

Radio Resource Allocation for OFDMA Cognitive Cooperative System

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Abstract— In this paper, the problem of wireless resource allocation in OFDMA-based cognitive cooperative network is investigated. The objective is maximizing the throughput of primary users in a system in which cognitive users act as a relay for primary users. In our proposed scheme, while primary users achieved desire rate, they dedicate some subcarrier for cognitive users. We propose two different algorithms for cognitive users' subcarrier allocation. Finally, we indicate the performance improvement of primary users in both proposed subcarrier allocation algorithms. The numerical results show the second approach in which both sum rate and each primary user rate are considered surpass the first one. In the first algorithm, only sum rate of all primary users is taken into account.

Keywords— resource allocation, OFDMA, cognitive radio, cooperative, throughput

I. INTRODUCTION

Recently, as a result of emergence of a variety of applications, the requirement for wireless services has been on raised rapidly. Today the spectrum is overcrowded and there are a few spaces available for future wireless applications [1].

The Federal Communications Commission (FCC) indicates that many portions of the licensed spectrum are not used efficiently [2]. In order to fully utilize the valuable spectrum more efficiently, Cognitive Radios (CR) concept was introduced by J. Mitola in his PhD dissertation in 2000 [3]. Cognitive users are allowed to reuse underutilized frequency bands if they avoid introducing interference to primary users by adjusting their transmission parameters.

Since cooperative communication provides multiuser diversity, it improves the performance of wireless networks. In cooperative communication, antennas of different users are shared to make virtual antenna array. Several cooperative strategies are proposed such as amplify-and forward (AF) protocol decode-and-forward (DF) protocol and coded cooperation (CC) protocol [4]-[5].

Due to flexibility orthogonal frequency division multiplexing (OFDM) in allocating radio resource, CR systems employ it commonly. The resource allocation for OFDM-based cognitive radio networks has been widely

studied at physical (PHY) layer in the term of subcarrier, bit, and power allocation [6]-[8]. The algorithms proposed in [6] maximize the weighted sum of cognitive user rates under the constraint of multiple primary users' interference temperature. In [7], the authors proposed the optimal allocating of transmission time and power to minimize overlap between Primary Users (PU) and Cognitive Users (CU).

The resource allocation algorithm proposed in [8], maximize the throughput in a multiuser OFDM-based CR system with cognitive user power, fairness, integer bit loading as well as a maximum inference power that can be tolerated by the primary user.

Combinations of cognitive radio and cooperation concept are proposed in [9] in which cognitive users help primary users as a relay. The authors investigate the system performance from MAC layer point of view. Improving system throughput through the problem of resource allocating in cognitive relaying system has received little attention.

In this paper, we consider a scenario in which cognitive users help primary users send their data. Cognitive users apply amplify-and-forward (AF) protocol for cooperation in which the relay amplified signal received from source and transmits the amplified signal to the destination. We investigate uplink resource allocation for primary users in an OFDMA-based cognitive relaying network to maximize the throughput of primary users by nonlinear optimization problems with total transmit power constraint for primary users. Since cognitive users dedicate power for relaying primary users' data, we assume that if primary users achieve their desired rate, they try to vacate some subcarriers for the use of cognitive users. It means that while we consider the desired rate of primary users, we have another constraint on the number of subcarriers used by primary users. If cooperative transmission with the constraint on the number of subcarrier doesn't surpass non cooperative scenario, primary users will return to non cooperative scenario. However, if cognitive users allocate enough power, this case will not happen and through the help of cognitive users, throughput of the studied system will be better than non cooperative scenario. In non cooperative scenario, we do not have restriction on the number of subcarriers used by primary users.

In proposed algorithms, we obtain two necessary conditions for power and subcarrier allocation through the set of Lagrange

multipliers and using a Karush-Kuhn-Tucker (KKT) condition. Considering the constraint on the number of primary users' subcarriers, the problem becomes intractable. Thus, to solve it, we first disregard this condition and solve a problem then we dedicate some subcarrier for cognitive users, to do this we propose two different algorithms. The first algorithm chooses the best K_{th} subcarriers for primary users considering the constraint of minimum sum rate of primary users and then uses two different methods to power allocation for them. The remaining subcarriers will be used by cognitive users. The second algorithm dedicates subcarriers for cognitive users while satisfying a minimum rate for each primary user and minimum sum rate of all primary users. Simulations results are provided to compare validate our proposed scheme.

