

Performance Analysis of Decode-and-Forward scheme Multiple-Hop MIMO Relay System

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Abstract—Multi-hop MIMO relay systems have been discussed in several published papers. However, the diversity as well as the capacity is obtained when the signal to noise ratio (SNR) is assumed to be fixed. In this paper, to obtain a high channel capacity, we express a propagation process of multi-hop MIMO relay system with decode-and-forward (DF) method by numerical formula, and the position of each relay, meaning the distance between each transceiver is optimized. The channel capacity of DF system is compared to that of AF system. In addition, the limitation of channel capacity when the number of relay tends to infinity is considered.

Index Terms—Multiple-hop MIMO relay system, Decode-and-Forward, Optimizing position, Upper bound of channel capacity, Infinite relay number

I. INTRODUCTION

In recent years, multiple-input multi-output (MIMO) technology using multiple antennas at both the access point (base station) and user terminal sides has become a popular research field of next-generation mobile communication systems. The increase of system capacity under finite frequency bandwidth has made the MIMO system unique and efficient in data transmission.

In terms of the scientific underpinnings, a research on MIMO can be divided into three following areas, namely, 1) array antennas and adaptive signal processing for the implementation of antenna configurations and control methods, 2) information theory and coding schemes (space-time coding) for the implementation of an efficient data transmission, and 3) radio wave propagation for the modeling of MIMO channel [1]-[8].

The Gaussian MIMO relay channel with fixed channel conditions, and derive upper bounds and lower bounds that can be obtained numerically by convex programming. The upper bound and a lower bound on the ergodic capacity is found. The sufficient conditions for achieving the ergodic capacity; and in particular, for the case where all nodes have the same number of antennas, the capacity can be achieved under certain signal to noise ratio (SNR) condition [9]. The capacity of a particular large Gaussian relay network is determined in the limit as the number of relays tends to infinity. Upper bounds are derived from cut-set arguments, and lower bounds follow from an argument involving uncoded transmission. It is shown that in cases of interest, upper and lower bounds coincide in the limit as the number of relays tends to infinity. Hence, a new example where a simple cut-set upper bound is achievable, and one more example where uncoded transmission achieves

optimal performance, are provided[10].

In the future, it is believed that the MIMO service area will become fashionable. Thus, an expansion of a service area to an isolated area is anticipated. Based on this idea, the authors have proposed a general idea of a MIMO relay system that can maintain the ability of high-speed and/or high-reliability data transmission. A MIMO relay system can relay radio signals from a MIMO service area to an isolated area. In general, when a MIMO relay system has only one relay, the whole channel in the MIMO relay system is equivalent to a MIMO multi-keyhole environment. In a multi-keyhole environment, the probability density functions (PDFs) of singular values of channel response matrix or eigenvalues of correlation matrix are important from a viewpoint of system designing such as transmission characteristic meaning channel capacity and bit error rate analysis. When the number of the relay antenna elements is less than the number of the transmit and receive antenna elements, the channel capacity of MIMO relay system is lower than the one of the original MIMO system. In addition, when the number of the relay antenna elements equals the number of the transmit and receive antenna elements or more, a MIMO relay system can provide the same averaged channel capacity as an original MIMO system. In other word, although the number of the relay antenna elements is larger than the transmit and receive antenna elements, the channel capacity of a MIMO relay system cannot exceed the one of an original MIMO system[11],[12]. A multi-hop relay network with multiple antenna terminals in a quasi-static slow fading environment also be considered[13]. However, these researchs assumed the SNR at receiver(s) to be fixed, the distance between transceiver as well as the transmit power of transmitter(s) are not considered. In the multiple-hop MIMO relay system, when the distance between the transmitter (Tx) and the final receiver (Rx) is fixed, the distance between the Tx to a relay (RS), a RS to a RS, a RS to the Rx called the distances between transceivers, is shorten, the SNR, consequently the channel capacity is increased. However, according to the number of relay and the position of the relays, the channel capacity is changed. Hence, to obtain the upper bound of the channel capacity, the position of each relay meaning the distance between transceivers need to be optimized. We have analyzed performance of multiple-hop relay system with amplify-and-forward (AF)[14]. In this paper, we analyze the performance of system with decode-and-forward (DF) and to compare to system with AF. In addition, the limitation

of capacity when the number of relay tends to infinity is considered.

