

Co-channel Interference and Coverage Probability of a Pilot Signal in the IS-95 CDMA Channel

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Abstract— This paper works on several methods to get sum of log-normal distribution to calculate CCI. We investigate the details of the Wilkinson's method and the Schwartz and Yeh's method. Those are applied to determine cell coverage probability for pilot signal in the IS-95 CDMA forward channel analytically. The cell coverage is also simulated by Monte Carlo simulation by computer generated random variable of the co-channel interference. The results of each method are compared.

Keywords— Co-channel interference, coverage probability, pilot signal, CDMA, forward channel.

I. INTRODUCTION

For cellular radio systems the radio link performance is usually limited by interference rather than noise and, therefore, the probability of co-channel interference (CCI) is of primary concern. The variations in average received carrier to noise interference ratio due to path loss and shadowing will determine the probability of CCI. The shadowing has a more significant effect on the probability of CCI than fading.

Based on the assumption that the received signals are affected by shadowing, fading and path loss only, the total co-channel interference is therefore modeled as a composition of individual interference signals, whose local mean power levels follow log-normal variation. It is well accepted that the distribution of the sum of log-normal random variables can be approximated by another log-normal distribution, and several methods have been proposed for computing the mean value and standard deviation in decibel units of the resulting distribution. The Wilkinson's [1] and the Schwartz and Yeh's methods [2] are among the most popular ones. This paper works on the details and computer implementation of several methods to get sum of log-normal distribution to calculate CCI. Then it is applied to the simulation of cell coverage probability for pilot signal in the IS-95 CDMA forward channel. It is assumed that the signal strength is log-normally distributed in CDMA channel. The CCI to the pilot signal in the CDMA forward channel is computed analytically by the Wilkinson's and the Schwartz and Yeh's methods, so the cell coverage probability for pilot signal with CCI can be obtained. The Monte Carlo simulation is performed to find the cell coverage in the CDMA forward channel by computer generated random variable of the co-channel interference. The results according to the each method are compared.

II. REVIEW OF MODELING CO-CHANNEL INTERFERENCE

Suppose that a receiver suffers N interference signals arriving from co-channel mobiles or base stations. With assumption that the effects of small scale fading are averaged out, the local mean power level I_i of the i -th signal undergoes log-normal variation. Then, the local mean power level can be modeled by using decibel units as

$$X_i = 10 \log_{10} I_i = m_{X_i} + \chi_i \quad (\text{in dBm})$$

where m_{X_i} is the area mean power and χ_i is a zero mean normally distributed random variable in dB with standard deviation σ_{X_i} , also in dB, due to the shadowing caused by large obstacles. The area mean m_{X_i} is usually modeled as a function of the T-R separation d_i , path loss exponent n , transmitted power P_{T_i} , in dBm, and transmitter and receiver antenna gains G_{T_i} and G_{R_i} , both in dB.

Under the reasonable assumption that the individual signals I_i add in coherently, the total interference signal is modeled as the sum of N log-normally distributed signals

$$I = \sum_{i=1}^N I_i$$

I can be approximated by another log-normal distribution [9], or, equivalent normal distribution can be considered as below.

$$X = 10 \log_{10} I$$

The mean m_X and standard deviation σ_X of X completely characterize the normal distribution of X , and they can be computed by the Wilkinson's [1] and the Schwartz and Yeh's [2] methods. In determining correlation between interference signals, a new random variable Y_i is defined as

$$Y_i = \ln I_i \text{ (logarithmic unit)}$$

with mean value m_{Y_i} , and standard deviation σ_{Y_i} given, respectively, by

$$m_{Y_i} = \lambda m_{X_i} \text{ and } \sigma_{Y_i} = \lambda \sigma_{X_i} \text{ (logarithmic unit)}$$

where $\lambda = \ln(10)/10$. Then, we have [3]

$$I = e^{Y_1} + e^{Y_2} + \dots + e^{Y_N} \approx e^Z = 10^X \quad (1)$$

where Z (in logarithmic units) and X (in dB) are normally distributed, and $Z = \lambda X$. The Wilkinson's and the Schwartz & Yeh's methods then compute the mean value and standard deviation of Z or X from the mean values and standard deviations of the summation of Y_i . It is generally assumed that the individual signals I_i are correlated to each other. Uncorrelated shadowing is simply a specific case for the general correlated shadowing assumption. In order to consider the correlated interference signals case, let us define the correlation coefficient $r_{Y_i Y_j}$ of Y_i and Y_j by

$$r_{Y_i Y_j} = \frac{E\{(Y_i - m_{Y_i})(Y_j - m_{Y_j})\}}{\sigma_{Y_i} \sigma_{Y_j}}$$