The rest of the paper is organized as follows. In Section II, the system model is described and the problems for the given system are formulated. In Section III, the proposed algorithms and condition for optimum solution are presented. Simulation results are provided in Section IV. Finally we conclude in Section V.

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II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System model

We consider resource allocation for the Uplink transmission of a single cell OFDMA-based cognitive relaying network in which a Base Station (BS) serves N primary users and M cognitive users. Assume that all the channel state information (CSI) can be properly obtained at BS.

It is assumed that there are K OFDM subcarriers in system. Each subcarrier is allocated to only one user and the length of each subcarrier is much less than the coherent bandwidth of channel. Thus, the channel response on each subcarrier is flat. It is assumed that system is time-slotted which time slot duration equal to an OFDM symbol. To synchronous all users, the beginning and the end of each time slot are known by them.

The transmission of primary users is protected by enforcing the cognitive users don't use carriers of primary users and send their data in unutilized subcarriers. Moreover cognitive users have to exit transmission whenever primary users come back. Cognitive users try not to interfere with primary users and also improved the performance of primary users by relaying data of primary users.

We assume that communication of a primary user takes place in two time slots. In first time slot, a primary user sends data to BS in its own subcarriers. In this time slot, cognitive users can receive data of the primary users. In second time slots, for each subcarrier cognitive user that has

highest relay-BS channel gain is chosen to be relay. The relay amplifies primary user data with coefficient A and then sends to BS.

The received signals from subcarriers k of n^{th} primary user (PU_n) at BS and m^{th} cognitive users (CU_m) in the first time slot are respectively given by:

$$\begin{aligned} y_{PU_n,BS}^k &= \sqrt{P_{n,k}} H_{PU_n,BS}^k x_{PU_n}^k + Z_{PU_n,BS} \\ y_{PU_n,CU_m}^k &= \sqrt{P_{n,k}} H_{PU_n,CU_m}^k x_{PU_n}^k + Z_{PU_n,CU_m} \end{aligned} \quad (1)$$

Where $P_{n,k}$, $x_{PU_n}^k$ are respectively defined as transmit power and signal of PU_n on subcarrier k . Generally, $H_{a,b}^k$ denotes the channel gain between node a and node b at subcarrier k which for different a and b is assumed to be zero-mean, independent, circularly symmetric complex Gaussian random variables. $H_{a,b}^k$ incorporates the effects of path loss and shadowing. $Z_{PU_n,BS}$ and Z_{PU_n,CU_m} are additive white Gaussian noise (AWGN) with variance N_0 . In the second time slot, the received signal of selected relay for subcarriers k of n^{th} primary user ($r_{n,k}$) is:

$$\begin{aligned} y_{r_{n,k},BS}^k &= H_{r_{n,k},BS}^k (y_{PU_n,r_{n,k}}^k) + Z_{r_{n,k},BS} \\ &= H_{r_{n,k},BS}^k A (H_{PU_n,r_{n,k}}^k x_{PU_n}^k + Z_{PU_n,r_{n,k}}) + Z_{r_{n,k},BS} \end{aligned} \quad (2)$$

The coefficient A is chosen in such ways that transmit power of $r_{n,k}$ is equal to $P_{r_{n,k}}$.

Consequently after Maximal Ratio Combining (MRC) at BS, the received signal-to-noise ratio (SNR) of PU_n at subcarrier k , $\gamma_{n,k}$, can be written as:

$$\begin{aligned} \gamma_{n,k} &= \frac{P_{n,k} |H_{PU_n,BS}^k|^2}{N_0} + \frac{\frac{P_{n,k} |H_{PU_n,CU_m}^k|^2}{N_0} \cdot \frac{P_{r_{n,k}} |H_{CU_m,BS}^k|^2}{N_0}}{\frac{P_{n,k} |H_{PU_n,CU_m}^k|^2}{N_0} + \frac{P_{r_{n,k}} |H_{CU_m,BS}^k|^2}{N_0} + 1} \\ &= \eta_{CU_m,BS}^k + \frac{\eta_{PU_n,CU_m}^k \eta_{CU_m,BS}^k}{\eta_{PU_n,CU_m}^k + \eta_{CU_m,BS}^k + 1} \end{aligned} \quad (3)$$

