

Characteristics of Sequential Detection in Cognitive Radio Networks

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Abstract—Sequential Analysis is an effective detection procedure for spectrum sensing in Cognitive Radio (CR) Networks. On average, given P_{fa} (probability of false alarm) and P_{md} (probability of mis detection) and low SNR regime, it requires less independent samples for the primary users (PU) detection comparing to the Neyman-Pearson (NP) test. This paper deal with the evaluation of the cumulants of the distribution (PDF) of random time of the sequential analysis, however, for simplicity we consider only Gaussian approximation. It is assumed that the PU and secondary users (SU) are sharing the same frequency bandwidth and for spectrum sensing SU's apply incoherent diversity combining of diversity branches with fading described by the Generalized Gaussian (GG) model.

I. INTRODUCTION

Cognitive Radio (CR) has attracted a significant research effort during the past decade. Operation of CR assumes some sort of coexistence of licensed (primary) users (PU) and secondary (unlicensed) user (SU) in the same time-space-frequency resource [1]-[6]. In one modality SU cannot transmit at all if PU is active. Alternatively, SU can transmit even if PU is active, however level of its radiated must be limited below prescribed level [1], [2]. In both cases one has determine presence or absence of PU and identifying spectrum opportunities for transmission. Spectrum hole detection is quantified by specifying probabilities P_{FA} , and P_{MD} , minimum detectable signal level and/or minimum SNR for sensing. At the same time complexity and computational effort as well as power consumption must be minimized. While most of the efforts in analysis of detection is focused on the Neyman-Pearson detectors [7], it is well known [8], that sequential detection provides substantially faster operation on average. The expression for average detection time can be easily obtained for the case of low SNR [8].

Obtaining other statistical characteristics, such as complete probability density of the decision time, is rather difficult. Approximation by the Wald PDF or by first few cumulants were suggested only in case of significantly different thresholds of detection, *i.e.* for $P_{FA} \ll P_{MD}$. The latter is not always applicable for CR network. In addition, modern receivers tend to use multiple antennas and it is important to understand effects of combining on performance of the

sequential detector. We consider situation of non-coherent combining of two branches, each suffering from Generalized Gaussian fading as described in [9]-[11].

The paper is organized as follows. Section II is dedicated to describe the Generalized Gaussian Channel Model. Basics of sequential analysis are presented at section III. Section IV is dedicated to the cumulants of the PDF of the time of sequential analysis. At section V general results of the sequential analysis of the incoherent diversity detection for general Gaussian channel model are presented. Section VI is devoted to conclusion remarks.

II. GENERALIZED GAUSSIAN CHANNEL MODEL

Majority of existing fading channel models are based on a statistical representation of the magnitude and phase (or in-phase and quadrature components) of a complex channel transfer function. Often, such modelling could be accomplished by postulating Gaussian PDF for both the in-phase and quadrature components x and y [9], *i.e.* their joint PDF $p(x, y)$ is given by :

$$p(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp \left[-\frac{(x - m_x)^2}{2\sigma_x^2} - \frac{(y - m_y)^2}{2\sigma_y^2} \right], \quad (1)$$

where σ_x^2 , σ_y^2 and m_x , m_y are variances and expectations of x and y quadrature components respectively. Defining the magnitude of $x + jy$ as $\gamma = \sqrt{x^2 + y^2}$ and the phase as $\phi = \arctan \frac{y}{x}$ one obtains

$$p(\gamma) = \frac{\gamma}{2\pi\sigma_x\sigma_y} \int_0^{2\pi} \exp \left\{ -\frac{(\gamma \cos \phi - m_x)^2}{2\sigma_x^2} - \frac{(\gamma \sin \phi - m_y)^2}{2\sigma_y^2} \right\} d\phi. \quad (2)$$

It is well known that (2) describes a large number of practical important distributions, such as Beckman, Hoyt, Rice and Rayleigh and truncated Gaussian, by varying four parameters: $\{m_x, m_y\}$ and $\{\sigma_x^2, \sigma_y^2\}$ [9], [10]. Therefore, PDF given by (2) is known as “four-parametric distribution” [10].