[1][2] show that the Schwartz & Yeh's method directly computes mean value and standard deviation in dB of the sum of log-normal random variables, while the Wilkinson's method obtains the moments of X by computing the first two moments of $I = 10^{X/10}$. Furthermore, the Schwartz & Yeh's method computes the exact moments of X for $N=2$ summands. For more than two summands, the method is recursively applied, assuming that the sum of two log-normal RVs is log-normally distributed. On the other hand, the Wilkinson's method computes mean value and standard deviation assuming that all the summands of log-normally distributed random variables is also log-normally distributed.

The advantage of the Wilkinson's method is on its simplicity. However, it has been reported that the Wilkinson's method breaks down when the standard deviation of separate interference is large than 4dB. Unfortunately, for cellular radio applications the standard deviation of log-normal shadowing typically ranges from 6 to 12dB. However, we are usually interested in the tails of both the complementary and cumulative distribution functions. It has been shown that the Wilkinson method, despite its simplicity, can approximate the tails of the cdf and cdff functions with reasonable accuracy.

In this paper, we introduce the results of determined values of mean and standard deviation by both the Wilkinson's and the Schwartz & Yeh's methods. Each method is applied to the pilot signal of IS-95 CDMA forward channel described in the Section III to compare each performance.

A. Wilkinson's Method

According to Wilkinson's method, the mean value and standard deviation of Z in the above equation are determined by matching the first and second moments of I with those of the sum of i_{th} interference I_i under the assumption that the power sum is log-normally distributed [1].

$$m_X = (1/\lambda) \left(2 \ln u_1 - \frac{1}{2} \ln u_2 \right)$$

$$\sigma_X = (1/\lambda) \sqrt{\ln u_2 - 2 \ln u_1}$$

where,

$$u_1 = \exp(m_Z + \sigma_Z^2/2) = \sum_{i=1}^N \exp(m_{Y_i} + \sigma_{Y_i}^2/2)$$

$$u_2 = \exp(2m_Z + 2\sigma_Z^2)$$

$$= \sum_{i=1}^N \exp(2m_{Y_i} + 2\sigma_{Y_i}^2) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \exp \left[\frac{1}{2} (\sigma_{Y_i}^2 + \sigma_{Y_j}^2 + 2r_{Y_i Y_j} \sigma_{Y_i} \sigma_{Y_j}) \right]$$

B. Schwartz & Yeh's Method

Schwartz and Yeh's method is also based upon the assumption that the power sum is log-normally distributed. However, unlike the Wilkinson's method, the Schwartz and Yeh's method uses exact expressions for the first two moments of the sum of two log-normal random variables. Nesting and recursion techniques are then used to find exact values for the first two moments for the sum of N log-normal random variables [2].

The sum of N log-normal random variables in (1) can be rewritten as

$$Z = \ln(e^{Y_1} + e^{Y_2} + \dots + e^{Y_N})$$

Let Z_k denote $\ln(e^{Z_{k-1}} + e^{Y_k})$, where Z_{k-1} is assumed to be normally distributed. The Schwartz and Yeh's method then recursively computes the mean m_{Z_k} and standard deviation σ_{Z_k} of Z_k , for $k=2, 3, \dots, N$. The mean and standard deviation in decibel units of the total sum are therefore given by $m_X = \lambda m_{Z_N}$ and $\sigma_X = \lambda \sigma_{Z_N}$, respectively. With the notation in [1], m_{Z_k} and σ_{Z_k} , for $k=2, 3, \dots, N$ are given by

$$m_{Z_k} = m_{Z_{k-1}} + G_1(m_{W_k}, \sigma_{W_k})$$

$$\sigma_{Z_k}^2 = \sigma_{Z_{k-1}}^2 - G_1^2(m_{W_k}, \sigma_{W_k}) + G_2(m_{W_k}, \sigma_{W_k})$$

$$+ 2 \frac{(r_{Z_{k-1}, Y_k} \sigma_{Y_k} - \sigma_{Z_{k-1}}) \sigma_{Z_{k-1}}}{\sigma_{W_k}^2} G_3(m_{W_k}, \sigma_{W_k})$$

