

Frequency Offset Cancellation in OFDM Systems Using Special $(1 - D - D^2)$ Correlative Codes

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Abstract — This paper concentrates on the correlative coding used to cancel the frequency offset in orthogonal frequency division multiplexing (OFDM) systems in mobile radio applications, such as 802.11 (WLAN), 802.16 (WiMAX), and digital video broadcasting (DVB) systems, e.g., DVB-CS2. This frequency offset may result due to the time variations of the channel during one OFDM symbol interval, or the transmitter and receiver are not phase locked, which in turn destroys orthogonality between different subcarriers, leading to power leakage among the subcarriers, known as Inter-Carrier Interference (ICI). A carrier – to – interference ratio (CIR) will be derived for the $(1 - D - D^2)$ correlative codes and the modified scheme, and hence an expression for the bit error probability for the modified scheme will be given. A comparison between the different correlative coding orders will be held considering the CIR and bit error probability for each scheme at different normalized frequency offset values, and then for different bit energy to noise ratios.

Keywords — Carrier – to – interference ratio (CIR); correlative coding, inter – carrier interference (ICI); frequency offset; orthogonal frequency division multiplexing (OFDM).

I. INTRODUCTION

In OFDM systems, a serial bit stream is split into parallel streams that modulate orthogonal subcarriers. OFDM symbols have relatively long time duration, and hence a narrow bandwidth. Consequently, OFDM is robust against channel multipath dispersion, resulting in decreased equalizers complexity in high delay spread channels or high data rates. However, the increased symbol duration makes an OFDM system more sensitive to the time variations of mobile radio channels. In particular, the effect of the Doppler spreading destroys the orthogonality between the subcarriers resulting in inter-carrier interference (ICI), due to power leakage among subcarriers.

Several publications have proposed different schemes to suppress the frequency offset. Some have considered the self – cancellation method [1], or using the conjugate method, where each symbol and it's conjugate are sent on two consecutive subcarriers [2], but both methods decreased the bandwidth efficiency. Other

publications introduced the concept of frequency – domain equalization [3], where special equalizers are used to compensate the frequency offset. In [4], a time – domain windowing technique is used, which is equivalent to frequency – domain equalization. In this technique, the pulse is shaped using certain time windowing functions, which results in a relatively small side ripples in the frequency domain. A $(1 - D)$ correlative coding is demonstrated in [5].

In this paper, an expression for the carrier – to – interference ratio (CIR) of the $(1 - D - D^2)$ correlative codes and the proposed scheme will be derived, and a comparison between the three correlative coding schemes will be made, showing their CIR and bit error probability for various normalized frequency offset values. The results will show that the proposed scheme has higher CIR for a relatively larger frequency offset.

This paper is organized as follows: In section II a model of the OFDM system is described. An expression for ICI of the different orders of correlative codes will be derived in section III. A comparison between the different order correlative codes will be made with the simulations in section IV. Suggested trends and future work are given in section V. Finally a conclusion is given in section VI.

II. OFDM SYSTEM

An OFDM system with N subcarriers is shown in Fig. 1. In an OFDM system, a high bit rate stream of bits, with bit duration T_b , is transferred by a serial to parallel converter into parallel streams of lower bit rate and increased bit duration of NT_b , which makes it more robust against delay spread in multipath channels. The parallel streams then modulate N orthogonal subcarriers to satisfy:

$$\frac{1}{NT_b} \int_0^{NT_b} \exp(j2\pi f_i t) \exp(j2\pi f_j t) dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (1)$$

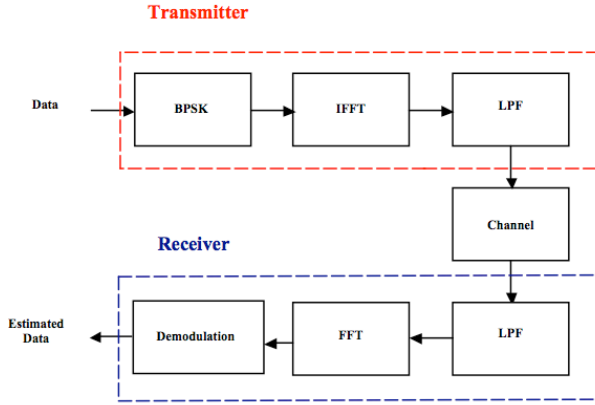


Fig. 1. A block diagram of an OFDM system.