The following series representation of $p(\gamma)$ are often useful in practical calculations:

$$p(\gamma) = \frac{\gamma}{\sigma_x \sigma_y} e^{-\frac{m_y^2 + \gamma^2}{2\sigma_y^2} - \frac{m_x^2}{2\sigma_x^2}} \sum_{k=0}^{\infty} \frac{H_{2k}(\alpha)}{(2k)!! 2^k} \cdot \gamma^k \left(\frac{\sigma_y^2}{m_y} \right)^k \left(\frac{1}{\sigma_x^2} - \frac{1}{\sigma_y^2} \right) I_k \left(\frac{\gamma m_y}{\sigma_y^2} \right), \quad (3)$$

where

$$\alpha = \frac{m_x}{\sqrt{2\sigma_y^2} \sqrt{\frac{1}{\sigma_x^2} - \frac{1}{\sigma_y^2}}}; \quad (\sigma_x^2 \leq \sigma_y^2)$$

Here $I_0(z)$ is a modified Bessel function of zero order and $H_n(x)$ are Hermitian polynomials [12].

Let us introduce the following normalized parameters, which are useful in further calculations :

$$q^2 = \frac{m_x^2 + m_y^2}{\sigma_x^2 + \sigma_y^2}; \quad \beta^2 = \frac{\sigma_x^2}{\sigma_y^2}, \quad \phi_0 = \arctan \frac{m_y}{m_x} < \gamma^2 > = \gamma_0^2 + \sigma_x^2 + \sigma_y^2; \quad \gamma_0^2 = m_x^2 + m_y^2. \quad (4)$$

It is often beneficial to approximate full four-parametric distribution with the less general but yet very flexible Nakagami distribution [9]. The equivalent parameter m of Nakagami distribution is given by:

$$m = \frac{(1 + \beta^2)^2 (1 + q^2) 2}{2[1 + \beta^4 + 2q^2(1 + \beta^2)(\beta^2 \cos \phi_0 + \sin^2 \phi_0)]}. \quad (5)$$

Beckman distribution follows from (3) when $m_y = 0, \gamma_0 = |m_x|$; while Hoyt PDF appears when $\sigma_x^2 \neq \sigma_y^2, \sigma_0 = m_x = m_y = 0$. Rayleigh and truncated Gaussian follows when $\sigma_0 = 0$ and $\sigma_x^2 \rightarrow 0, m_x = m_y = 0$ respectively. It is worth to mention that Nakagami distribution is only an approximation to the four-parameter case, but it adequately represents the of the variation of the functional four-parameter PDF form.

III. SEQUENTIAL ANALYSIS OF A. WALD

In contrast to NP test where the logarithm of the Maximum Likelihood Ratio (MLR) is compared to a single threshold Λ_0 at a predefined and fixed observation interval T , the sequential test of A. Wald [8] compares MLR to two thresholds Λ_1 and Λ_2 sequentially, until certain condition is not satisfied. In both cases parameters of the test are calculated based on required P_{FA} and P_{MD} as described in [13], [8]. However, in NP test duration of testing is chosen based on desired probabilities of errors, while in the Wald test the decision time is allowed to fluctuate, resulting in the fastest (on average) detector. As a results, a better average decision time could be achieved. The threshold in question can be calculated (upper bounded) as [8]

$$\Lambda_2 = \ln \frac{P_{md}}{1 - P_{fa}} = \ln C, \quad \Lambda_1 = \ln \frac{1 - P_{md}}{P_{fa}} = \ln D. \quad (6)$$

The decision is not made as long as the MLR is between the thresholds Λ_1 and Λ_2 . Decision in favor of the hypothesis $\mathcal{H}_0$ (absence of PU) is made if the MLR becomes smaller than Λ_1 and the hypothesis $\mathcal{H}_1$ (presence of PU) is admitted if MLR

exceeds Λ_2 . So, in contrary to NP test, the time instant for decision making for sequential analysis is not fixed and it is generally a random variable.

It is possible to obtain average number of samples (mean decision time) for accepting one of the two hypothesis, depending on which one of the two hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$ is correct [8]:

$$M\{\nu|\mathcal{H}_0\} = \frac{1}{A} \left[(1 - P_{FA}) \ln \frac{P_{MD}}{1 - P_{FA}} + P_{FA} \ln \frac{1 - P_{MD}}{P_{FA}} \right] \quad (7)$$

$$M\{\nu|\mathcal{H}_1\} = \frac{1}{B} \left[P_{MD} \ln \frac{P_{MD}}{1 - P_{FA}} + (1 - P_{MD}) \ln \frac{1 - P_{MD}}{P_{FA}} \right].$$

Here

$$A = M\{\ln \ell(\cdot)|\mathcal{H}_0\}; \quad B = M\{\ln \ell(\cdot)|\mathcal{H}_1\};$$

and $\ell(\cdot)$ is a MLR after ν steps.