Where, $\eta_{a,b} \triangleq \frac{P_a |H_{a,b}^k|^2}{N_0}$ is SNR of link between node a and

node b , where node a as transmitter dedicates power P_a for transmission data. In the case of independent and identically distributed zero-mean, circularly symmetric complex Gaussian input, the maximum mutual information between n^{th} primary user and BS at subcarrier k under cooperation is given by [10]:

$$I_{PU_n} = \sum_{k=1}^K b_{n,k} \log_2(1 + \gamma_{n,k}). \quad (4)$$

Where $b_{n,k}$ is a variable that indicates presence of n^{th} primary user on subcarriers k .

B. Problem formulation

Our objective is to maximize the sum rate for primary users subject to total transmit power of each primary user and the number of subcarriers that used by primary users constraints. The last constraint only applies when the desired rate of primary users is achieved. Thus, if we assumed that in cooperative transmission primary users achieved their desired rate, we can formulate our problem as:

$$\max_{P,b} \sum_{n=1}^N \sum_{k=1}^K b_{n,k} \log_2(1 + \gamma_{n,k}) \quad (5)$$

$$\sum_{n=1}^N \sum_{k=1}^K b_{n,k} \leq K_{th} \quad \forall k \quad (6)$$

$$\sum_{k=1}^K P_{n,k} \leq P_n \quad (7)$$

$$\sum_{n=1}^N b_{n,k} \leq 1 \quad (8)$$

$$P_{n,k} \geq 0 \quad (9)$$

$$b_{n,k} \geq 0 \quad (10)$$

Where K_{th} is the maximum number of subcarrier that used by primary users that is, primary users vacant $K_{CU} \triangleq K - K_{th}$ subcarriers for cognitive users and P_n denotes the total allowed power of n^{th} primary user. The optimization variables are primary subcarrier allocation indicator vector, i.e. $b = [b_{11}, b_{12}, \dots, b_{1K}, b_{21}, \dots, b_{NK}]^T$ and primary power allocation vector, i.e. P . (6) represents that primary users should allocate minimum subcarriers to cognitive users. The number of K_{th} should be determined in a way that in spite of constraint (6), the performance of primary users improves. We will show that despite this limit, primary users gain advantages of cognitive user cooperation. Constraint (7) corresponds to total power that used by n^{th} primary user. Inequality (8) follows from assumption that a subcarrier can be allocated to at most one user.

III. PROPOSED ALGORITHM

An optimal solution to integer programming problem in (6) is computationally complex. To become the optimization problem more tractable, the noise in PU_n -relay link, i.e.

$Z_{PU_n, r_{n,k}}$ is neglected, therefore $\gamma_{n,k}$ can be written as:

$$\gamma_{n,k} \approx P_{n,k} \frac{|H_{PU_n, d}^k|^2}{N_0} + P_{r_{n,k}} \frac{|H_{r_{n,k}, d}^k|^2}{N_0} \quad (11)$$

$$\gamma_{n,k} \approx a_{n,k} P_{n,k} + c_{n,k},$$

$$\text{where } a_{n,k} = \frac{|H_{PU_n, d}^k|^2}{N_0}, c_{n,k} = \frac{P_{r_{n,k}} |H_{r_{n,k}, d}^k|^2}{N_0}.$$

Despite this approximation the optimal solution is difficult thus; we try to find suboptimal solutions for this problem. At first we solve this problem by disregarding condition (6) and

solve a problem then we suggest two different algorithms for cognitive users' subcarrier allocation. The details will be given later. Considering the approximation in (11), the expression (5)-(10) form a convex optimization problem because the objective function is convex in optimization variables, and all constraints are linear in them. The convex program can be solved using general solution techniques; however we use an approximately optimal solution base on Lagrangian theory [11].

