

# The Effect of Measurement Error on the Estimation of Channel Parameters and a Novel Way to Prevent Estimation Failure

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**Abstract**— For the mobile radio channel parameter estimation, the Space-Alternating Generalized Expectation-maximization (SAGE) algorithm is a popular scheme to estimate the individual path parameters, i.e., the relative delay, incidence azimuth, Doppler frequency, and complex amplitude of impinging waves. Because of the constraint of bandwidth of channel sounding signal and the limitation of the number of antennas, it is impossible to separate every individual path exactly especially when the paths are close in the spatial domain as well as in the time domain. Furthermore the measurement errors including measurement noise and thermal noise make it more difficult to separate each path in the received sounding signal constructed with multipath delay. In this study we modelled the measurement errors of channel sounding system as an Additive White Gaussian Noise (AWGN) and present the effects of measurement errors on the estimation of channel parameters using SAGE algorithm. We demonstrate that the measurement error causes the estimation failure and the impact of the error on the antenna pattern is most pronounced. We introduce a novel method to prevent the estimation failure when using SAGE algorithm. The simulation results are presented to compare the estimation results between with and without proposed method.

**Keywords**— MIMO, Channel Modelling, Channel Parameter Estimation, SAGE, Estimation

## I. INTRODUCTION

In communication system researches to achieve high data rate and the Quality of Service (QoS) various communication algorithms are proposed. In order to verify the performance of a communication algorithm in a certain environment it is very important to provide a proper communication channel model. In the specular channel models such as WINNER model, the channel is modeled as the sum of individual paths which is direct delta function in time domain. To get parameter estimates of the individual path from the received signal various signal processing methods can be used. For the circular antenna arrays the performance of the Space-Alternating Generalized Expectation-maximization (SAGE) algorithm [1] is superior to any other algorithms considered in [2]. However it has a resolution limits to get the signals for real individual path due to the constraint of bandwidth and the number of antenna arrays even if in noiseless environment. If

there are some noises and measurement errors it is very hard to get the channel parameters of individual paths from superimposed received signals.

In this paper we demonstrate the effect of noise and measurement errors on channel parameter estimates using SAGE algorithm. We discuss on the case that the transmitter has an Omni-antenna and the receiver has a Circular-antenna whose number of elements is eight. Thus we estimate the azimuth of arrival (AoA) only. The azimuth of departure (AoD) can be estimated in the same manner when the transmitter has more number of antenna elements.

The paper organized as follows. Section II gives a brief description about the SAGE algorithm. Section III presents the influence of noise and measurement errors on the channel parameter estimation. In section IV, we demonstrate the ‘estimation failure’ and introduce a novel method to prevent it.

## II. SAGE ALGORITHM

### A. Signal Model

The transmitted signal  $u(t)$  consists of the repeated sounding sequence which is commonly Pseudo Noise (PN) sequence. The received signal can be written as

$$X(t) = \sum_{l=1}^L s(t; \theta_l) + \sqrt{\frac{N_0}{2}} W(t) . \quad (1)$$

$$s(t; \theta_l) = \alpha_l \exp(j2\pi v_l t) C(\Theta_l) u(t) . \quad (2)$$

where  $\theta_l \triangleq [\tau_l, \theta_l, v_l, \alpha_l]$  is a parameter vector of  $l$  th wave which consists of relative delay  $\tau$ , incidence azimuth  $\theta$ , Doppler frequency  $v$ , and complex amplitude  $\alpha$ .  $C(\Theta_l) = [c_1(\Theta_l), \dots, c_8(\Theta_l)]^T$  denotes the steering vector of receiver array and  $W(t) \triangleq [w_1, \dots, w_8]^T$  denotes the complex white Gaussian noise for eight receiver antennas.