The rest of the paper is organized as follows. We introduce the system model in Section II. Section III shows the upper bound of channel capacity when the distance between each transceiver is optimized. In Section IV we seek the limitation of channel capacity when the total number of antenna element tends to infinity. Finally, Section V concludes the paper.

II. MULTIPLE-HOP MIMO RELAY SYSTEM

A. Concept of system

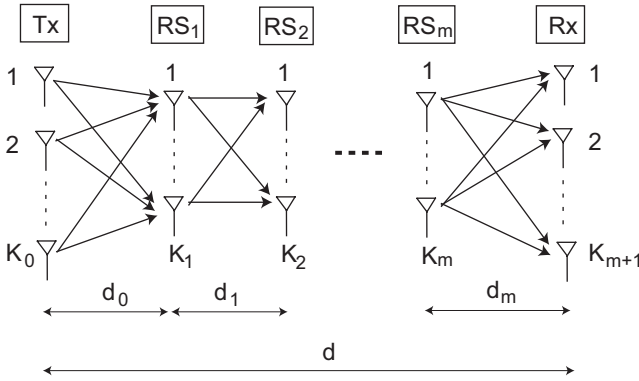


Fig. 1. Concept of the multiple-hop MIMO relay system

Fig. 1 shows the multiple-hop MIMO relay system with m relays intervened. Here, M , N and K_i ($i = 1 \dots m$) denote the number of the antenna elements at the Tx, the Rx and each relays. Let the distances between transceivers are d_i ($i = 0 \dots m$). Let us fix the distance between the Tx and the Rx d . The signal is transmitted from the Tx to the 1th relay. At the 1th relay, the signal is decoded, encoded and transmitted to the 2th relay. Similarly, the signal is transmitted over and over until to the final receiver. We assumed the transmit power of the Tx (E_{Tx}) as well as the total transmit power of RS (E_{RS}) are fixed regardless total relay number. In order to simplify the composition of relay, the total transmit power of relays is equally divided into each relay. Moreover, the transmit power of each relay is equally divided into each antenna element. In other site, as described in Section I, if the number of antenna elements at the least one relay is smaller than the other, this relay will be the bottleneck of the system and the channel capacity of the system will be decreased. Therefore, to obtain high channel capacity, we assume the number of antenna elements at each relay is the same as that at the transceivers.

B. Channel capacity

Let $\mathbf{H}_{ii+1}$ denotes a $i+1 \times i$ channel matrix between the i^{th} relay and the $i+1^{\text{th}}$ relay. Since the path loss is taken into consideration, $\mathbf{H}_{ii+1}$ is the composite matrix. We model $\mathbf{H}_{ii+1}$ as

$$\mathbf{H}_{ii+1} = \sqrt{l_{ii+1}} \mathbf{H}_{wii+1}, \quad i = 0, \dots, m, \quad (1)$$

where $\mathbf{H}_{wii+1}$ is a matrix with independent and identically distributed (i.i.d.), zero mean, unit variance, circularly symmetric complex Gaussian entries, and l_{ii+1} represents path loss between the i^{th} relay and the $i+1^{\text{th}}$ relay. The path loss is described in detail in the following section.

Let the channel capacity between the i^{th} relay and the $i+1^{\text{th}}$ relay is C_i , and C denotes the channel capacity of the multiple-hop MIMO relay system.