$$\text{where } f_i = \frac{i-1}{NT_b}, \quad (i = 1, 2, \dots, N)$$

IFFT is used to generate these orthogonal subcarriers efficiently. The baseband-transmitted signal can be represented as:

$$s(t) = \frac{1}{\sqrt{NT_b}} \sum_{k=1}^N s_k e^{j2\pi f_k t}, 0 \leq t \leq NT_b, \quad f_k = \frac{k-1}{NT_b}. \quad (2)$$

where $s_k = \pm\sqrt{2E_s}$, for the case of binary phase shift keying (BPSK) which will be used throughout the analysis, and E_s denotes the average energy of the symbol s_k . A cyclic prefix is added after the OFDM symbol to decrease the effect of ICI, decreasing the bandwidth efficiency in return.

The received signal, assuming only frequency offset, takes the form:

$$r(t) = \frac{1}{\sqrt{NT_b}} \sum_{k=1}^N s_k e^{j2\pi(f_k + \Delta f)t} + n(t) \quad (3)$$

where Δf denotes the frequency offset, and $n(t)$ is the channel noise which is assumed to be AWGN.

The received stream is then passed through a bank of parallel correlators, where each correlator is tuned to one of the N subcarriers. One way of implementing this scenario is by using FFT and then passing through integrators over the symbol duration: $0 \rightarrow NT_b$. The streams are then passed through a parallel to serial convertor resulting the estimated high bit rate data stream.

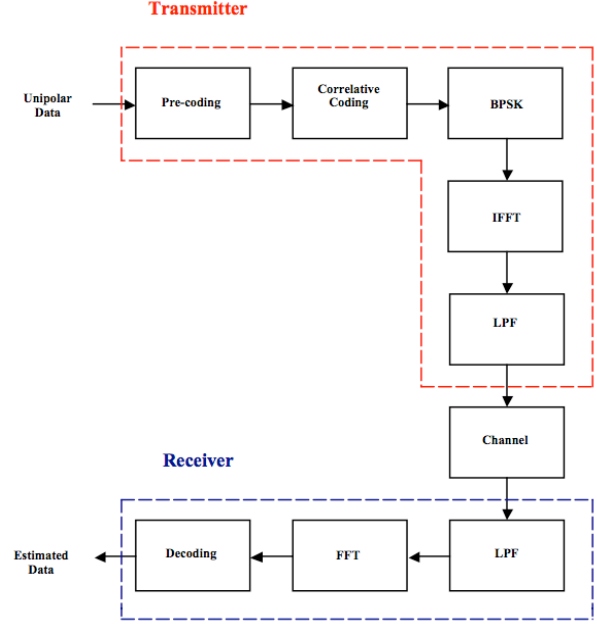


Fig. 2. A block diagram of an BPSK – OFDM system using correlative coding.

III. CIR EXPRESSIONS

The block diagram of a BPSK – OFDM system using correlative coding is shown in Fig. 2. The input signal sequence a_k , where k is the subcarriers' index with $k = 1, 2, \dots, N-1$, takes the values $-1, +1$, that fulfill the zero mean and independence conditions [5]. Using a coding correlation polynomial $F(D) = (1 - D)$, where D denotes the unit delay of the subcarrier index K , the coded symbols is expressed as:

$$b_k = a_k - a_{k-1}, \quad k \in \{1, N\}. \quad (4)$$

The coded symbols modulate N orthogonal subcarriers. The symbol b_k takes three possible values $\{-2, 0, 2\}$. Thus equation (5) introduced a correlation between successive symbols (b_k, b_{k-1}), and the independence condition is not maintained. Precoding is performed before the BPSK modulation to avoid the error propagation in the decoding process due to correlative coding [5].

