

Analysis of ASTC in a Correlated Rayleigh Fading Channel with Imperfect Channel Estimation

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Abstract—In this paper, we analyze the performance of an Algebraic Space Time Codes (ASTC), called the Golden code. Due to its Algebraic construction based on Quaternionic algebra, the code has a full rate, full diversity, non-vanishing constant minimum determinant for increasing spectral efficiency, uniform average transmitted energy per antenna and good shaping. In this paper, we analyze the performances of the Golden codes in correlated Rayleigh channel. We consider a coherent demodulator and we analyze the channel estimation error impact.

Index Terms—Golden code, Minimum Mean Square Error, Least Squares Estimation, Rayleigh channel

I. INTRODUCTION

In order to achieve a very high spectral efficiency over wireless channels, we need multiple antennas at both transmitter and receiver ends. Therefore, a significant interest for these antennas has been studied to develop systems that offer both high capacity and high data speed. Researchers have recently begun to investigate the capacity and Bit Error Rate (BER) performance of such systems. In multi antenna communication systems, investigations of N transmit and M receive antenna systems showed that the capacity of such systems increases linearly with the minimum of N and M [1], [2]. High data rates are obtained by simultaneously sending signals from several transmit antenna. Under quasi-static fading channels, the maximum combined transmit- receive diversity order equals to $M \times N$. A variety of Space-Time codes exists in the literature. In [3], Alamouti proposed a new modulation scheme over $N = 2$ transmit and $M = 2$ receive antennas where a rate of one symbol Per Channel Use (PCU) with $2N$ diversity was achieved. A new family of codes based on cyclic division algebras was constructed in [1]. The so called Quaternionic space time (ST) codes have good performance but they suffer from the non uniform distribution of the energy in the codeword. To alleviate this problem, a new family of codes, called perfect ST codes for $N = 2$ was created. These codes are characterized by a good energy efficiency given by a uniform energy distribution and transmitted constellations have no shaping loss. This paper is organized as follows. In section II, we present the main criteria of the proposed ASTC chain. The third section focuses on the channel estimation methods in MIMO context. The section IV presents the simulation results.

II. SYSTEM AND CHANNEL MODEL

A. System Model

We consider a coherent system over a non selective correlated Rayleigh fading MIMO channel 2×2 , with imperfect channel estimation. The baseband system model used in this paper is shown in Fig. 1. The binary source sequence b_i of length L is modulated using the QAM-4 modulator, then encoded using the Golden code mapper. Each information sequence is encoded by the ASTC encoder into two streams constellations as follows

$$\mathbf{v}_{ni,ni+1} = [s(2n_{i+1})-3, s(2n_{i+1})-2, s(2n_{i+1})-1, s(2n_{i+1})] \quad (1)$$

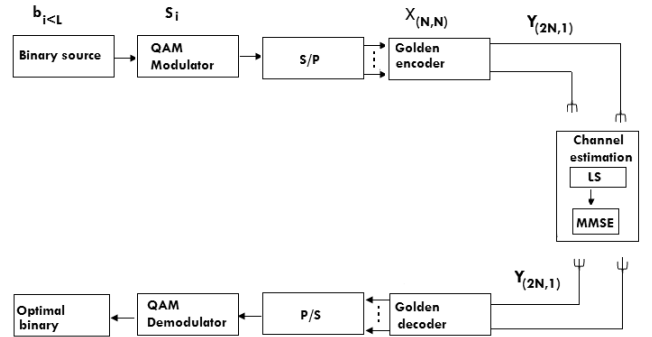


Figure 1. Block diagram of Joint Coded ASTC system

The output of the encoder can be written in matrix form as follow [1], [2]

$$\mathbf{X}_{N \times N} = \frac{1}{\sqrt{p}} \begin{pmatrix} \alpha (\mathbf{v}(1) + \theta \mathbf{v}(2)) & \alpha (\mathbf{v}(3) + \theta \mathbf{v}(4)) \\ \bar{\alpha} (\mathbf{v}(3) + \bar{\theta} \mathbf{v}(4)) & \bar{\alpha} (\mathbf{v}(1) + \bar{\theta} \mathbf{v}(2)) \end{pmatrix} \quad (2)$$

where

$$\theta = \frac{1 + \sqrt{5}}{2} \quad \bar{\theta} = \frac{1 - \sqrt{5}}{2} \quad \alpha = 1 + i - i\theta \quad \bar{\alpha} = 1 + i - i\bar{\theta}$$