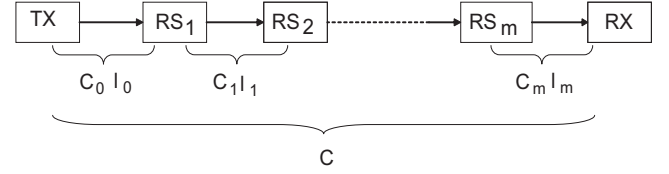


Fig. 2. Channel capacity of multiple-hop MIMO relay system

The access of every relay in this system is assumed to be controlled accurately, and hence an absence of the interference is assumed. Therefore, when the signal $\mathbf{S}_{i-1}$ is transmitted from the $i-1^{\text{th}}$ relay, the received signal at the i^{th} relay is expressed as [9],

$$\mathbf{S}_i = \mathbf{H}_{i-1i} \sqrt{\mathbf{P}_{i-1}} \mathbf{S}_{i-1} + \mathbf{n}_i. \quad (2)$$

Here, $\mathbf{P}_i = \text{diag}(p_{i1}, p_{i2}, \dots, p_{iK_i})$ is the transmit power matrix and be assumed to be subject to a constraint

$$\text{Tr}(\mathbf{P}_i) = \begin{cases} E_{Tx} & (i = 0), \\ \frac{E_{RS}}{m} & (i \neq 0), \end{cases}$$

where $\text{Tr}(\cdot)$ and p_{ij} denote the trace and transmit power of j^{th} antenna element of i^{th} relay, respectively. $\mathbf{n}_i$ is the noise vector with i.i.d., zero mean, σ^2 variance.

The other site, the channel capacity is as follows,

$$C_i = \log_2 \left(\det \left(I_{K_{i+1}} + \frac{SNR_{i+1}}{K_{i+1}} \mathbf{H}_{ii+1} \mathbf{H}_{ii+1}^H \right) \right), \quad \text{for } i = 0, \dots, m, \quad (3)$$

where I_{K_i} is $K_i \times K_i$ unit matrix. Since the channel capacity C_i ($i = 0, \dots, m$) is independent of the other one, the channel capacity of the system is expressed as

$$C = \min(C_0, C_1, \dots, C_m). \quad (4)$$

In addition, $\mathbf{H}_{ii+1} \mathbf{H}_{ii+1}^H$ ($i = 0, \dots, m$) can be assumed to be an unit matrix. Consequently, the SNR can be examined instead of a channel capacity. Thus the upper bound of the channel capacity of the system is obtained when the channel capacity of every receivers meaning the SNR of every receivers are equal. Therefore, it is necessary to optimize the distances between each relay in order to obtain the upper bound of the channel capacity of the system.

C. Path loss

The system channel matrix is proportional to $\mathbf{P}_i$ and l_i ($i = 0, \dots, m$). We assume that the total transmit power of relays is fixed whereas the number of relay is changed, and the transmit power of each relay is kept the same. Therefore,

the path loss plays an important role in the channel model. At scheme 1, we assume the path loss is only attenuated by the second power of the propagation distance. Thus the path loss becomes

$$l_i = \left(\frac{\lambda}{4\pi d_i}\right)^2, \quad (5)$$

where λ is the wavelength.

However, the path loss is necessary to consider being attenuated by the reflection, since there are a lot of transmit obstacle, such as tall building, in propagation environment. The power of signal is reduced when the reflection is occurred. An amount of the reduction by one time of reflection is called reflection factor. It is natural that the reflection factor is changed according to matter of obstacles, the angle of reflections and so on. However, in this paper, the reflection factor of all reflections is assumed to be the same and denoted by a . The path loss in this case is indicated for scheme 2, and expressed as [15]

$$l_i = \left(\frac{\lambda a^{t_i}}{4\pi d_i}\right)^2, \quad (6)$$

where t_i is the number of reflection while a signal is transmitted between i^{th} and $i+1^{\text{th}}$ relays. In addition, in order to obtain the number of reflection, the propagation environment coefficient D_i is defined as the average distance from a transmitter or from a reflection point to the next reflection point or a receiver. In other words, it is the average of line-of-sight (LOS) distance between transceiver. Therefore, the number of reflection between each transceivers can be expressed as $t_i = \frac{d_i}{D_i}$. Consequently, the path loss in (6) can be rewritten as

$$l_i = \left(\frac{\lambda a^{\frac{d_i}{D_i}}}{4\pi d_i}\right)^2. \quad (7)$$

III. BOUND OF CHANNEL CAPACITY

A. Optimizing distance

1) *DF scheme*: In order to obtain the upper bound of channel capacity of system, all of SNR are necessary to be equal.