## B. Channel Parameter Estimation using the SAGE Algorithm

Then the log-likelihood function of  $\theta_i$  given an observation  $X(t)$  over observation window  $D_0$  is [3]

$$\Lambda(\theta; y) \triangleq \frac{1}{N_0} \left[ 2 \int_{D_0} \Re\{s^H(t'; \theta) y(t')\} dt' - \int_{D_0} \|s(t'; \theta)\|^2 dt' - \int_{D_0} \|y(t')\|^2 dt' \right]. \quad (3)$$

where  $\theta \triangleq [\theta_1, \dots, \theta_L]$  is a set of parameters.  $\Re\{\cdot\}$  and  $\|\cdot\|$  denotes the real part and the norm of the argument respectively. The maximum likelihood estimate (MLE) of  $\theta$  reads

$$(\hat{\theta})_{ML} = \arg \max_{\theta} \{\Lambda(\theta; y)\}. \quad (4)$$

The calculation of MLE is very computational since the dimension of  $\theta$  is large. The SAGE algorithm reduces the complexity of MLE. The SAGE algorithm updates current state of parameters using expectation-step (e-step) and maximization-step (m-step) in iterative manner. The SAGE algorithm decomposes the superimposed signal to individual signals for each path in the e-step and estimates the parameters from the individual signal for each path in the m-step. At initialization state the SAGE algorithm decomposes the observation signal into individual signal for each path using successive cancellation scheme [4]. At first the SAGE algorithm estimate parameters for the first dominant path assuming that the received signal came through a single path channel. Then the SAGE algorithm reconstructs the signal for the first path with the estimated parameters and gets the signal for second path by subtracting the reconstructed signal for first path from the received signal. In this way, the SAGE algorithm can get an observation signal for individual paths by subtracting the previous reconstructed signals and estimate parameters for each path. After initialization state the SAGE algorithm perform iteration state. In iteration state the observation signals and parameters for each path are updated. The MLE of an observation signal for a single path and the updating procedure of the parameter estimates of one path are expressed as [5]

$$(\hat{\gamma}_l, \hat{\phi}_l, \hat{v}_l)_{ML}(x_l) = \arg \max_{[\gamma, \phi, v]} \{z(\gamma, \phi, v; x_l)\}. \quad (5)$$

where

$$z(\gamma, \phi, v; x_l) \triangleq \int_{D_0} u^*(t' - \gamma) \exp\{-j2\pi v t'\} \cdot C^H(\phi) x_l(t') dt'. \quad (6)$$

The SAGE algorithm has the remarkable property that for

any generated sequence the sequence of log-likelihood values cannot decrease (monotonicity property) [6].

## III. THE INFLUENCE OF NOISE AND MEASUREMENT ERRORS

The SAGE algorithm does not estimate parameters jointly but separately. Thus it might find a local maximum instead of global maximum if the situation is not ideal. Actually there are some constraints that increase the chance to fail to find the real values of parameters. The bandwidth of the sounding signal restricts the time resolution and the number of antenna elements determines the spatial domain resolution. Because of these resolutions it is impossible to separate exact paths in the expectation-step of SAGE algorithm. Errors in separating paths could result that the estimated values of parameters are far from real values.

### A. BECS system

The Band Exploration and Channel Sounding system (BECS) is designed for wideband MIMO channel sounding using multiple antennas [7]. In this real measurements system there are measurement errors as well as thermal noise. Since it has a system response the path components of received signal can be written as

$$s(t; \theta_l) = \alpha_l \exp(j2\pi v_l t) C(\Theta_l) b(t). \quad (7)$$

where  $b(t)$  is system impulse response (SIR). To estimate channel parameters we need to measure SIR which might have some noise. Furthermore SIR is time-varying since SIR includes the responses of cables whose responses change as they move. The steering vector of receiver array  $C(\Theta_l)$  also needs to be measured since the responses of real antenna elements are different from theoretical values.

In measuring SIR and the steering vector it is very hard to get the real values. There should be some noise and measurement errors. We model these noise and errors as an Additive White Gaussian Noise (AWGN) and demonstrate the influence of these noise and errors in estimating channel parameters using SAGE algorithm.