We note here that the encoder can transmit 4 symbols on each antenna at the same time, whereas the Alamouti [3] encoder can only code 2 symbols at a time. The best ASTC is for $p = \sqrt{5}$ the received matrix is:

$$\mathbf{Y}_{N \times N} = \mathbf{H}_{N \times N} \cdot \mathbf{X}_{N \times N} + \mathbf{W}_{N \times N} \quad (3)$$

Where $\mathbf{X}$ is the transmitted codeword of duration T , $\mathbf{H}$ is the

channel matrix and $\mathbf{W}$ is the i. i. d. Gaussian noise matrix. Subscripts indicate the dimensions of the matrices. In this paper, we consider square ($N = T = 2$) linear dispersion to have full-rate square codes using QAM constellation. We can re-express the total code word $\mathbf{X}$ of length L at time $(ni, ni+1)$ as the follows:

$$\mathbf{x}_{ni} = \frac{1}{\sqrt{5}} \begin{pmatrix} (\alpha(s(2n_{i+1})-3 + \theta s(2n_{i+1})-2))_{(ni,1)} \\ (\bar{\alpha}(s(2n_{i+1})-1 + \bar{\theta} s(2n_{i+1})))_{(ni+1,1)} \\ (\alpha(s(2n_{i+1})-1 + \theta s(2n_{i+1})))_{(ni,2)} \\ (\bar{\alpha}(s(2n_{i+1})-3 + \bar{\theta} s(2n_{i+1})-2))_{(ni+1,2)} \end{pmatrix} \quad (4)$$

To obtain the lattice representation of the system above we should re-express the total transmitted system over a non selective channel $\mathbf{H}_{ni}$ with time variation:

$$\mathbf{H}_{ni} = \frac{1}{\sqrt{5}} \begin{pmatrix} h_{ni}^{11} & h_{ni+1}^{21} & 0 & 0 \\ h_{ni}^{12} & h_{ni+1}^{22} & 0 & 0 \\ 0 & 0 & h_{ni}^{11} & h_{ni+1}^{21} \\ 0 & 0 & h_{ni}^{12} & h_{ni+1}^{22} \end{pmatrix} \quad (5)$$

where h_{ni} is the channel coefficient at time n_i . To decode the received signal we use the MMSE decoder. The solution of the linear MMSE is given by [4]:

$$\hat{\mathbf{x}}_{MMSE} = \left(\frac{1}{SNR} \mathbf{I} + \hat{\mathbf{H}}^H \cdot \hat{\mathbf{H}} \right)^{-1} \hat{\mathbf{H}}^H \cdot \mathbf{y} \quad (6)$$

where $\mathbf{H}$ is a $L \times 4$ matrix, $\mathbf{y}$ is a $L \times 1$ vector and $\mathbf{I}$ is the identity $L \times L$ matrix. The decision vector for each 4 transmitted symbol is then:

$$\hat{\mathbf{s}}_{MMSE} = \Phi^{-1} \times \hat{\mathbf{x}}_{MMSE} \quad (7)$$

where

$$\Phi = \begin{pmatrix} \alpha & \alpha\theta & 0 & 0 \\ 0 & 0 & i\bar{\alpha} & i\bar{\alpha}\bar{\theta} \\ 0 & 0 & \alpha & \alpha\theta \\ \bar{\alpha} & 0 & 0 & 0 \end{pmatrix} \quad (8)$$

B. Channel Model: Correlated Rayleigh fading channel

In this work, we suppose that the encoded signal is transmitted over a non selective Rayleigh fading channel. We adopt here the Clarke channel model. The received signal is the sum of q waves, we take into account the Doppler shifts effect. To obtain a correlated Rayleigh fading channel, the autocorrelation function of $\{h_k^j\}$ process is given by:

$$\begin{aligned} r_h &= E[h_k^j h_{k-q}^j] \\ &= \sigma_h^2 \exp(j2\pi f_c q) J_0(2\pi f_m q T_s) \end{aligned} \quad (9)$$

where J_0 is the Bessel function with zero order, f_m is the maximum Doppler shift and j is the antenna's number.

III. CHANNEL ESTIMATION

The basic assumption was that the receiver has perfect channel knowledge. However, this does not apply to real situations. In such cases, it becomes necessary to estimate the channel parameters. In this section, we present a channel estimation method for systems using pilot symbols.