The received signal may be expressed as the sum of the desired signal C_k , and the undesired ICI signal I_k

$$r_k = C_k + I_k \quad (5)$$

where $C_k = b_k S(0)$

$$I_k = \sum_{l=1, l \neq k}^N b_l S(l-k)$$

$$S(l-k) = \frac{\sin(\pi\epsilon)}{N \sin\left(\frac{\pi}{N}(\epsilon + l-k)\right)} \cdot \exp\left(j \frac{\pi}{N}((N-1)\epsilon - (l-k))\right) \quad (6)$$

where $\epsilon = \Delta f/NT_b$ denotes the normalized frequency offset.

As the modulation technique used is BPSK, this will yield $E[a_k] = E[b_k] = E[C_k] = E[I_k] = 0$.

From [5], it can be easily shown that the CIR can be expressed as:

$$CIR = \frac{\sin^2(\pi\epsilon)/(\pi\epsilon)^2}{\sum_{k=2}^N |S(l)|^2 - P_I} \quad (7)$$

$$\text{where } P_I = \frac{1}{2} \sum_{l=3}^N [S(l)S^*(l-1) + S(l-1)S^*(l)]$$

and $S(l)$ is given by (6) when $k=0$.

Using a coding correlation polynomial $F(D) = (1-D-D^2)$ the coded symbols may be expressed by

$$b_k = a_k - a_{k-1} - a_{k-2}, \quad k \in \{1, N\}. \quad (8)$$

which yields an CIR that can be expressed by:

$$CIR = \frac{\sin^2(\pi\epsilon)/(\pi\epsilon)^2}{\sum_{k=2}^N |s_k|^2 - P_I - P_{II}} \quad (9)$$

where $P_I = 0$

$$P_{II} = \frac{1}{3} \sum_{l=3}^N [S(l)S^*(l-2) + S(l-2)S^*(l)]$$

The proposed scheme results a better CIR by using weighted $(1-D-D^2)$ correlative coding (modified) to improve the Interference cancellation in (9). So in the proposed coding correlation polynomial $F(D) = (2-D-D^2)$, the coded symbols may be expressed as:

$$b_k = 2a_k - a_{k-1} - a_{k-2}, \quad k \in \{1, N\}. \quad (10)$$

which results in a CIR that can be expressed by:

$$CIR = \frac{\sin^2(\pi\epsilon)/(\pi\epsilon)^2}{\sum_{k=2}^N |s_k|^2 - P'_I - P'_{II}} \quad (11)$$

where

$$P'_I = \frac{1}{6} \sum_{l=3}^N [S(l)S^*(l-1) + S(l-1)S^*(l)]$$

$$P'_{II} = \frac{1}{3} \sum_{l=3}^N [S(l)S^*(l-2) + S(l-2)S^*(l)]$$

IV. DISCUSSION AND COMPARISON

The CIR for ordinary OFDM and $(1-D)$ correlative codes for different normalized frequency offset (ϵ) is given in Fig. 3. It can be seen that for a given ϵ , the CIR of the $(1-D)$ correlative codes is better than the ordinary OFDM by about 3.5 dB.

A comparison between the different correlative coding schemes is given in Fig. 4, considering the CIR for various values of ϵ . The $(1-D-D^2)$ can be used where small ϵ takes place, as it has the best CIR for $0 < \epsilon < 0.14$. For larger values of ϵ , the modified scheme has better CIR than the other schemes. It can be noted that a peak occurs at $\epsilon = 0.12$ for the $(1-D-D^2)$ and at $\epsilon = 0.18$ for the modified scheme, this is due to a complete interference cancellation at these values.

The modulated symbols b_k for the different correlative coding schemes are given in Table I. It's obvious that the values take the form of M-ary ASK and not the BPSK. This is due to the correlation that the correlative coding has created between the BPSK independent symbols. As a result, a viterbi based decoder will be used at the receiver.