where,

$$\begin{aligned}
m_{W_k} &= m_{Y_k} - m_{Z_{k-1}} \\
\sigma_{W_k} &= \sqrt{\sigma_{Y_k}^2 + \sigma_{Z_{k-1}}^2 - 2r_{Z_{k-1}, Y_k} \sigma_{Y_k} \sigma_{Z_{k-1}}} \\
G_1(m_{W_k}, \sigma_{W_k}) &= m_{W_k} \Phi\left(\frac{m_{W_k}}{\sigma_{W_k}}\right) + \frac{\sigma_{W_k}}{\sqrt{2\pi}} e^{-\frac{m_{W_k}^2}{2\sigma_{W_k}^2}} \\
&\quad + \sum_{k=1}^{\infty} C_k [F(\sigma_{W_k}, m_{W_k}, k) + F(\sigma_{W_k}, -m_{W_k}, k)] \\
G_2(m_{W_k}, \sigma_{W_k}) &= (m_{W_k}^2 + \sigma_{W_k}^2) \Phi\left(\frac{m_{W_k}}{\sigma_{W_k}}\right) + [m_{W_k} + \log(4)] \frac{\sigma_{W_k}}{\sqrt{2\pi}} e^{-\frac{m_{W_k}^2}{2\sigma_{W_k}^2}} \\
&\quad + 2 \sum_{k=1}^{\infty} C_k (m_{W_k} - k\sigma_{W_k}^2) + F(\sigma_{W_k}, m_{W_k}, k) \\
&\quad + \sum_{k=2}^{\infty} B_k [F(\sigma_{W_k}, m_{W_k}, k) + F(\sigma_{W_k}, -m_{W_k}, k)] \\
G_3(m_{W_k}, \sigma_{W_k}) &= \sigma_{W_k}^2 \sum_{k=2}^{\infty} (-1)^k [F(\sigma_{W_k}, m_{W_k}, k) + F(\sigma_{W_k}, -m_{W_k}, k+1)] \\
F(\sigma_{W_k}, m_{W_k}, k) &= e^{-\frac{km_{W_k} + \frac{k^2\sigma_{W_k}^2}{2}}}{\sigma_{W_k}} \Phi\left(\frac{m_{W_k} - k\sigma_{W_k}^2}{\sigma_{W_k}}\right) \\
\Phi(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt \\
C_k &= \frac{(-1)^{k+1}}{k} \\
B_k &= \frac{2(-1)^{k+1}}{k+1} \sum_{j=1}^k \frac{1}{j}
\end{aligned}$$

The mean and standard deviation of X are then given by $m_X = \lambda m_{Z_N}$ and $\sigma_X = \lambda \sigma_{Z_N}$.

III. IS-95 CDMA FORWARD CHANNEL [4]

IS-95 CDMA forward channel constitutes 63 forward channels available for use. It includes one for pilot channel, another one for synchronization channel, 0~7 for paging channels, and 55~62 for traffic channels. Other configuration may be available according to the convenience. The detail of each channel is not considered here.

The CDMA proposed standard specifies a forward link CDMA waveform design that uses a combination of frequency division, pseudorandom code division, and orthogonal signal multiple access techniques. Frequency division is employed by dividing the available cellular spectrum into nominal 1.25MHz bandwidth channels. Normally, a cellular system would be implemented in a service area within a single radio channel until demand requires employment of additional channels.

Pseudorandom noise (PN) binary codes are used to distinguish signals from different base stations received at a mobile station. All CDMA signals in the system share a quadrature pair of PN codes. Signals from different cells and sectors are distinguished by time offsets from the basic code. This relies on the property of PN codes that the auto-correlation (when averaged over a few bit times) has an average which approaches zero for all time offsets greater than

a single code chip time (approximately 1μsec). The PN chip rate is 1.2288MHz, or exactly 128 times the 9600 bps information transmission rate.

All signals transmitted from a cell in a particular CDMA radio channel share a common PN code phase. They are distinguished at the mobile station receiver by using a binary orthogonal code based on Walsh function (also known as Hadamard matrices). The Walsh function is 64 PN code chips long and represents 64 different orthogonal codes. Orthogonality provides nearly perfect isolation between the multiple signals transmitted by the base station.

A. Channel Interference of IS-95 CDMA Forward Channel

On the forward link, the mobile unit suffers from interference from other cell as well as from the home cell. The power spectral density of these two components is defined as I_{oc} and I_{sc} , respectively. In addition, the thermal noise contributes to the received signal. The one-sided power spectral density of the thermal noise is N_o . Fig. 1 shows the forward link model.