$$SNR_i = SNR_j, \quad \text{for } i \neq j, \quad i, j = 0 \cdots m, \quad (8)$$

where

$$\begin{aligned} SNR_0 &= \frac{E_{Tx} l_0}{M\sigma^2}, \\ SNR_i &= \frac{E_{RS} l_i}{mM\sigma^2}, \quad \text{with } i = 1 \cdots m, \end{aligned}$$

where M is the number of antenna element at transmitter, each relay and final receiver.

By solving (8), the optimized distances become,

$$\begin{aligned} d_i &= d_j, \quad \text{for } i \neq j, \quad i, j = 1 \cdots m, \\ d_0 &= d - md_i. \end{aligned}$$

In case of scheme 1, the propagation loss shown in (5) is substituted in (8). By solving this equation, the optimized

distances between transceivers are obtained and be expressed as,

$$d_i = \frac{d\sqrt{E_{RS}}}{\sqrt{mE_{Tx}} + m\sqrt{E_{RS}}}, \quad \text{for all } i = 1, \cdots, m, \quad (9)$$

$$d_0 = \frac{d\sqrt{mE_{Tx}}}{\sqrt{mE_{Tx}} + m\sqrt{E_{RS}}}. \quad (10)$$

In the case of scheme 2, to analyze performance of this system easily, let D denote the average of D_i ($D = \sum_{i=0}^m D_i / (m+1)$) meaning the average of LOS distance between the Tx and Rx . The path loss shown in (7) is substituted in (8). The equation becomes,

$$\sqrt{\frac{mP_{Tx}}{P_{RS}}} d_i = (d - md_i) a^{\frac{(m+1)d_i - d}{D}}. \quad (11)$$

Let

$$g(d_i) = a^{\frac{(m+1)d_i - d}{D}}.$$

The Taylor expansion is approximately used for $g(d_i)$, and the (11) is expressed as

$$b_4 d_i^4 + b_3 d_i^3 + b_2 d_i^2 + b_1 d_i + b_0 = 0, \quad (12)$$

where

$$\begin{aligned} b_0 &= d, \\ b_1 &= \frac{d(m+1)\ln(a)}{D} - m - \sqrt{\frac{mP_{Tx}}{P_{RS}}} a^{\frac{d}{D}}, \\ b_2 &= \frac{d}{2} \left(\frac{m+1}{D} \ln(a)\right)^2 - \frac{m(m+1)\ln(a)}{D}, \\ b_3 &= \frac{d}{6} \left(\frac{m+1}{D} \ln(a)\right)^3 - \frac{m}{2} \left(\frac{m+1}{D} \ln(a)\right)^2, \\ b_4 &= -\frac{m}{6} \left(\frac{m+1}{D} \ln(a)\right)^3. \end{aligned}$$

Therefore, according to solving (12) by using Galois theory [16], d_i is obtained. The case analyzing performance of this system with different D_i is similar. The Taylor expansion is approximately used for a term $a^{\frac{-2d_i}{D_i}}$. Then, the partial differential equation with respect to each d_i is obtained, and each d_i can be obtained similarly. However, in this case, we have made the Taylor expansion, solving partial differential $m+1$ times to obtain each d_i .