Furthermore, since the decision time $T = \nu T_s$ (here T_s is the sampling interval and ν is a number of steps to make a decision) is a random variable, it could be described by its PDF $p_T(t)$. Exact shape of this PDF is not known, although some approximations are suggested by Wald [8] for the asymptotic cases $D = \text{const}$, $C \rightarrow 0$ and $C = \text{const}$, $D \rightarrow \infty$: the approximate solution is known as the Wald's PDF. Estimates of the variance σ_N^2 are also known under the assumption that $P_{fa} \ll P_{md}$. The original book [8] also contains a procedure for determining the characteristic function and the moments of the PDF $p_T(t)$.

Derivations of the parameters of the distribution $p_T(t)$ are based on the notion of the so-called *Operation Characteristic* (OC) $L(a)$ which is defined as [8] for the case of small value of a_1 (low SNR case):

$$L(a) = \frac{D^{h(a)-1}}{D^{h(a)} - C^{h(a)}}, \quad (8)$$

Here a is the parameter of hypothesis testing such that $a = a_0$ corresponds to the hypothesis $\mathcal{H}_0$ while $a = a_1$ corresponds to the hypothesis $\mathcal{H}_1$. In detection in fading problems a is simply proportional to SNR of the PU, so that $a_0 = 0$ if PU is absent and equal to the average link SNR (a_1) in the case of $\mathcal{H}_1$. Furthermore, $h(a)$ is a unique non-zero root of the equation [8]:

$$\int_{-\infty}^{\infty} \left[\frac{p_{a_1}(x)}{p_{a_0}(x)} \right]^{h(a)} p_a(x) dx = 1 \quad (9)$$

where $p_a(x)$ is PDF of the observation based on the parameter a and $p_{a_1}(x)/p_{a_0}(x)$ is a likelihood ratio for two hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$. It was shown at [8] that $h(a_1) = -1$ and $h(a_0) = 1$.

IV. CUMULANT ANALYSIS OF THE PDF OF THE RANDOM TIME OF SEQUENTIAL ANALYSIS OR RANDOM SAMPLE SIZE

Since the analysis time T and the number of samples ν needed to make decision are related through $T = \nu T_s$, where T_s is a fixed sampling interval, we will focus on determining

parameters of distribution of ν . The average time (7) can be expressed in terms of OC $L(a)$ as [13], [8]:

$$M_a\{\nu\} = \frac{L(a) \ln C + 1 - L(a) \ln D}{M_a\{\xi\}}, \quad (10)$$

where C and D are thresholds defined in (7) and

$$M_a\{\xi\} = \int_{-\infty}^{\infty} \ln \frac{p_{a_1}(x)}{p_{a_0}(x)} p_a(x) dx, \quad (11)$$

Here

$$\xi(x) = \ln \frac{p_{a_1}(x)}{p_{a_0}(x)}$$

is the log-likelihood ratio for the two hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$.

The cumulant generating function $\Psi_T(\vartheta) = \ln \Theta_T(\vartheta)$ where

$$\Theta_T(\vartheta) = \int_{-\infty}^{\infty} p_T(x) \exp(j\vartheta x) dx, \quad (12)$$

can be represented by a series with coefficients κ_i being cumulants of $p_T(t)$ [14]

$$\Psi_T(\vartheta) = \ln \Theta_T(j\vartheta) = \sum_{i=1}^{\infty} \frac{\kappa_i}{i!} (j\vartheta)^i. \quad (13)$$

Given $\Theta_T(\vartheta)$ or $\Psi_T(\vartheta)$, cumulants in question could be found as

$$\kappa_i = j^{-i} \frac{d^i}{d\vartheta^i} \ln \Theta_T(\vartheta) |_{\vartheta=0} = j^{-i} \Psi^{(i)}(0).$$

In particular

$$\kappa_1 = -j\Psi'_T(0), \quad \kappa_2 = -\Psi''_T(0) = D_a \quad (14)$$

If only two cumulants κ_1 and κ_2 are taken into account, one obtains Gaussian approximation of the real distribution $W_T(t) \approx N(\kappa_1, \kappa_2)$; while if first four cumulants κ_1 - κ_4 are taken into account the so called curtosis approximation is obtained. The latter is more accurate approximation for values near the mean, however it is poor in approximating tails of the distribution (however it could be used in obtaining exponentially tight bounds). Given the complexity of calculating higher order cumulants we focus here only on the Gaussian approximation.