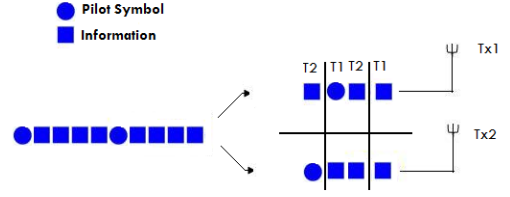


Figure 2. Space time partition of pilot symbol

A. Estimation at pilot position

We use, at pilot location the LS (Least Squares) estimation method [5]–[7]. The pilot insertion scheme is shown in Fig. 2. The estimate channel can be obtained by minimizing the error matrix $\mathbf{E}_k^2$ per symbol where $\mathbf{E}_k^2$ is given by [6]

$$\mathbf{E}_k^2 = (\mathbf{y}_k^p - \mathbf{x}_k^p \mathbf{h}_k^p)^H (\mathbf{y}_k^p - \mathbf{x}_k^p \mathbf{h}_k^p) \quad (10)$$

where $\mathbf{y}_k^p$ is the received vector with the k^{th} pilot symbol, $\mathbf{x}_k^p$ is the k^{th} transmitted pilot sample, and $\mathbf{h}_k^p$ is the channel coefficient for the k^{th} symbol. The least squares solution for the estimate channel is obtained by:

$$\mathbf{h}^{LS} = [(\mathbf{x}_k^p)^H \mathbf{x}_k^p]^{-1} (\mathbf{x}_k^p)^H \mathbf{y}_k^p \quad (11)$$

where $(\cdot)^H$ denotes the Hermitian operator, p indicate pilot symbols. Thus the estimate channel coefficients at 4 consecutive symbol pilot positions are denoted by :

$$\mathbf{h}^{LS} = \begin{pmatrix} \mathbf{x}_k(1)^p & \mathbf{x}_k(2)^p & 0 & 0 \\ 0 & 0 & \mathbf{x}_k(1)^p & \mathbf{x}_k(2)^p \\ \mathbf{x}_k(3)^p & \mathbf{x}_k(4)^p & 0 & 0 \\ 0 & 0 & \mathbf{x}_k(3)^p & \mathbf{x}_k(4)^p \end{pmatrix}^\dagger \mathbf{y}_{ni}^{LS} \quad (12)$$

where $\dagger$ is the pseudo-inverse operator.

B. LMMSE estimation

We use the MMSE (Minimum Mean Squares Estimation) method in order to optimize the LS estimation, by minimizing the following mean-square error [5]

$$\varepsilon^2 = E \left[(\tilde{\mathbf{h}}^{LS} - \hat{\mathbf{h}})^H (\tilde{\mathbf{h}}^{LS} - \hat{\mathbf{h}}) \right] \quad (13)$$

To describe the LMMSE channel estimation, let us define the sample response (coefficient) of the desired channel by w . The channel will process the real-valued stationary process $\mathbf{x}_n$ to produce an estimate $\hat{\mathbf{d}}(n)$ of the desired real-valued signal $\mathbf{d}(n)$:

$$\mathbf{d}(n) = \sum_{m=0}^{L-1} \tilde{\mathbf{h}}_m^{LS} x_{n-m} \quad (14)$$

where L is the number of filter coefficients. Without loss of generality, we assume that the processes $\mathbf{x}_n$ and $\mathbf{d}(n)$, have zero mean values. The output of the filter is equal to the convolution of the input and the filter coefficients. Hence, we obtain

$$\hat{\mathbf{d}}(n) = \sum_{m=0}^{L-1} \omega_m x_{n-m} = \mathbf{w}^T \mathbf{x}_n \quad (15)$$

where $w = [\omega_1 \omega_2 \dots \omega_{L-1}]^T$. The MMSE estimator is given by:

$$\begin{aligned} J(w) &= E\{e^2(n)\} \\ &= E\{[\mathbf{d}(n) - w^T \mathbf{x}_n]^2\} \\ &= E\{\mathbf{d}^2(n)\} - 2w^T E\{\mathbf{d}(n) \mathbf{x}_n\} + w^T E\{\mathbf{x}_n \mathbf{x}_n^T\} w \\ &= \sigma_d^2 - 2w^T \mathbf{r}_L + w^T \mathbf{R}_L w \end{aligned} \quad (16)$$

where

$$\begin{aligned} w^T \mathbf{x}_n &= \mathbf{x}_n w^T \\ \mathbf{r}_L &= [r_{dx}^0, r_{dx}^1, \dots, r_{dx}^{L-1}]^T \end{aligned} \quad (17)$$

The $\mathbf{R}_L$ Matrix is the correlation matrix of the input data, $\mathbf{r}_L$ is the cross-correlation vector between $\mathbf{d}(n)$ and $\mathbf{x}_n$ and σ_d^2 is the variance of signal $\mathbf{d}(n)$.