2) *AF scheme*: As described in [14], in order to optimizing the distance between transceiver, the channel capacity of the Rx is expressed.

$$C = \log_2 \left(\det(\mathbf{I}_M (1 + \frac{1}{\sigma^2} \frac{1}{M(\frac{1}{l_0 E_{Tx}} + \frac{m}{E_{RS}} \sum_{i=1}^m \frac{1}{l_i})})) \right). \quad (13)$$

When the function $f(m)$ is defined as

$$f(m) = \frac{1}{l_0 E_{Tx}} + \frac{m}{E_{RS}} \sum_{i=1}^m \frac{1}{l_i}, \quad (14)$$

the function $f(m)$ is minimized instead of maximizing the channel capacity. Consequently, the optimized distance in the case of scheme 1 is obtained by substituting (5) in (14) and

the partial differential equation of $f(m)$ with respect to d_0 and d_i .

$$\begin{aligned} d_0 &= \frac{E_{Tx}d}{E_{Tx} + E_{RS}}, \\ d_i &= \frac{E_{RS}d}{m(E_{RS} + E_{Tx})}, \quad i = 1, \dots, m. \end{aligned} \quad (15)$$

In the case of scheme 2, the path loss in (7) is substituted in (14), and the Taylor expansion is approximately used for $a^{\frac{d_i}{D_i}}$. Consequently, the optimized distance is obtained as same as the case of DF. The detail is described in [14].

B. Bound of channel capacity

As described in [14], the channel capacity of different D_i with optimized distance of average D is considerably lower than the capacity of average D . However, the difference of channel capacity in both cases of D and D_i is not so much. It can be explained that although the propagation environment between each transceiver is different, by using the optimization distance method mentioned before, the distance is shorten in environment with small D_i . Consequently, no relay becomes bottleneck, and the channel capacity reaches close to that of D . Additionally, according to the central limit theorem, the channel capacity of D_i is lower than that of D , because the distance dispersion in the case of D_i is bigger than that of the case of D . In real environment, it can be considered that the propagation environment between each transceiver is different, therefore the optimized distances should be decided by the method with different D_i . However, the difference in channel capacity of both cases is not so much and the calculation in case of D is much simpler than that in case of D_i , thus the optimization distance method with average D is used to evaluate channel capacity from now on.

The system parameters are summarized in Table I. Figs. 3, 4 show the channel capacity in both cases of AF and DF with propagation environment coefficient D is 1000 m and 4000 m, respectively.

TABLE I
NUMERICAL PARAMETERS

Antenna elements at Tx, Rx, RS	4
Transmit power of Tx [mW]	100
Total transmit power of RS [mW]	100
Noise power [mW]	$6.12e-011$
Reflection factor	0.38
Distance between $Tx-Rx$ [m]	3000

As shown in Figs. 3 and 4, the optimized distance in the scheme 2 is more effective, and higher channel capacity can be obtained in both D , 4000 m and 1000 m. In other words, the optimized distance in scheme 2 is always more effective regardless of the propagation environment. Additionally, in the same system model as well as the propagation environment, the channel capacity of DF scheme is higher than that of AF scheme. However, the structure of relay in DF scheme is more complex than that in AF scheme, and the power for encode/decode electric circuit of DF relay not be considered.

