$

According to the above real-time information filling algorithm, the missing information can be effectively repaired.

C. Stationary float car data series

Time series analysis method only applies to stationary series prediction, but the traffic flow data of float car system has a periodic trend. Thus in the course of data pre-processing, periodic trend must be removed, so as to make the series stationary.

The conventional methods of removing the impact of periodicity are calculating the difference of two consecutive period data and then building the ARIMA model[9][10]. But after a large number of experiments, it is confirmed that for the existing float car data, these methods cannot effectively remove the impact caused by periodicity.

The above problems are mainly caused by the reason that the traffic flow not only has the weekly periodic feature, but also has the daily tidal feature. For the same link, the variation of its speed is similar on the same day of the week, but the arrival time of morning and evening rush hours is not always the same. Moreover, when incidents happen, the difference of two consecutive period data will be impacted by the information of incidents, which means irregular incident information is mixed in while removing periodic trends. So, removing the periodic trends of traffic flow by the difference of two consecutive period curves is not effective.

Therefore, a feasible method is present to eliminate periodic trends of series, which makes use of the difference of real-time traffic data and the historical mean of the same moment.

It is proved that even after eliminating periodic trends, real-time data of the whole day still has non-stationary trend. The changing trend item can be fitted in the form of polynomial and extracted by least square method.

Suppose $x(i)$ is a sampling function with the same interval (Δ), that is, $x(i) = (i\Delta)^k$, $i = 0, 1, \dots, N-1$, and it is fitted by the polynomial $\hat{x}(i)$:

$$\hat{x}(i) = \sum_{k=0}^K (i\Delta)^k c_k \quad (16)$$

The points collection of $\hat{x}(i)$ must be the estimation of polynomial composition in $\hat{x}(i)$. Define the error of polynomial coefficients $E(c)$ as

$$E(c) = \sum_{i=0}^{N-1} [x(i) - \sum_{k=0}^K (i\Delta)^k c_k]^2 \quad (17)$$

It can be minimized by differential equation, and then

$$\frac{\partial E}{\partial c_j} = \sum_{i=0}^{N-1} 2[x(i) - \sum_{k=0}^K (i\Delta)^k c_k] [-(i\Delta)^j] = 0 \quad (18)$$

where, $(j = 0, 1, \dots, K)$, that is,

$$\sum_{k=0}^K c_k \sum_{i=0}^{N-1} (i\Delta)^{k+j} = \sum_{j=0}^{N-1} (i\Delta)^j x(i) \quad (19)$$

During the calculation, we set $\Delta = 1$ to improve calculation accuracy. Usually when $K \leq 3$, the trend items can be extracted, making the time series excluding trends

meet the requirement of stationary. After a large number of experiments, it is found that when $K = 1$, $N = 3$, other trends can be eliminated.

D. Dynamic modelling

The final aim of time series modeling is to predict. Time series $\{x_t\}$ prediction is, with the known x_t , predicting the future value x_{t+l} , where $l > 0$. The predicted value is represented as $\hat{x}_t(l)$ and is named the prediction of l -step ahead from start time t .

The aim of prediction is to minimize the prediction error of the time series future value. As the prediction error is a random variable, its variance must be minimized. So the chosen predicted value $\hat{x}_t(l)$ must make

$$E[\hat{x}_t(l)]^2 = E[x_{t+l} - \hat{x}_t(l)]^2 \quad (20)$$

minimum. This prediction is named minimum variance prediction of the future time $t + l$ at time t .

The minimum variance prediction equation is obtained through the conditional expectation of x_{t+l} :

$$\hat{x}_t(l) = E[x_{t+l} | x_t, x_{t-1}, \dots, x_1] \quad (21)$$

The general form of prediction equation is

$$E[x_{t+l}] = \phi_1 E[x_{t+l-1}] + \dots + \phi_p E[x_{t+l-p}] \quad (22)$$

where,

$$E[x_{t-j}] = x_{t-j}, j = 0, 1, 2, \dots \quad (23)$$

$$E[x_{t+j}] = \hat{x}_t(j), j = 1, 2, \dots \quad (24)$$

$$E[a_{t-j}] = a_{t-j}, j = 0, 1, 2, \dots \quad (25)$$

$$E[a_{t+j}] = 0, j = 1, 2, \dots \quad (26)$$

In the conventional time series modeling methods of traffic flow prediction, the process of solving model parameters is offline, and the model parameters are fixed during the prediction. These cannot well adapt to the requirement of the short-term traffic flow dynamic prediction with strong uncertainty. So the dynamic modeling method is proposed in this paper. When the prediction model error exceeds 20%, the time series modeling function is dynamically called to estimate the model and its parameters again, so as to make the model adjust by the change of traffic flow in time and increase prediction accuracy.