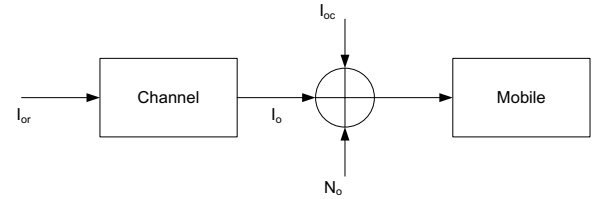


Figure 1. IS-95 CDMA forward link model.

The channel is modeled as a k-path frequency selective with mean-square strength α_i , $i=1,2,\dots,k$. Assume that the power spectral density of the combined signal out of the cell power amplifier is I_{or} . The output of the channel is then

$$I_o = I_{or} T(r) \sum_{i=1}^k \alpha_i$$

where $T(r)$ is transmitted path loss.

1) Interference from Home Cell (I_{sc}): This component is mainly due to multipath and is dominant for mobiles close to their home cell. Multipath will make channels no longer orthogonal to a desired channel. Neglecting the effects of thermal and other cells interference, for a mobile with M demodulators, the interference from the home cell will be

$$I_{sc} = \frac{I_o}{\sum_{i=1}^M \alpha_i / (1 - \alpha_i)}$$

2) Interference from Other Cell (I_{oc}): Using a single frequency in a CDMA system makes a co-channel interference, especially in a small coverage area radio systems. It is well accepted that the co-channel interference can be represented by a log-normally distributed random variable,

and the significant interference comes only from the first and second tiers (18 cells surrounding the cell under consideration).

3) The Total Received Noise Density (I_t): The average of the total received noise density at the mobile is composed of three components: thermal noise, interference from the mobile's home cell, and interference from other cells.

$$I_t = N_0 + I_{sc} + I_{oc}$$

For a mobile at the edge of the cell with transmission loss of -140dB, cell Power of 25 watt (44dBm), and three-ray equal strength model channel, typical values of the above parameters are $N_0 = -166$ dBm/Hz, $I_o/N_0 = 9$ dB, $I_{sc}/N_0 = 7$ dB, and $I_{oc}/N_0 = 12$ dB [4]. According to the [5], the interference from the home cell for a mobile at the edge of a cell can be assumed as $I_{oc}/I_{sc} = 2.5$ dB.

B. Analytical Model for Pilot Channel Power of IS-95 CDMA Forward Channel in Multiple Cell Environment with Log-Normally Distributed CCI

Fig. 2 shows the configuration of multiple cell environments for simulation. The number in the center of each hexagonal cell is a cell identification number.

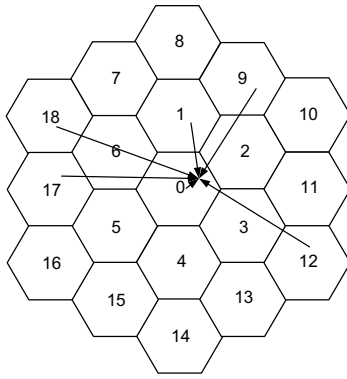


Figure 2. Configuration of the multiple cell environment.

- The mobile received signal to noise ratio of the pilot signal from the home cell [4][7] where,

- E_c : Chip energy
- I_t : Total interference density for a received mobile
- n : Propagation constant
- P_t : Transmit Power from a base station
- β_p : The ratio of the pilot strength to the total transmit power of a cell
- G_t : Transmitter Antenna gain
- G_r : Receiver Antenna gain
- W : The CDMA channel bandwidth
- h : The chip transmit rate

- λ : Carrier wavelength
- σ : Standard deviation of log-normal shadowing
- N_0 : Thermal noise spectral density
- I_{oc} : Interference spectral density, other cells
- I_{sc} : Interference spectral density, own cell
- X : random variable of signal power
- d_f : the close-in reference distance
- d_i : the distance from the i th base station to a received mobile station

C. Coverage Probability for Pilot Channel with CCI

For the case of multiple interferers of Fig.2, we first obtain the appropriate values for the mean μ_z and the variance σ_z^2 for the log-normal interference. Then the probability of CCI can be readily obtained as follow.

$$\Delta_{(dB)}(d) = \Omega_{(dB)}(d_0) - Z_{(dB)}(d_1, d_2, \dots, d_{18})$$

Where, $\Omega_{(dB)}(d_0)$ is log-normal distribution with mean of μ_Ω , and variance of σ_Ω^2 for the pilot signal from the home cell. $Z_{(dB)}(d_1, d_2, \dots, d_{18})$ is the sum of log-normal interference from the 18 surrounding cell of a received mobile with mean of μ_z , and variance of σ_z^2 . Then, $\Delta_{(dB)}(d)$ has mean and variance as [3]

$$\mu_\Delta(d) = \mu_\Omega - \mu_z$$

$$\sigma_\Delta^2(d) = \sigma_\Omega^2 + \sigma_z^2$$

Therefore, the probability of CCI is

$$O(d) = Q\left(\frac{\mu_\Omega(d_0) - \mu_z - \gamma_{(dB)}}{\sqrt{\sigma_\Omega^2 + \sigma_z^2}}\right)$$

Where, γ is a threshold value for a received pilot signal.