It is shown in [8] that (9) leads to the following expression for the characteristic function $\Theta_T(j\vartheta)$:

$$\theta_\nu(j\vartheta) = \frac{D^{t_2(\vartheta)} - D^{t_1(\vartheta)} + C^{t_1(\vartheta)} - C^{t_2(\vartheta)}}{C^{t_1(\vartheta)} D^{t_2(\vartheta)} - D^{t_1(\vartheta)} C^{t_2(\vartheta)}}, \quad (15)$$

where $t(\vartheta)$ are roots of the equation:

$$g(t) = \int_{-\infty}^{\infty} \left[\frac{p_{a_1}(x)}{p_{a_0}(x)} \right]^{t(\vartheta)} p_a(x) dx = e^{-j\vartheta}, \quad (16)$$

which satisfy the following conditions:

$$\lim_{\vartheta \rightarrow 0} t_1(\vartheta) = 0, \quad \lim_{\vartheta \rightarrow 0} t_2(\vartheta) = h(a). \quad (17)$$

Using (16) and expanding the exponent under the integral sign into the Taylor series one obtains the following relationship between moments $M_N(\xi)$ of the log-likelihood ξ and values of $t_{1,2}(\vartheta)$:

$$g(t) = \int_{-\infty}^{\infty} e^{t(\vartheta)\xi(x)} p_a(x) dx, \quad (18)$$

and

$$\ln g(t) = M_a\{\xi\} t(\vartheta) + \frac{D_a\{\xi\}}{2} t^2(\vartheta) + \dots = -j\vartheta. \quad (19)$$

The algorithm for evaluating any cumulant κ_i can be accomplished using the following steps:

- 1) Differentiate the logarithm of (15) i -times and evaluate it when $\vartheta \rightarrow 0$.
- 2) Differentiating (16) or (19) i -times and evaluate it when $\vartheta \rightarrow 0$ taking into account that $t_1(0) = 0, t_2(0) = h(a)$.
- 3) Obtain recurrent expressions for $t_k^{(i)}(0)$ from $t_k^{(i-1)}(0), t_k^{(i-2)}(0), \dots$ ($k = 1, 2$).
- 4) This will define $\psi^{(i)}(0)$ and, therefore, κ_i

After some simple but laborious calculus one obtains the following values of derivatives of t_1 and t_2 :

$$\begin{aligned} t'_1(0, a_1) &= -\frac{j}{M_{a_1}\{\xi\}} & t'_1(0, a_0) &= -\frac{j}{M_{a_0}\{\xi\}} \\ t'_2(0, a_1) &= -\frac{j}{M_{a_0}\{\xi\}} & t'_2(0, a_0) &= -\frac{j}{M_{a_1}\{\xi\}} \\ t''_1(0, a_1) &= \frac{D_{a_1}\{\xi\}}{M_{a_1}^3\{\xi\}} & t''_1(0, a_0) &= \frac{D_{a_0}\{\xi\}}{M_{a_0}^3\{\xi\}} \\ t''_2(0, a_1) &= \frac{D_{a_0}\{\xi\}}{M_{a_0}^3\{\xi\}} & t''_2(0, a_0) &= \frac{D_{a_1}\{\xi\}}{M_{a_1}^3\{\xi\}} \end{aligned} \quad (20)$$

In the case of κ_1 this immediately results in expressions (7) for the mean values (or first cumulants). The variance $D_a = \kappa_2$ can then be found from (20) and (15) to produce:

$$\begin{aligned} D_{a_0}\{\xi\} &= \left[\frac{(D^2 - D + C - DC) \ln(C)}{M_{a_0}^3\{\xi\}} + \frac{(C^2 - C + D - DC) \ln(D)}{M_{a_0}^3\{\xi\}} \right] D_{a_0}\{\xi\} + \left[\frac{2C - 2DC}{M_{a_1}\{\xi\} M_{a_0}\{\xi\}} + \right. \\ &\quad \left. \frac{DC + C - D - 1}{M_{a_1}^2\{\xi\}} \right] \ln^2(C) + \left[\frac{4DC - 2C - 2D}{M_{a_1}\{\xi\} M_{a_0}\{\xi\}} + \right. \\ &\quad \left. \frac{2 - 2DC}{M_{a_1}^2\{\xi\}} \right] \ln(D) \ln(C) + \left[\frac{2D - 2DC}{M_{a_1}\{\xi\} M_{a_0}\{\xi\}} + \right. \\ &\quad \left. \frac{DC - D + C - 1}{M_{a_0}^2\{\xi\}} \right] \ln^2(D). \end{aligned} \quad (21)$$