### B. Influence of SNR of Received Signal

Table 1 shows the simulation environments. We generate a channel with a single path whose AoA is 100 degree and the received signal for eight elements of circular receiving antenna by adjusting steering vector of receiver array. Then we estimate the channel parameters using SAGE algorithm. Figure 1 shows the power delay profile (PDP) of original generated channel (black line), the PDP of reconstructed channel (red line) with channel parameters estimated by SAGE algorithm and the residual power (blue line) remained after subtracting reconstructed channel from the original channel. If the signal to noise ratio (SNR) of received signal is infinite and there is no measurement error, the estimates of parameters are exact so that the reconstructed channel must be

the same as the original channel. In figure 1 and figure 2 the SNR of received signal is 20 dB and 0 dB respectively and parameters are estimated by SAGE algorithm with the same SIR and steering vector as original with no noise. As the SNR decreases the noise floor level of PDP increases. If there is another path whose power is lower than -45 dB against dominant peak path, it is impossible to find the path using SAGE algorithm in 0 dB SNR situation.

### C. Influence of Errors in SIR

Figure 3 shows the case that the SNR of received signal is 20 dB and the SNR of SIR is 10 dB. In this case the residual power shows different from figure 1 and figure 2. If there is another path whose power is lower than -20 dB against dominant peak path, the SAGE algorithm will estimate wrong path instead of real path in this situation, because the SAGE would initialize the delay parameter for second path on the peak of residual signal.

### D. Influence of Errors in The Steering Vector of Antenna Array

Figure 4 shows the case that the SNR of received signal is 20 dB and the SNR of the steering vector of antenna array is 10 dB. In this case the residual power still remains on the same delay as the dominant path. In this situation the SAGE algorithm might find ghost path on the same delay for another path.

TABLE 1. THE SIMULATION ENVIRONMENTS

| Simulation Parameters          | Value      |
|--------------------------------|------------|
| Number of transmitter antennas | 1          |
| Number of receiver antennas    | 8          |
| Number of path                 | 1          |
| AOA of path                    | 100 degree |
| Fading coefficient of path     | 1          |
| Relative delay of path         | 5 [chip]   |
| Doppler frequency of path      | 0 Hz       |
| Order of sounding PN sequence  | 12         |

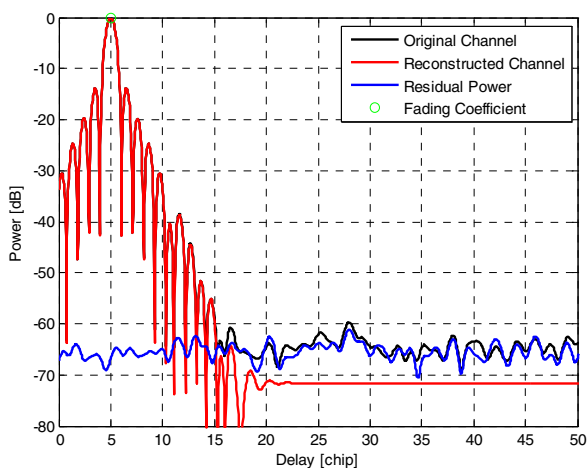


Figure 1. The PDP of the original channel and the reconstructed channel where the SNR of received signal is 20 dB

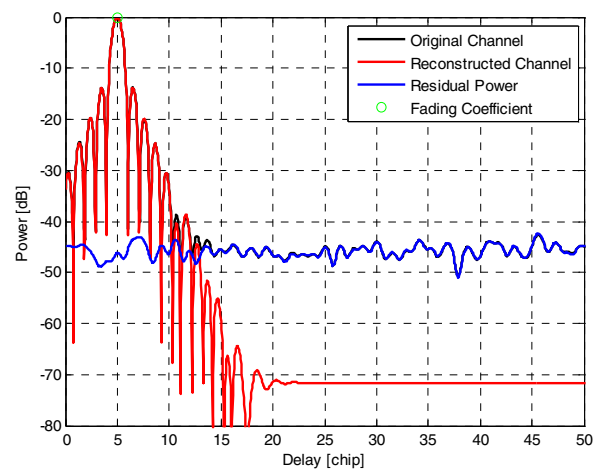


Figure 2. The PDP of the original channel and the reconstructed channel where the SNR of received signal is 20 dB