IV. EXPERIMENTS

In this section, several experiments using float car data are performed to verify the outlier algorithms and to evaluate the prediction method we proposed.

A. Outliers processing experiments

Randomly select two links obviously including outliers, start the program of eliminating outlier, and then analyse the results.

As is showed in Fig.2, the eliminating outlier algorithm successfully removed the four apparent outliers at 3:19, 4:04, 12:59 and 18:39 respectively. The data at 5:04 and 5:09 seems abnormal, but according to the result of normal test, they are not abnormal, which indicates the processing results by float car information processing system are fluctuant seriously in the early morning.

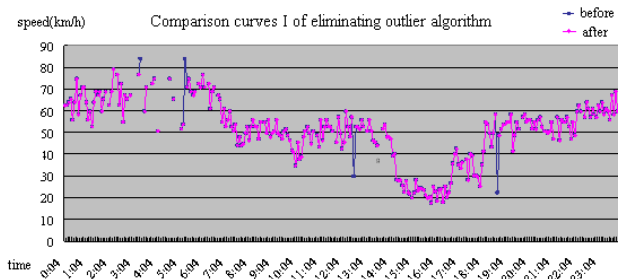


Figure 2. Comparison curves I of eliminating outlier algorithm

It can be seen from Fig.3 that the eliminating outlier algorithm successfully eliminate two outliers at 10:34 and 13:24 respectively. Meanwhile, it can be seen that the processing results by float car information processing system are seriously missing and fluctuant.

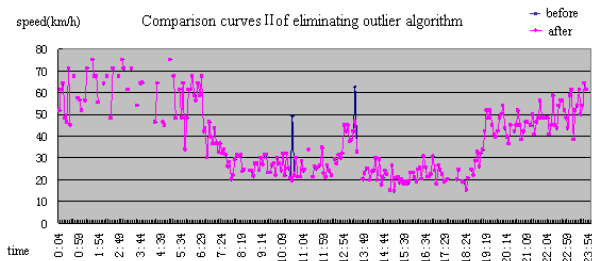


Figure 3. Comparison curves II of eliminating outlier algorithm

B. Prediction experiments

The percentage error (PE) is used to measure the accuracy of prediction. It can be calculated by:

$$PE = \frac{x_t - \hat{x}_t}{x_t} \quad (27)$$

where the prediction value of travel time at time t is denoted by $\hat{x}_t$; the measurement value of travel time at time t is denoted by x_t ; the number of samples is denoted by n .

For example, we select "East 3rd Ring Road" in Beijing as an experimental link. First, we made comparison curves of measurement and prediction speeds as Fig.4 to observe the accuracy intuitively. Then the percentage errors of all intervals as Fig.5 were calculated to obtain a quantification of prediction accuracy.

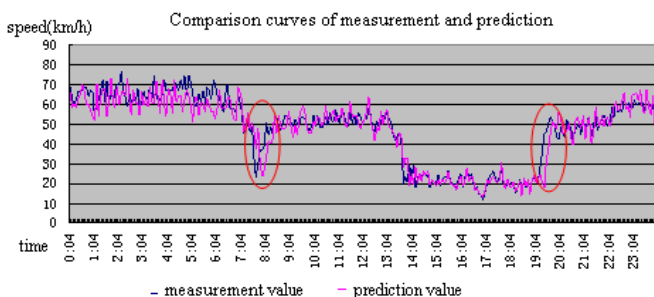


Figure 4. Comparison curves of measurement and prediction

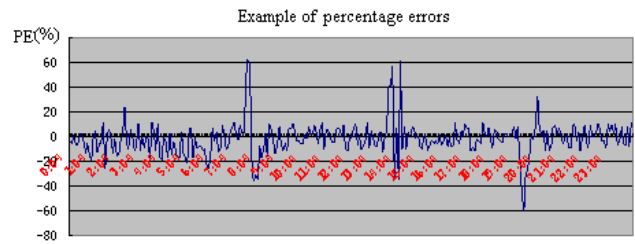


Figure 5. Example of percentage errors

The float car data of a whole week in Beijing was chose as experiment data and we finally got a summary of percentage errors as shown in Table.1:

Table 1. PE summary of time series

No.	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Average
1	19.78%	17.33%	22.02%	24.00%	18.92%	17.00%	19.49%	19.79%
2	19.36%	13.09%	22.36%	23.63%	20.09%	16.81%	19.68%	19.29%
3	19.30%	13.30%	22.90%	22.16%	21.73%	18.03%	18.73%	19.45%
4	17.75%	17.66%	22.70%	18.00%	15.40%	30.09%	21.67%	20.47%
5	14.87%	14.49%	24.36%	16.11%	23.08%	22.85%	18.64%	19.20%
6	14.67%	18.90%	21.60%	25.83%	23.36%	19.59%	18.59%	20.36%
7	17.90%	14.26%	13.81%	20.67%	16.44%	17.20%	13.87%	16.31%
8	17.55%	15.50%	16.36%	21.04%	17.26%	18.39%	13.40%	17.07%
9	13.88%	21.17%	19.49%	17.69%	15.04%	15.34%	11.68%	16.33%
10	11.00%	14.07%	15.46%	12.09%	10.09%	17.06%	10.64%	12.92%
Ave	16.61%	15.98%	20.11%	20.12%	18.14%	19.24%	16.64%	18.12%

In order to make a comparison, we used the Kalman Filter method to do the same experiment which is described above. The summary of percentage errors of Kalman Filter is shown in Table.2. It could be seen that the PE of our time series method is much smaller than Kalman Filter method.

Table 2. PE summary of Kalman Filter

No.	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Average
1	28.68%	26.22%	23.87%	31.00%	27.19%	29.45%	25.34%	27.39%
2	26.39%	20.08%	23.26%	29.40%	24.15%	26.22%	27.11%	25.23%
3	25.30%	18.50%	19.56%	29.65%	30.01%	25.17%	23.49%	24.53%
4	25.64%	27.66%	27.61%	27.11%	20.97%	24.99%	22.07%	25.15%
5	27.64%	19.84%	23.46%	23.47%	24.10%	23.01%	22.81%	23.48%
6	27.54%	21.63%	22.50%	23.18%	23.00%	23.54%	25.44%	23.83%
7	20.50%	17.12%	17.18%	21.37%	21.33%	18.61%	20.47%	19.51%
8	20.41%	24.32%	19.82%	20.51%	21.76%	16.99%	17.29%	20.16%
9	18.31%	23.21%	19.10%	19.58%	18.79%	17.84%	18.49%	19.33%
10	17.00%	18.25%	16.91%	20.92%	20.66%	19.71%	15.51%	18.42%
Ave	23.74%	21.68%	21.33%	24.62%	23.20%	22.55%	21.80%	22.70%

By the analysis of experiment results, it could be seen that although the peaks of traffic is delayed, the method we proposed can still predict accurately.

V. CONCLUSION

This paper presents a mechanism of dynamically extracting traffic periodic trends, which is through the statistical analysis of historical data accumulated by float car traffic information processing system. According to the inherent characteristic of

traffic information, a travel time prediction method based on a series of improvements of time series are proposed. Experiments using float car data are performed and the results verified the outlier algorithms and the prediction method. During the next stage of research, we will further improve our method to remove the delay of prediction in the peaks of traffic and to make the method more efficient in order to supply faster service of prediction for the real-time systems.

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REFERENCES

- [1] Gibson S., McCartney M. Traffic dynamics. Teaching Mathematics Applications [J], 2004, 23(2): 89-96.
- [2] Lu L., Wang X. Time-scale domain analysis ITS data via wavelet transformation [C]. Traffic and Transportation Studies Proceedings of ICTTS, 2002: 1232-1239.
- [3] Wang J., Wei Y., Philip S. Mining Asynchronous Periodic Patterns in Time Series Data[C]. IEEE Transactions on Knowledge and Data Engineering, 2003, 15(3): 613-628.
- [4] Zhang X., Rice J.A. Short-term travel time prediction [J]. Transportation Research Part C, Vol.11, 2003: 187-210.
- [5] Billings D, Jiann-Shiou Yang, "Application of the ARIMA models to urban roadway travel time prediction - a case study," Man and Cybernetics, 2006, pp.2529 - 2534.
- [6] Chen H.B., Grant M., Susan. Use of Sequential Learning for Short-term Traffic Flow Forecasting [J]. Transportation Research (Part C), 2001, 9(5): 319-336.
- [7] Visa A. Data Mining on Time Series of Sequential Patterns [J]. Proceedings of SPIE the International Society for Optical Engineering, 2002: 166-171.
- [8] Box G.E.P, Reinsel G.C. Time Series Analysis, Forecasting and Control [M]. 3rd edition, Englewood Cliffs: Prentice Hall, 1994: 1-98.
- [9] A study of travel time modeling via time series analysis Jiann-Shiou Yang; Control Applications, 2005. CCA 2005. Proceedings of 2005 IEEE Conference on 28-31 Aug. 2005 Page(s):855 - 860
- [10] Application of the ARIMA Models to Urban Roadway Travel Time Prediction - A Case Study Billings, D.; Jiann-Shiou Yang; Systems, Man and Cybernetics, 2006. SMC '06. IEEE International Conference on Volume 3, 8-11 Oct. 2006 Page(s):2529 - 2534