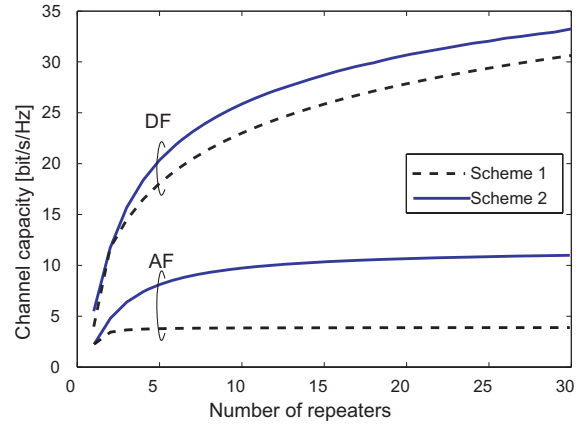


Fig. 3. Channel capacity of system in case of $D=1000$ m

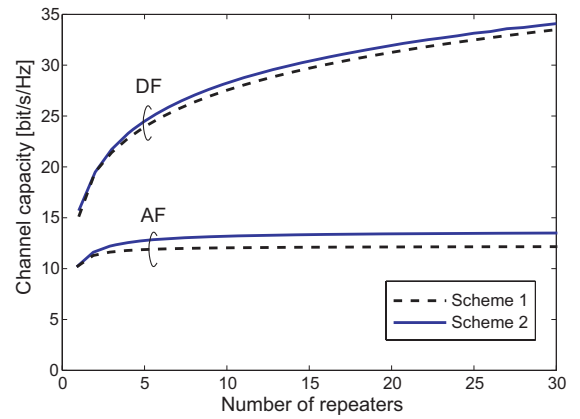


Fig. 4. Channel capacity of system in case of $D=4000$ m

IV. THE LIMITATION OF CHANNEL CAPACITY

As expressed in Figs. 3 and 4, the channel capacity of both AF and DF scheme is increased when the number of relay is increased. When the number of relay tends to infinity, the limitation of channel capacity is need to be precisely. When the number of relay is infinite, it means the distance between each transceiver is enough small to decide that $a^{\frac{d_i}{D_i}}$ becomes one regardless of D_i . Consequently, the path loss of any propagation environment becomes the path loss of scheme 1.

A. DF scheme

According to path loss in 9, the channel capacity of system is expressed as

$$C = M \log_2 \left(1 + \left(\frac{\lambda}{4\pi} \right)^2 \frac{1}{M\sigma^2} \left(\sqrt{E_{Tx}} + \sqrt{mE_{RS}} \right)^2 \right). \quad (16)$$

Therefore, when the number of relay tends to infinite, the channel capacity also tends to infinite.

B. AF scheme

Similarly, the path loss in (15) is substituted in (14), therefore, the minimum of $f(m)$ can be expressed as,

$$f(m) = \left(\frac{4\pi}{\lambda}\right)^2 \frac{d^2}{E_{Tx} + E_{RS}}. \quad (17)$$

And the channel capacity is obtained.

$$C = M \log_2 \left(1 + \left(\frac{\lambda}{4\pi}\right)^2 \frac{1}{M\sigma^2} \frac{E_{Tx} + E_{RS}}{d^2} \right). \quad (18)$$

As a result, when the number of relay infinitely increases and the parameter is summarized in Table I, the channel capacity converges in 15.12[bit/s/Hz]. Fig. ?? shows the upper bound of channel capacity when total transmit power of the Tx and RS is changed, other parameters are kept the same.

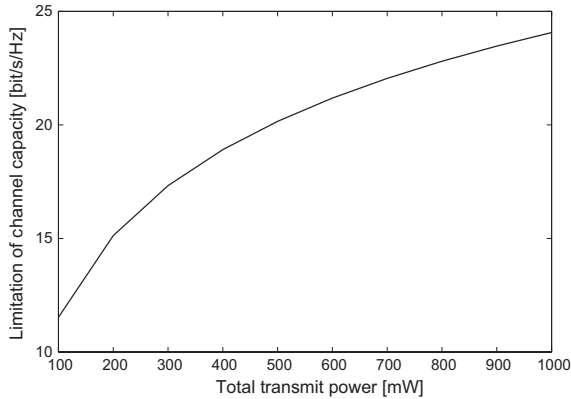


Fig. 5. Limitation of channel capacity when total transmit power is changed

Although the channel capacity of DF scheme tends to infinity when the number of relay tends to infinity, the signal processing at DF relay takes time longer than that of AF relay. Thus, the delay of system with DF relay should be considered, and based on real-time scenario, the number of DF relay is restricted.

V. CONCLUSION

In this paper, we examined the performance of a multiple-hop MIMO relay system with the decode-and-forward method. For a large system channel capacity of a wireless network, the positions of all relays were optimized. The channel capacity of DF system is higher than that of AF system, however the structure of relay in DF scheme is more complex than that in AF scheme, and the power for encode/decode electric circuit as well as the delay of DF relay not be considered. In this paper, we have analyzed the system under Gaussian channel model, this assumption allows for considerable simplification of the analysis. However, under this channel model, the feature of DF scheme have not described clearly. In the future, the performance of this system is need to be analyzed under channel model which is close to practice.