The above equation (17) can be written in the following matrix form, called the Wiener-Hopf equation

$$\mathbf{R}_L w_{L \times 1}^0 = \mathbf{r}_L \quad (18)$$

$$w^0 = \mathbf{R}_L^{-1} \mathbf{r}_L = \hat{\mathbf{h}}_{LMMSE} \quad (19)$$

where w^0 is an $L \times 1$ vector.

C. Estimation at non pilot position

We assume that the maximum Doppler shift f_m is small enough to suppose that the channel is slow fading over the duration of one MIMO symbol. We can estimate the channel response at a data symbol location using a decision feedback equalizer [7]. In this case, the channel transfer function is estimated using past estimated values. First, channel values at already estimated locations are used to make an initial decision on the transmitted code word.

$$\tilde{\mathbf{x}}_{ni} = (\mathbf{h}_{ni}^{LMMSE})^\dagger \mathbf{y}_{ni}^{LMMSE} \quad (20)$$

After that, the channel estimation is updated by:

$$\tilde{\mathbf{h}}_{ni}^{LMMSE} = [(\tilde{\mathbf{x}}_{ni})^H \tilde{\mathbf{x}}_{ni}]^{-1} (\tilde{\mathbf{x}}_{ni})^H \mathbf{y}_{ni} \quad (21)$$

IV. SIMULATION RESULTS

To investigate the performance of the proposed space time code, a series of Monte Carlo simulations were carried out. Transmitted symbols belong to 4-QAM. For all simulations the BER curves were averaged over 100000 Monte Carlo trials.

A. Golde code versus Alamouti in an ideal channel condition

This section compares the BER performances of the Golden code and the classical Alamouti code. Figure 3 shows, that for a high SNR, the Gold code have a good performance in terms of BER. However, for a SNR near to 10dB, the BER remains close to Alamouti BER, despite, we are coding 4 symbols at the same time, however we code only 2 symbols with Alamouti, then the gain still significant in terms of rate (full rate). These results lead us to deal with real channel conditions mainly if we use a correlated Rayleigh channel with

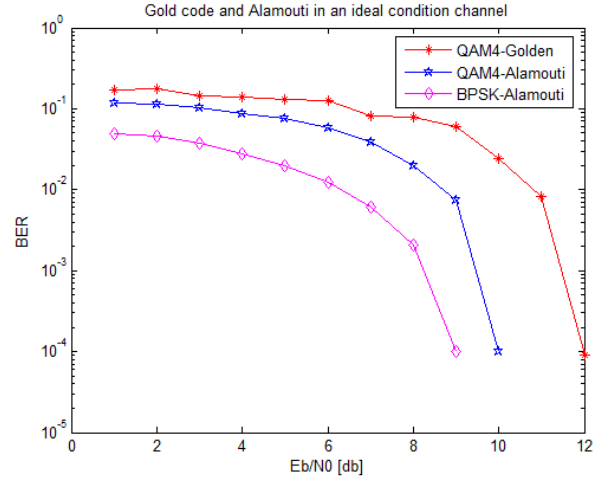


Figure 3. Golden code versus Alamouti in an ideal condition channel

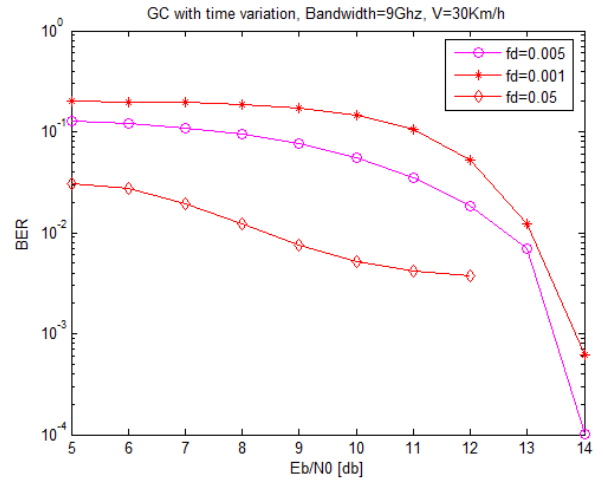


Figure 4. BER performance for Golden Code, with MMSE decoder, 4-QAM modulation, Band width =9 Ghz

a severe decoder like MMSE and with an unknown channel coefficients.