An upper boundary of the bit error probability for the proposed correlative coding will be obtained from the transfer function of the finite state machine related to the proposed scheme [6], and will be expressed in the presence of ICI as:

$$P_e \leq \frac{Q\left(\sqrt{5E_b/2\left(\frac{N_o}{2} + \sigma_{ICI}\right)}\right)}{\left[1 - 2\exp\left(-E_b/4\left(\frac{N_o}{2} + \sigma_{ICI}\right)\right)\right]^2} \quad (12)$$

where E_b denotes the average bit energy, N_o is the average power of the noise, assuming an AWGN, and σ_{ICI} is the average power of the ICI, which is given by the expressions of CIR for the different schemes, and is assumed to follow a Gaussian distribution. The relation between the average signal-to-noise ratio per bit (E_b/N_o), and the bit error probability is shown in Fig. 5. It can be shown that the E_b/N_o of the modified $(1-D-D^2)$ correlative coding is better than the $(1-D-D^2)$ correlative codes and the gain is noticeably increasing as the (E_b/N_o) increases. For example, the gain improvement of the modified $(1-D-D^2)$ is about 3 dB at a bit error probability of 10^{-3} , and about 4 dB at a bit error probability of 10^{-4} .

TABLE I

	Output symbols	M
$1 - D$	-2, 0, 2	3
$1 - D - D^2$	-3, -1, 1, 3	4
Modified $1 - D - D^2$	-4, -2, 0, 2, 4	5

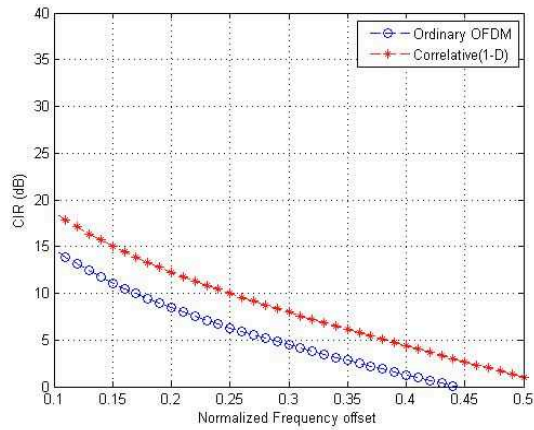


Fig. 3. Comparison between ordinary OFDM and $(1 - D)$ correlative coding using BPSK with respect to CIR for different normalized frequency offset values.

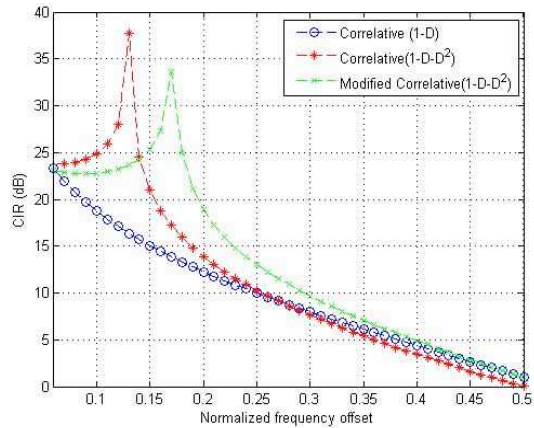


Fig. 4. Comparison between $(1 - D)$, $(1 - D - D^2)$, and modified $(1 - D - D^2)$ correlative codes using BPSK with respect to CIR for different normalized frequency offset values.

A comparison between the $(1 - D - D^2)$, and the modified $(1 - D - D^2)$ at different values of the normalized frequency offset (ϵ) and (E_b/N_o) of 5 dB and 10 dB are shown in Fig. 6 and Fig. 7 respectively. At an (E_b/N_o) of 5 dB, the bit error probability of the $(1 - D - D^2)$ is 8 times that of the modified $(1 - D - D^2)$ at $\epsilon =$

0.15, and 3 times at $\epsilon = 0.3$, while at (E_b/N_o) of 10 dB, the bit error probability of the $(1 - D - D^2)$ is almost the same as that of the modified $(1 - D - D^2)$ at $\epsilon = 0.15$, and about 10^7 times at $\epsilon = 0.3$. These results are consistent with the results obtained from the CIR, that the modified $(1 - D - D^2)$ is better than the $(1 - D - D^2)$ at high ϵ .