IV. SIMULATION AND RESULTS

Simulation is performed by creating numerous snapshots for the distribution of mobile stations in a cell. For analytical simulation, the mean value and the standard deviation of the co-channel interference are computed by the Wilkinson's method and the Schwartz and Yeh's method through computer simulation. The process is carried out in a cell of IS-95 CDMA channel to get a coverage probability for a pilot channel. For empirical simulation, co-channel interference is simulated by Monte-Carlo simulation with computer generated random variables. The results of each simulation are compared. It is assumed 18 around identical hexagonal cells centered one cell (home cell). Typical value for the I_{sc} [5] which is mainly due to multi-path effect is used. The link budget which is used in the simulation is as below [4][5][6][8].

- One source cell and 18 around (2 clusters) identical hexagonal cells with same radius, $R = 5$ km
- n (Propagation constant)=4

- P_T (Transmit Power from a base station)=25W
- β_p (The ratio of the pilot strength to the total transmit power of a cell)=0.2
- β_i (The ratio of the sum of traffic, synch., paging strength to the total transmit power of a cell)=0.8
- G_t (Transmit Antenna gain) = 1
- G_r (Receiver Antenna gain) = 1
- W (The CDMA channel bandwidth): 1.2288MHz
- H (The chip transmit rate): 1.2288Mchips/sec)
- F (Carrier frequency): 900MHz
- σ (Standard deviation of log-normal shadowing): 8dB
- N_0 (Thermal noise spectral density): -166dBm
- I_{oc}/I_{sc} : 2.5dB

Fig. 3 shows the results of the analytical simulation for the coverage probability of pilot signal of which the CCIs are computed by the Wilkinson's method. In a hexagonal cell, the coverage area is shown as a green circle. The simulations are performed for the coverage probabilities in which the received pilot signal power to noise ratio is greater than -13dB. Fig.3 shows the areas of which the probabilities are greater than 50%, 70%, 80%, and 90%, and the corresponding coverage areas in a cell are determined as 97.7%, 63.8%, 46.1%, and 29.1%, respectively.

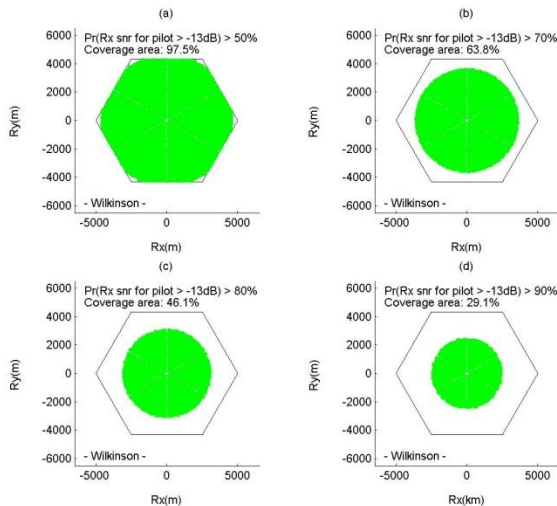


Figure 3. Coverage area of pilot signal computed by the Wilkinson's method.

Fig. 4 shows the results of the simulation for the coverage probability for pilot signal of which the CCIs are computed by the Schwartz & Yeh's method in a similar way with the case of Wilkinson's method. The simulations are performed for the coverage probabilities in which the received pilot signal power to noise ratio is greater than -13dB. Fig.4 shows the areas of which the probabilities are greater than 50%, 70%, 80%, and 90%, and the corresponding coverage areas in a cell are determined as 84.5%, 54.7%, 41.8%, and 28.2%, respectively. Fig. 5 shows the empirical result by Monte Carlo simulation with computer generated random variables of co-channel interference. The simulations are performed for the coverage

probabilities in which the received pilot signal power to noise ratio is greater than -13dB. Fig.3 shows the areas of which the probabilities are greater than 50%, 70%, 80%, and 90%, and the corresponding coverage areas in a cell are determined as 85.0%, 54.8%, 41.5%, and 27.5%, respectively.