B. Doppler effect on Gold code performance

Figure 4, shows that when $fd = 0.001$, and $fd = 0.005$, the gain obtained is about 0.8 dB for a BER of 10^{-3} . This gain is less significant when the normalized Doppler frequency increases.

C. Golden code with LS and LMMSE estimations

Figure 5, presents the performance for an unknown channel. For LS estimator, the BER is close to 10^{-4} for about 23 dB, however, in case of the LMMSE method estimation at 22dB, the BER is about 10^{-4} , so the gain is around 2dB for the LMMSE estimator. The LS estimator suffers from a high mean-square error and does not take into account the channel correlation property. These drawbacks can be addressed by the

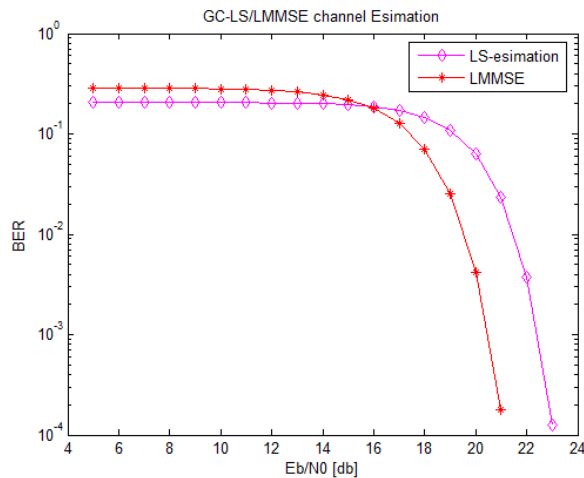


Figure 5. Golden Code with channel estimation and MMSE decoder, $f_d=0.005$, Band width =9 Ghz, $V=30\text{Km/h}$

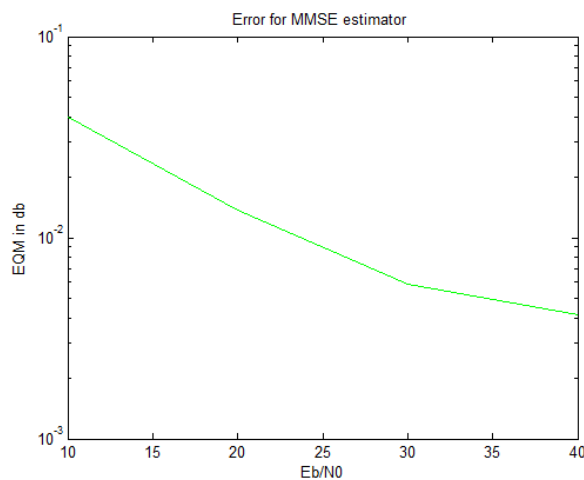


Figure 6. BER performance for the MMSE Error estimation

MMSE channel estimation technique, which helps reducing the estimation error.

Figure 6 shows the LMMSE error estimation for a set of SNR. We note that at 40 dB, the error is the lowest.

In conclusion, we note that the best algebraic space time code, Gold code, shows a good performance with LMMSE estimator and MMSE receiver when we work with 4-QAM constellation and with a correlated Rayleigh fading channel.

V. CONCLUSION

In this paper, we have proposed a MIMO transmission system, based on Algebraic space time coding. We have used the Golden code for antenna systems. Numerical results show that this code has a reasonable BER when working in a realistic environment. We know that the Golden code has good properties. in fact it is full rank, full rate, and non-vanishing determinant for an increasing rate. Based on theoretical and

simulation results, we demonstrate that it is interesting to implement the Golden code in the future generation of cellular networks. The difficulties to deal with a realistic condition push us to think how we can use that code when working on a selective channel without losing any of his spectral performance [6], [7].

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