The Lagrangian of the convex program is obtained by introducing Lagrangian multipliers $\mu \leq 0$, $\gamma \leq 0$, $\nu \leq 0$, $\kappa \leq 0$ for constraints (7)-(10) respectively. It should be noted that we assume $b_{n,k} \in [0,1]$ but it can be shown that the relaxation of $b_{n,k}$ from 0 or 1 to $[0,1]$ does not change the optimal value. This leads to:

$$L(P, b, \mu, \gamma, \nu, \kappa) = \sum_{n=1}^N \sum_{k=1}^K b_{n,k} \log_2(1 + a_{n,k} P_{n,k} + c_{n,k})$$

$$+ \sum_{n=1}^N \mu_n \left(\sum_{k=1}^K P_{n,k} - P_n \right) + \sum_{k=1}^K \gamma_k \left(\sum_{n=1}^N b_{n,k} - 1 \right)$$

$$- \sum_{n=1}^N \sum_{k=1}^K \nu_{n,k} P_{n,k} - \sum_{n=1}^N \sum_{k=1}^K \kappa_{n,k} b_{n,k}. \quad (12)$$

The Karush-Kuhn-Tucker (KKT) optimality conditions are given by the set of constraints (7)-(10).

$$\frac{\partial L(P, b, \mu, \gamma, \nu, \kappa)}{\partial P_{n,k}} = b_{n,k} \times \frac{a_{n,k}}{1 + a_{n,k} P_{n,k} + c_{n,k}} + \mu_n - \nu_{n,k} = 0 \quad (13)$$

$$\frac{\partial L(P, b, \lambda, \mu, \gamma)}{\partial b_{n,k}} = \log_2(1 + a_{n,k} P_{n,k} + c_{n,k}) + \gamma_k - \kappa_{n,k} = 0 \quad (14)$$

$$\mu_n \left(\sum_{k=1}^K P_{n,k} - P_n \right) = 0 \quad (15)$$

$$\gamma_k \left(\sum_{n=1}^N b_{n,k} - 1 \right) = 0 \quad (16)$$

$$P_{n,k} \nu_{n,k} = 0 \quad (17)$$

$$b_{n,k} \kappa_{n,k} = 0. \quad (18)$$

From (14), for subcarrier k and two primary users PU_1 and PU_2 , we have:

$$\frac{\partial L(P, b, \lambda, \mu, \gamma)}{\partial b_{PU_1, k}} - \frac{\partial L(P, b, \lambda, \mu, \gamma)}{\partial b_{PU_2, k}} =$$

$$\log_2(1 + a_{PU_1, k} P_{PU_1, k} + c_{PU_1, k}) + \gamma_k - \kappa_{PU_1, k}$$

$$- \log_2(1 + a_{PU_2, k} P_{PU_2, k} + c_{PU_2, k}) - \gamma_k + \kappa_{PU_2, k} = 0. \quad (19)$$

Now, from (18), if PU_1 , PU_2 share the subcarrier k with each other, we have $\kappa_{PU_1, k} = 0$, $\kappa_{PU_2, k} = 0$. Thus, according to (14), (19) can be expressed as:

$$\log_2(1 + a_{PU_1, k} P_{PU_1, k} + c_{PU_1, k}) = \log_2(1 + a_{PU_2, k} P_{PU_2, k} + c_{PU_2, k}). \quad (20)$$

From (18), on one hand if subcarrier k is only used by at most one primary user such as PU_1 , $\kappa_{PU_1,k} = 0$. On the other hand, since subcarrier k is not assigned to PU_2 and Lagrangian multipliers are negative $\kappa_{PU_2,k} \leq 0$. Thus, we have:

$$\log_2(1 + a_{PU_1,k} P_{PU_1,k} + c_{PU_1,k}) > \log_2(1 + a_{PU_2,k} P_{PU_2,k} + c_{PU_2,k}). \quad (21)$$

Finally, according to (20) and (21), subcarrier k should be allocated to user i selected by,

$$i = \max_x \log_2(1 + a_{x,k} P_{x,k} + c_{x,k}). \quad (22)$$

Similary the optimal transmit power allocation for subcarrier k of n^{th} PU using the Karush–Kuhn–Tucker condition, is given by:

$$P_{n,k} = \left(\Delta - \frac{1}{\Gamma_{n,k}} \right)^+. \quad (23)$$

Where $\Gamma_{n,k} = a_{n,k} (1 + c_{n,k})^{-1}$ and Δ is the water-filling level which makes the total power of primary user n equal to power constraint.