REFERENCES

- [1] K. Miyashita, T. Nishimura, T. Ohgane, Y. Ogawa, Y. Takatori and K. Cho, "High data-rate transmission with eigenbeam space division multiplexing(E-SDM) in a MIMO channel," Proc. IEEE VTC 2002-Fall, vol. 3, pp. 1302-1306, Sept. 2002.
- [2] J. B. Andersen, "Array gain and capacity for known random channels with multiple element arrays at both ends," IEEE J. Set. Areas Commun., vol. 18, no. 11, pp. 2172-2178, 2000.
- [3] J. F. Kepler, T. P. Krauss and S. Mukthavaram, "Delay spread measurements on a wideband MIMO channels at 3.7GHz," 2002 IEEE VTC-Fall, Vancouver, Sept. 2002.
- [4] D. S. Shiu, G. J. Foschini, M. J. Gans and J. M. Kahn, "Fading correlation and its effect on the capacity of multi-element antenna systems," IEEE Trans. Commun., vol. 48, no. 3, pp. 502-513, 2000.
- [5] D. Chizhik, G. J. Foschini, M. J. Gans and R. A. Valenzuela, "Key-holes, correlations, and capacities of multi-element transmit and receive antennas," IEEE Trans. Wireless Commun., vol. 1, no. 2, pp. 361-368, 2002.
- [6] D. Gesbert, H. Bolcskei, D. A. Gore and A. J. Paulraj, "MIMO wireless channel: Capacity and performance prediction," Proc. GLOBECOM, vol. 2, pp. 1083-1088, Nov/Dec. 2000.
- [7] S. M. Alamouti, "A simple transmit technique for wireless communications," IEEE J. Set. Areas Commun., vol. 16, no. 8, pp. 1451-1458, 1998.
- [8] V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," IEEE Trans. Inf. Theory, vol. 44, no. 2, pp. 744-765, 1998.
- [9] B. Wang, J. Zhang and A. H. Madsen, "On the capacity of MIMO relay channel," IEEE Trans. Inf. Theory, vol. 51, no. 1, pp. 29-43, Jan. 2005.
- [10] Michael Gastpar, Martin Vetterli, "On the capacity of large Gaussian relay networks," IEEE Trans. Inf. Theory, vol. 51, no. 3, pp. 765-779, Mar. 2005.
- [11] Makoto Tsuruta and Yoshio Karasawa, "Multi-Keyhole Model for MIMO Repeater System Evaluation," IEICE Trans. Commun., vol. J89-B, no. 9, pp. 1746-1754, 2006.
- [12] G. Levin, S. Loyka, "On the outage capacity distribution of correlated keyhole MIMO channels," IEEE Trans. Inf. Theory, vol. 54, no. 7, pp. 3232-3245, Jul. 2010.
- [13] Deniz Giindüz, Mohammad A. Khojastepour, Andrea Goldsmith and H. Vincent Poor, "Multi-hop MIMO relay networks: Diversity-Multiplexing trade off analysis," IEEE Trans. wireless commun., vol. 9, no. 5, pp. 1738-1747, May. 2010.
- [14] Pham Thanh Hiep and Ryuji Kohno, "Optimizing Position of Repeaters in Distributed MIMO Repeater System for Large Capacity," IEICE Trans. Commun., Vol.E93-B, No.12, pp.-, Dec. 2010.
- [15] Naoki Kita, Wataru Yamada, Akio Sato, "Path loss prediction model for the over-rooftop propagation environment of microwave band in suburban areas," (in Japanese) IEICE Trans. Commun., vol. J89-B, no. 2, pp. 115-125, 2006.
- [16] Harold M. Edwards, "Graduate texts in Mathematics: Galois Theory", Springer, 1997