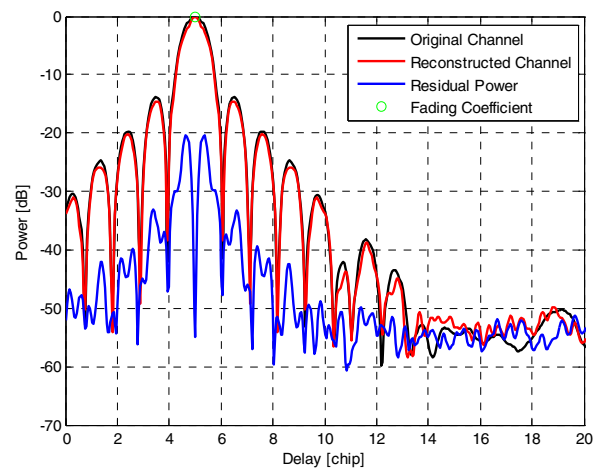


Figure 3. The PDP of the original channel and the reconstructed channel where the SNR of received signal is 20 dB and the SNR of the SIR is 10 dB

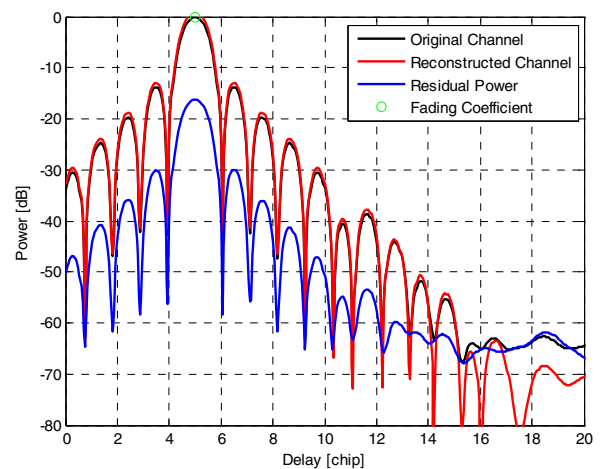


**Figure 4.** The PDP of the original channel and the reconstructed channel where the SNR of received signal is 20 dB and the SNR of the steering vector of antenna array is 10 dB

#### IV. A NOVEL WAY TO PREVENT ESTIMATION FAILURE

##### A. Estimation Failure in Estimate of Multi-path Parameters

In the real channel measurement scenarios, the SNR of received signal can be increased by increasing the transmit power and using a high order of PN sequence. The SNR of SIR also can be increased by increasing the transmit power and using phase stable cables. However increasing the SNR of the steering vector of antenna array is not easy.

In this section we investigate the results of estimate of multi-path channel parameters. Table 2 shows the original parameters of each path. The SNRs of received signal and SIR are 20 dB and the SNR of the steering vector of antenna array is 10 dB. Figure 5 shows the estimate results of one snap shot of estimation for delay and AoA of 5 paths. The location of real path are marked by '\*' and the size of '\*' corresponds to the magnitude of fading coefficient. The SAGE algorithm failed to estimate delay and AoA parameters for path-2, path-3 and path-5. This failure happens because the peak of residual power remaining after estimating the path-1 is greater than the power of other paths.

##### B. A Novel Way to Prevent Estimation Failure

The greater measurement error causes the higher residual power. If the peak of residual power is higher than the power of other paths the SAGE algorithm will find ghost path from the residual signal. Since the residual signal is come from random errors the chance to find proper channel parameters to fit to the residual signal is very low. Thus the power of residual signal remains still high even after finding several paths. This causes a successive failure of channel parameter estimation.

We introduce a novel way to prevent successive estimation failure. The key idea is to determine if the residual signal is come from other paths or any random noise. If the peak of the residual signal is come from random noise, reconstructed channel is hardly similar to residual signal. Thus, if the similarity between residual signal and reconstructed signal is smaller than a threshold the estimation might be failed. This idea can be implemented in the initialization state of SAGE algorithm by estimating noise on the steering vector.

Figure 6 shows the estimate results of 50 runs for delay and AoA of 5 paths with the same condition in figure 5 and figure 7 shows the estimate results of 50 runs for delay and AoA of 5 paths with the same condition in figure 5 with proposed method. With proposed method the failure of estimation of path-2 does not propagate to the estimations of other paths.