If $D \rightarrow \infty, C = \text{const}$ one obtains the well known Wald's formula [8]

$$D_{a_0}\{\nu\} = \frac{D_{a_0}\{\xi\}}{M_{a_0}^3\{\xi\}} \ln C. \quad (22)$$

Similarly,

$$D_{a_1}\{\xi\} = \left[\frac{(CD^2 - C^2D + C^2 - CD)\ln(C)}{M_{a_1}^3\{\xi\}} + \frac{(D^2 - CD^2 + C^2D - CD)\ln(D)}{M_{a_1}^3\{\xi\}} \right] D_{a_1}\{\xi\} + \left[\frac{2C - D^2 - 2DC}{M_{a_1}\{\xi\}M_{a_0}\{\xi\}} + \frac{C^2D - D^2C^2 + CD - CD^2}{M_{a_1}^2\{\xi\}} \right] \ln^2(C) + \left[\frac{4DC - 2CD^2 - 2CD}{M_{a_1}\{\xi\}M_{a_0}\{\xi\}} + \frac{2C^2D^2 - 2DC}{M_{a_1}^2\{\xi\}} \right] \ln(D)\ln(C) + \left[\frac{2C^2D - 2CD}{M_{a_1}\{\xi\}M_{a_0}\{\xi\}} + \frac{DC - C^2D + CD^2 - D^2C^2}{M_{a_1}^2\{\xi\}} \right] \ln^2(D). \quad (23)$$

and if $D = \text{const}$, $C \rightarrow 0$, one obtains

$$D_{a_1}\{\nu\} = \frac{D_{a_1}\{\xi\}}{M_{a_1}^3\{\xi\}} \ln D. \quad (24)$$

Equations for the special cases (22) and (24) are well known while the general equations (21) and (23) are valid for any relations between P_{FA} and P_{MD} which to the best of our knowledge are not reported in the literature. As it can be seen the second cumulant κ_2 for the fixed thresholds D and C to depend to $M_a\{\xi\}$ and $D_a\{\xi\}$ for different hypothesis $\mathcal{H}_0, \mathcal{H}_1$. The last one depends on the detection scenario $p_a(x)$ and will be considered at section V.

The probability that the duration of the Wald test exceeds some preset value $\nu_{\max}$ is calculated by

$$\int_{\nu_{\max}}^{\infty} p(\nu) d\nu \approx Q\left(\frac{\nu_{\max} - M_1}{\sigma}\right). \quad (25)$$

It is often more practical to use $\nu_{\max}$ instead of the average $M_a\nu$ provides a realistic figure for comparison with NP test. Moreover $\nu_{\max}$ can be used comparing characteristics of the classical sequential analysis considered here with new proposals of the sequential analysis (see, for example [15]). The results for those comparisons will be published elsewhere.

V. SEQUENTIAL ANALYSIS OF THE INCOHERENT DIVERSITY DETECTION FOR GENERALIZED GAUSSIAN FADING MODEL

A. Statistically Independent Fading in the Diversity Branches

For this case, the quadratic diversity addition is the maximum likelihood (ML) optimum incoherent algorithm in the case of known CSI at the receiver side of the link [9], [10]¹.

$$D \leq \sum_{k=1}^Q V_k^2 + \tilde{V}_k^2 \leq C, \quad (26)$$

where $V_k, \tilde{V}_k$ are normally distributed statistically independent signals of the quadratic channels (quadrature components), C, D are thresholds (see above), and $\tilde{V}_k$ is a Hilbert transform of V_k . Algorithm (26) for the case of close hypothesis is

¹If CSI is unknown or partially known, the solution is generalized ML ratio (see for example [4], etc)

optimum for criteria of minimum average sample size (time of analysis) as well but can be considered as quasi optimum for any level of SNR.