After determining transmits power for subcarriers of each primary user by using the water-filling in (23), we allocate subcarriers to appropriate primary user in which maximizes the rate given in (22). Thus, we have:

$$\max_x \log_2(1 + a_{x,k} \left(\Delta - \frac{1}{\Gamma_{x,k}} \right)^+ + c_{x,k}). \quad (24)$$

Now, we should consider the constraint (6) in resource allocation. After power and subcarrier allocation for primary users we assign some subcarrier for cognitive users according to two different proposed algorithms, in the first algorithm, we select K_{th} subcarriers which have max rate in resource allocation problem without constraint (6). It means that the way we assigned subcarriers to primary users will not change in this stage and we assume the same subcarrier allocation for this resource allocation problem. But, we consider different power allocation because, the sum of power allocations for K_{th} selected subcarriers not equal to defined power constraint in (7). Thus, to hold the power constraint for K_{th} selected subcarriers, we should increase power of selected subcarriers that can be done in two different ways. First, we consider the power allocation defined in (23) then increase them with constraint

rate $\left(\frac{\sum_{k \in T_n} P_{n,k}}{P_n} \right)^{-1}$, where T_n is a set of selected subcarriers for n^{th} primary user. P_n is the total allowed power of n^{th} primary user that define in (7).

Second, we apply water-filling method for selected subcarriers.

In the second algorithm, after power and subcarrier allocation for primary users, we should select K_{CU} subcarriers for use of cognitive users. Subcarriers selection is performed subject to constraint that each primary users' link obtains a specified data rate requirement. We consider that the requirement data rate for each primary user is equal to achieve data rate in non-cooperative scenario multiplied by a coefficient greater than one, i.e. α . To achieve higher rate in cooperative scenario, the coefficient α is considered.

$R_n = \sum_{k=1}^K b_{n,k} r_{n,k}$, R_{th_n} , and R_{n_CO} are the available data rate of n^{th} primary user, required data rate for n^{th} primary user, and the rate of n^{th} primary user in the cooperative mode, respectively. U_n is a set of subcarriers used by n^{th} primary user. We sort primary users in descending order. Thus, the primary user with the highest rate is considered first. The subcarrier of considered primary user which has lowest rate is dedicated to cognitive users. This dedication continues while sum rate of all primary users and rate of the primary user do not fall below minimum requirements. We summarize the procedure in algorithm 2.

Table 1. algorithm 2

Algorithm2 Subcarrier Selection for Cognitive Radio
Initialize : Power & Subcarrier allocation for PU_n for $n=1:N$ do While $R_n \geq \min\{R_{th_n} \times \alpha, R_{n_CO}\}$ & $K_{vacate} \leq K_{CU}$ if $(R_n - \min_{k \in U_n}(r_{n,k})) \geq R_{th_n} \times \alpha$ then $K_{vacant} = K_{vacant} + 1$ $R_n = R_n - r_{n,k}$ end if end While end for

IV. SIMULATION RESULTS

In this section, we present the performance of the proposed algorithms by computer simulations. We use an OFDMA-based system with 64 subcarriers. In our simulation, the number of primary users and the number of cognitive users are respectively 3 and 4. Channel gain for relay-BS, PU_n -BS and $PU_n - r_n$, are assumed to be zero-mean, independent, circularly symmetric complex Gaussian random variables with variances -10 dB. We assume that the desired rate is the sum rate of primary users achieved in non cooperative scenario.

Figure 1 shows the sum rate of primary users versus the SNR of relays for non cooperative scenario and cooperative scenario for algorithm 2. In non cooperative scenario, whole subcarriers are assigned to primary users. For comparison, we illustrate the cooperative scenario in which whole and partial numbers of subcarriers are assigned to primary users. From Figure.1, when primary users used all subcarriers, cooperative scenario outperforms non cooperative scenario. Even though, in

the cooperative scenario in which primary users do not use all subcarrier and have constraint on the number of subcarriers, it still surpasses the cooperative scenario.

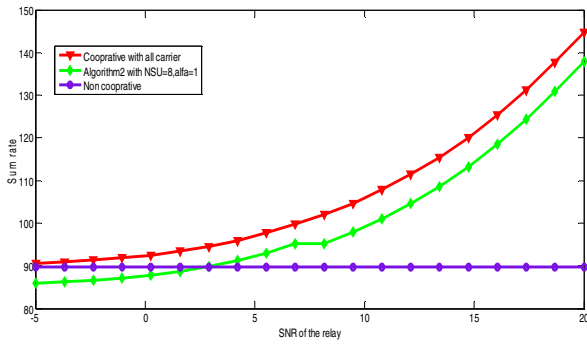


Figure 1. achieve rate for primary users for cooperative and non-cooperative scenario.