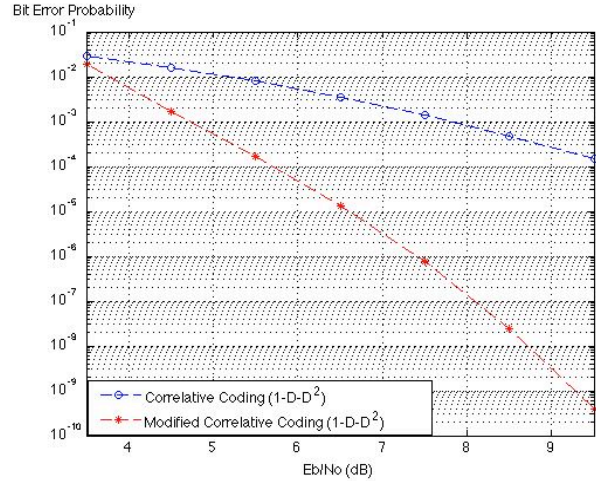
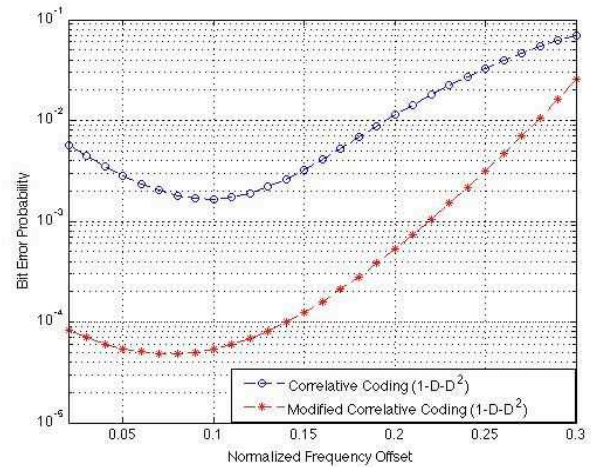


Fig. 5. Comparison between $(1 - D - D^2)$, and modified $(1 - D - D^2)$ correlative codes using BPSK with respect to bit error probability for different E_b/N_o values.

Fig. 6. Comparison between $(1 - D - D^2)$, and modified



$(1 - D - D^2)$ correlative codes using BPSK with respect to bit error probability for different values of ϵ at $E_b/N_o = 5$.

V. FUTURE WORK

The analysis of correlative coding can be held for higher orders of correlative coding, although this might increase the complexity of the OFDM system. Different modulation schemes can also be used in the analysis in a similar approach.

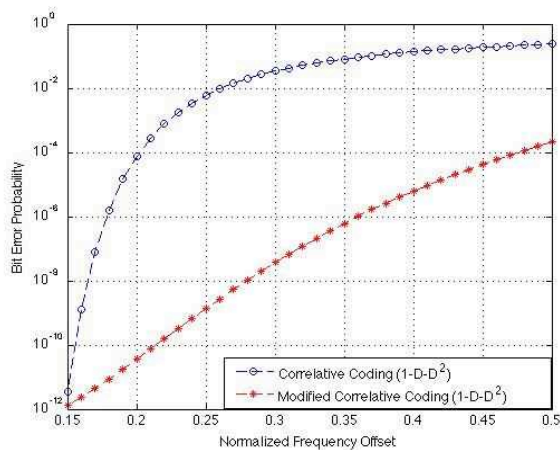


Fig. 7. Comparison between $(1 - D - D^2)$, and modified $(1 - D - D^2)$ correlative codes using BPSK with respect to bit error probability for different values of ε at $E_b/N_o = 10$.

VI. CONCLUSION

The performance of the OFDM systems using correlative coding is studied for different orders. Comparing the CIR for the different schemes, the modified scheme is found to improve the CIR for a more wide range of frequency offset than the conventional correlative coding scheme.

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