The parameters of quadrature components can be calculated as in [10]. In particular, For hypothesis $\mathcal{H}_1$ one obtains:

$$\begin{aligned} M_1\{V_k\} &= m_{x_k} \sqrt{2d_k} \sqrt{\frac{1 + 2h_{x_k}^2}{2h_{x_k}^2}} \\ M_1\{\tilde{V}_k\} &= m_{y_k} \sqrt{2d_k} \sqrt{\frac{1 + 2h_{y_k}^2}{2h_{y_k}^2}} \\ D_1\{\tilde{V}_k\} &= 2h_{x_k}^2, \quad D_1\{\tilde{V}_k\} = 2h_{y_k}^2 \end{aligned} \quad (27)$$

where

$$d_k = \frac{2E_k}{N_0}, \quad h_{x_k}^2 = \sigma_{x_k}^2 \frac{E_k}{N_0} \text{ and } h_{y_k}^2 = \sigma_{y_k}^2 \frac{E_k}{N_0}.$$

Similarly, in the case of the hypothesis $\mathcal{H}_0$ similar quantities are given by:

$$\begin{aligned} M_0\{V_k\} &= m_{x_k} \sqrt{2d_k} \frac{1}{2h_{x_k}^2} \sqrt{\frac{2h_{x_k}^2}{1 + 2h_{x_k}^2}} \\ M_0\{\tilde{V}_k\} &= m_{y_k} \sqrt{2d_k} \frac{1}{2h_{y_k}^2} \sqrt{\frac{2h_{y_k}^2}{1 + 2h_{y_k}^2}} \\ D_0\{V_k\} &= \frac{2h_{x_k}^2}{1 + 2h_{x_k}^2}, \quad D_0\{\tilde{V}_k\} = \frac{2h_{y_k}^2}{1 + 2h_{y_k}^2}. \end{aligned} \quad (28)$$

In the general case (arbitrary $h_{x_k}^2, h_{y_k}^2$), the quantity $\sqrt{\sum_{k=1}^Q V_k^2 + \tilde{V}_k^2}$ is four parametric (see section II); some simplifications can be done for $h_{x_k}^2, h_{y_k}^2 \gg 1$ (see for example [10]). However, this is not situation encountered in cognitive radios. Therefore, a good and simple approximation for low SNR is required for the four-parametric distribution. It has been shown that [10] that Nakagami PDF is a good fit, especially for low SNR scenario. In order to construct such approximation let us first define the parameters $\phi_{p_k}, \beta_{k_n}^2$ and $q_{k_n}^2$ by means of equations (27)-(28). The next step is to find the equivalent parameter m for each hypothesis $\mathcal{H}_1, \mathcal{H}_0$, applying eq. (5) from [5], $n = 0, 1$:

$$m_n^{(k)} = \frac{(1 + \beta_{k_n}^2)(1 + q_{k_n}^2)^2}{2[1 + \beta_{k_n}^4 + 2q_{k_n}^2(1 + \beta_{k_n}^2)(\beta_{k_n}^2 \cos \phi_{p_{k_n}} + \sin^2 \phi_{k_n})]} \quad (29)$$

One can derive that $V_k^2 + \tilde{V}_k^2$ is a Γ -distributed random variable for each of hypothesis $\mathcal{H}_0, \mathcal{H}_1$ with parameters:

$$\alpha_n^{(k)} = m_n^{(k)}, \quad \beta_n^{(k)} = \frac{\Omega_n^k}{m_n^{(k)}}, \quad n = 0, 1$$

The final PDF for ξ can be found with the results of [16].

In the following let us consider a corresponding to constant special a case for fading parameters independent on k . (29):

$$\frac{\Omega^k}{m_k^{(k)}} = \text{const}. \quad (30)$$

In this case $p(\xi)$ turns out to be a Γ -distribution with parameters [17]:

$$\alpha_\Sigma = \sum_{k=1}^Q m^{(k)}, \quad \beta_\Sigma = \frac{\sum_{k=1}^Q \Omega^{(k)}}{\sum_{k=1}^Q m^{(k)}} = \frac{\Omega_\Sigma}{m_\Sigma}. \quad (31)$$