**TABLE 2.** ORIGINAL CHANNEL PARAMETERS FOR 5 PATHS.

| Paths  | Fading Coefficient | Delay [chip] | AOA [degree] |
|--------|--------------------|--------------|--------------|
| Path 1 | 1.0                | 3.7          | 0            |
| Path 2 | 0.1                | 4.1          | 30           |

|        |      |      |     |
|--------|------|------|-----|
| Path 3 | 0.05 | 5.4  | 70  |
| Path 4 | 0.05 | 6.3  | 0   |
| Path 5 | 0.05 | 11.3 | 100 |

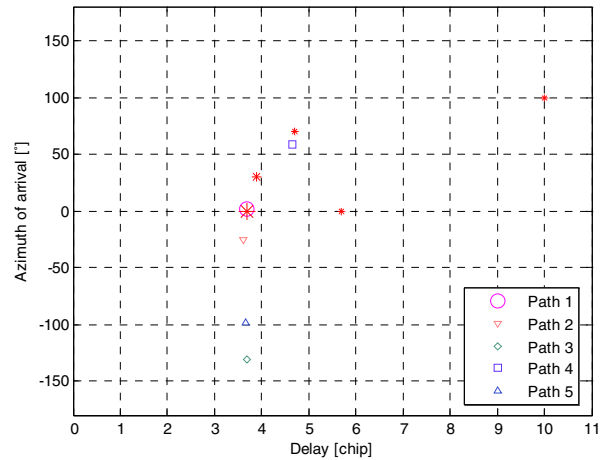


**Figure 5.** The estimate results of one snap shot of estimation for delay and AoA of 5 paths

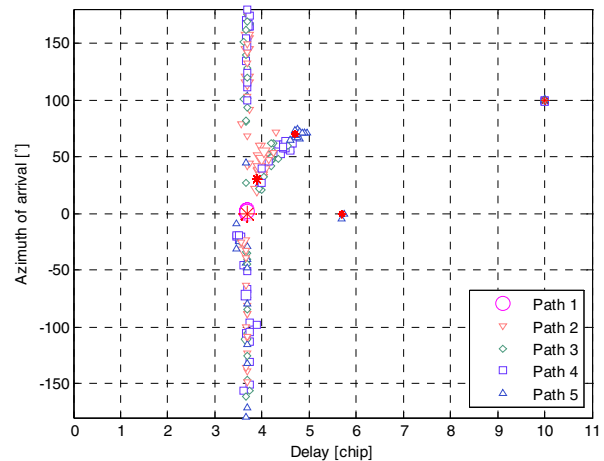


**Figure 6.** The estimate results of 50 runs for delay and AoA of 5 paths

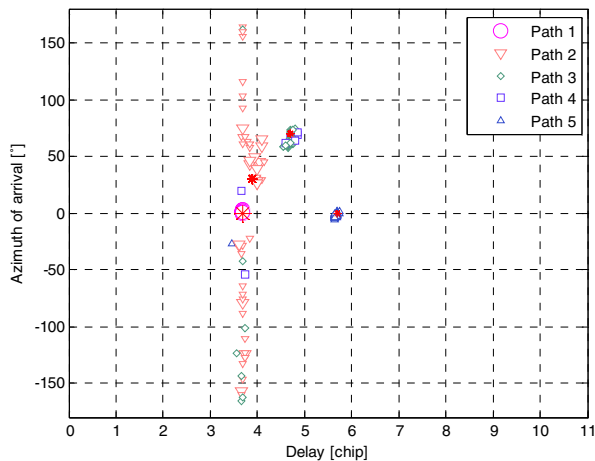


**Figure 7.** The estimate results of 50 runs for delay and AoA of 5 paths with proposed method

## V. CONCLUSIONS

In this study we modelled the measurement errors of channel sounding system as an Additive White Gaussian Noise (AWGN) and present the effects of measurement errors on the estimation of channel parameters using SAGE algorithm. We demonstrate that the measurement error causes the estimation failure and the impact of the error on the antenna pattern is most pronounced. We introduce a novel method to prevent the estimation failure when using SAGE algorithm. The simulation result shows that using the

proposed method can prevent propagating of the successive estimation failure.

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