Figure.3 shows sum rate of primary users versus the SNR of cognitive users for relaying for two different algorithms for cognitive users' subcarrier allocation. In this Figure, we assume $K_{th} = 56$ or $K_{CU} = 8$. From Figure.3, algorithm2 has better performance.

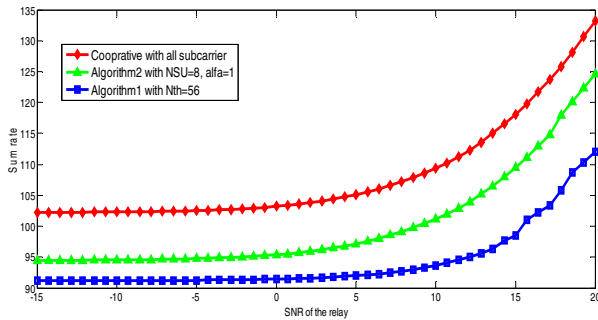


Figure 2. sum rate of PUs for two different algorithms for cognitive users' subcarrier allocation.

Figure 2 illustrates the sum rate of primary users versus SNR of the relay for different numbers of K_{th} for the first algorithm of subcarrier selection for primary users and use second methods of power allocation for selected subcarriers. As expected, by increasing the number of subcarriers used by primary users (K_{th}), the performance of primary users improves. However, the reduction of sum rate of primary users as a result of decreasing K_{th} can be compensated by the increasing the power of cognitive users dedicated for relaying.

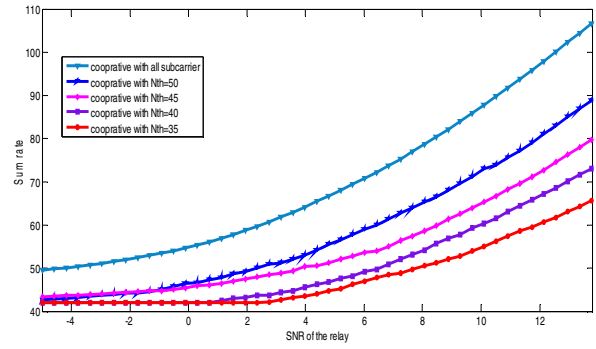


Figure 3. achieve rate of primary users for different numbers of selected subcarriers (use algorithm 1).

Figure.3 shows sum rate of primary users versus the SNR of cognitive users for relaying for the first algorithm of subcarrier selection for primary users. We compare two different methods that used for power allocation for selected subcarriers. In this Figure, we assume $K_{th} = 45$. From Figure.3, the second method in which power is allocated to selected subcarriers according to water-filling method has better performance, as expected. Although the first method is simpler than the second one, to get better performance, it is reasonable to apply the second method.

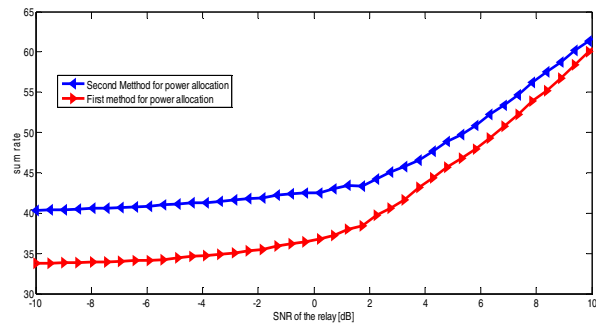


Figure 4. sum rate of PUs for two different methods of power allocation for selected subcarriers ((use algorithm 1).

V. CONCLUSION

In this paper, we have considered cognitive relay network in which cognitive users act as a relay for primary users. We have investigated the resource allocation for primary users with two constraints. It has been assumed that if primary users achieve their minimum sum rate, they use only limited number of subcarriers. Cognitive user cooperation has been shown to improve the sum rate of primary users. We have considered two different algorithms for cognitive users' subcarrier allocation and proposed two different methods of power allocation for selected subcarriers in first algorithm. It has shown that the first algorithm has better performance.

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