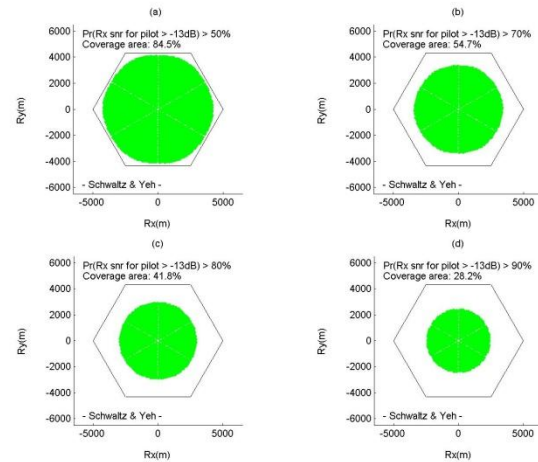


Figure 4. Coverage area of pilot signal computed by the Schwartz & Yeh's method.

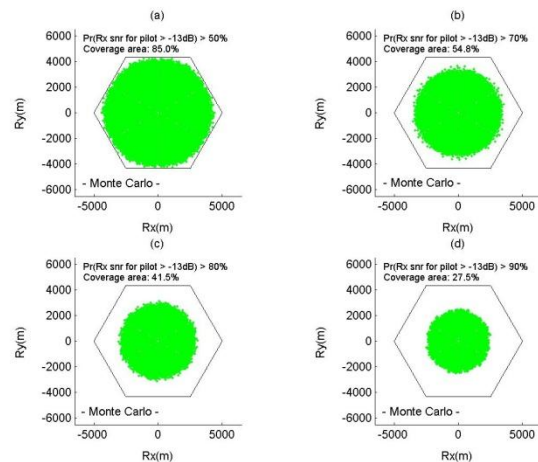


Figure 5. Coverage area of pilot signal computed by Monte Carlo simulation.

Fig.6 plots all the results of coverage probability in a cell vs. the probability of received pilot for each method. The dashed dot line (red) with '+' mark indicates the results of the Wilkinson's method, the dot line (blue) with '*' mark indicates the results of Schwartz & Yeh's method, and the thick solid line (black) indicates the Monte Carlo simulation results.

As we can see in the results, both the Wilkinson's and the Schwartz & Yeh's method provide reasonable approximation. The Schwartz & Yeh's method is more accurate than the Wilkinson's method. It is noted that the Wilkinson's method is well matched with other two results in the tail of the distribution, despite its simplicity. It is noted that the coverage areas by the Wilkinson's method appear larger than the Schwartz & Yeh's case.

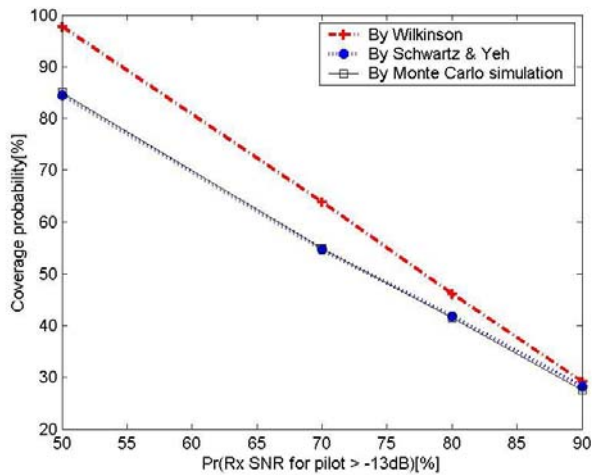


Figure 6. Coverage probability in a cell vs. the probability of received pilot signal.

V. CONCLUSIONS

The cell coverage probabilities for pilot channel of IS-95 CDMA forward channel in multiple cell environments under the assumption of log-normally distributed co-channel interference have been evaluated. The sum of log-normal distributions for co-channel interference is achieved

analytically by the Wilkinson's method and the Schwartz and Yeh's method, and an empirical simulation is also used to get sum of log-normal distribution for co-channel interference. The results of coverage probability according to each method are compared. Both of the Wilkinson's method and the Schwartz & Yeh's method provide reasonable approximation. The Schwartz & Yeh's method is more accurate than Wilkinson's method. It is noted that Wilkinson's method, which has the advantage of its simplicity, is well matched in the tail of a distribution.

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