Thus, analysis of (26) can be transformed to the problem of sequential analysis of the two simple hypothesis ($\mathcal{H}_0, \mathcal{H}_1$) with Γ -distributions with parameters $(\alpha_{\Sigma_1}, \beta_{\Sigma_1})$ and $(\alpha_{\Sigma_0}, \beta_{\Sigma_0})$. Let us evaluate the following parameters $M_{a_1}\{\xi\}, D_{a_1}\{\xi\}, M_{a_0}\{\xi\}, D_{a_0}\{\xi\}$ needed for evaluation of $M_a\{\nu\}$ and $D_a\{\nu\}$:

$$\begin{aligned} M_{a_1}\{\xi\} &= [\alpha_{\Sigma_0} \ln \beta_{\Sigma_0} - \alpha_{\Sigma_1} \ln \beta_{\Sigma_1} + \ln \Gamma(\alpha_{\Sigma_0}) - \ln \Gamma(\alpha_{\Sigma_1})] \\ &\times \left\{ (\alpha_{\Sigma_1} - \alpha_{\Sigma_0}) < \ln \xi >_{a_0} - \left(\frac{1}{\beta_{\Sigma_1}} - \frac{1}{\beta_{\Sigma_0}} \right) < \xi >_{a_0} \right\} \\ M_{a_0}\{\xi\} &= [\alpha_{\Sigma_0} \ln \beta_{\Sigma_0} - \alpha_{\Sigma_1} \ln \beta_{\Sigma_1} + \ln \Gamma(\alpha_{\Sigma_0}) - \ln \Gamma(\alpha_{\Sigma_1})] \\ &\times \left\{ (\alpha_{\Sigma_1} - \alpha_{\Sigma_0}) < \ln \xi >_{a_0} - \left(\frac{1}{\beta_{\Sigma_1}} - \frac{1}{\beta_{\Sigma_0}} \right) < \xi >_{a_0} \right\} \\ D_{a_1}\{\xi\} &= \\ &[(\alpha_{\Sigma_0} \ln \beta_{\Sigma_0} - \alpha_{\Sigma_1} \ln \beta_{\Sigma_1}) + \ln \Gamma(\alpha_{\Sigma_0}) - \ln \Gamma(\alpha_{\Sigma_1})]^2 \\ &\left\{ (\alpha_{\Sigma_1} - \alpha_{\Sigma_0})^2 < \ln^2 \xi >_{a_1} - 2(\alpha_{\Sigma_1} - \alpha_{\Sigma_0}) \left(\frac{1}{\beta_{\Sigma_1}} - \frac{1}{\beta_{\Sigma_0}} \right) \right. \\ &\times \left. < \xi \ln \xi >_{a_1} + \left(\frac{1}{\beta_{\Sigma_1}} - \frac{1}{\beta_{\Sigma_0}} \right)^2 < \xi^2 >_{a_1} \right\} - M_{a_1}^2\{\xi\} \\ D_{a_0}\{\xi\} &= \\ &[(\alpha_{\Sigma_0} \ln \beta_{\Sigma_0} - \alpha_{\Sigma_1} \ln \beta_{\Sigma_1}) + \ln \Gamma(\alpha_{\Sigma_0}) - \ln \Gamma(\alpha_{\Sigma_1})]^2 \\ &\left\{ (\alpha_{\Sigma_1} - \alpha_{\Sigma_0})^2 < \ln^2 \xi >_{a_0} - 2(\alpha_{\Sigma_1} - \alpha_{\Sigma_0}) \left(\frac{1}{\beta_{\Sigma_1}} - \frac{1}{\beta_{\Sigma_0}} \right) \right. \\ &\times \left. < \xi \ln \xi >_{a_0} + \left(\frac{1}{\beta_{\Sigma_1}} - \frac{1}{\beta_{\Sigma_0}} \right)^2 < \xi^2 >_{a_0} \right\} - M_{a_0}^2\{\xi\}, \end{aligned} \quad (32)$$

with auxiliary parameters given by:

$$\begin{aligned} < \ln \xi >_a = \ln \beta_\Sigma \cdot \Psi(\alpha_\Sigma) \\ < \ln^2 \xi >_a = \ln^2 \beta_\Sigma \Psi^2(\alpha_\Sigma) + \zeta(2, \alpha_\Sigma - 1) \\ < \xi \ln \xi >_a = (\beta_\Sigma \alpha_\Sigma \ln \beta_\Sigma) \Psi(\alpha_\Sigma + 1) \beta_\Sigma^2 \\ < \xi^2 >_a = \alpha_\Sigma (\alpha_\Sigma + 1) \beta_\Sigma^2 \end{aligned} \quad (33)$$

Here $\Psi(x)$ is the Euler Psi-function [18] and $\zeta(x, y)$ is the Reiman zeta function [18]. Substitution of (32), (33) into (11), (21), (23) provides with representation of dependance for $M_a\{\nu\}$ and $D_a\{\nu\}$ ($a = a_0, a_1$) from parameters of G.G. fading models, SNR, etc.

B. Special Case of Covariance Matrix for Diversity Branches

Let us consider the diversity combining algorithm assuming that diversity branches are subject to correlated fading, described by GG model. In particular we consider special case of the covariance matrix for quadrature components $\{V_{k-}\}^Q$ and $\{\tilde{V}_k\}^Q$: assume that across the branches all V_k or $\tilde{V}$ are correlated with the same coefficient R between the

same quadrature components and there are no cross-correlation between all $\{V_k\}$ and $\{\tilde{V}_k\}$.

Certainly, this assumption restricts the type of covariance matrix, but even though it might be useful for the evaluation of the influence of covariance on the characteristics of the sequential detection. It is well known (see [10], [14]), that with rotation of the coordinated system by the angle $\phi = \arctan \left\{ 2R \frac{\sigma_1 \sigma_2}{\sigma_1^2 - \sigma_2^2} \right\}$, where $\sigma_1, \sigma_2, D\{V_k\}$ and $D\{\tilde{V}_k\}$ for any two different k for two correlated branches at (26)².

Then as a result of rotating of the coordinate system one can get new Gaussian quadrature components with parameters

$$\hat{D}_{I,II} = \frac{2\sigma_1^2 \sigma_2^2 (1 - R^2)}{(\sigma_1^2 + \sigma_2^2) \left[1 \pm \sqrt{1 - (1 - R^2) \frac{4\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2}} \right]}, \quad (34)$$

where σ_1^2, σ_2^2 are the variances of the any quadrature components under consideration: $V_1(\tilde{V}_1)$ and $V_2(\tilde{V}_2)$ with correlation coefficient $R: D\{V_k\}, D\{\tilde{V}_k\}, D_{I,II}$ are new variances. In the same way for new expectations one can get:

$$\begin{aligned} \hat{M}_I &= m_1 \cos \phi - m_2 \sin \phi \\ \hat{M}_{II} &= m_1 \sin \phi - m_2 \cos \phi \end{aligned} \quad (35)$$

Here m_1, m_2 are expectations of the any quadrature component under consideration with correlation coefficient $R: M\{V_k\}, M\{\tilde{V}_k\}$, for different hypothesis $\mathcal{H}_1, \mathcal{H}_0$ (see (27), (28)). One have to notice that these new set of parameters (34) and (35) of the Gaussian quadrature components corresponds to statistically independent branches with the new values of fading parameters $m, \alpha_\Sigma, \beta_\Sigma$ for hypothesis $\mathcal{H}_1, \mathcal{H}_0$, while formulas (31)-(33) are still valid. So, recalculating their new α_Σ and β_Σ one can use the results of the previous analysis). It is worth to mention here, that from the theory of diversity combining it is well known, that correlation between the branches is influential mainly in the area of high SNR, which is not the case for closed hypothesis. Meanwhile, notice that in the case of low SNR, the $M_a\{\nu\}$ can be rather large (more than 8-10) so finally for both the cases of correlated or uncorrelated branches $W(\nu)$ will be tending to Gaussian PDF, but with different $M_a\nu$ and $D_a\{\nu\}$ what might affect the $\nu_{\max}$ value or not. Detailed analysis of this issue will be published elsewhere.

VI. CONCLUSIONS

In this paper we have presented a methodology for cumulant calculus of the PDF of random time (sample size) of sequential detection of PU in Cognitive Radio Networks in Generalized Gaussian Channels. For the practical interesting case of low SNR, a complete the analysis has been developed. We presented a methodology which allows to obtain any order cumulant of the PDF of the random time of analysis. For practical applications the so-called ‘‘Gaussian approximation’’ of the unknown PDF was suggested, allowing to limit the analysis to the first two cumulants. An expression for the variance of time of analysis valid for any relations between P_{fa} and P_{md}

²In the following, for simplicity is assumed $Q = 2$.

has been derived in general settings. As an example, we have analyzed a non-coherent detection in generalized fading.

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