

Performance Analysis of an (m, n) Energy Detector in Fading Environment with Double Threshold

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Abstract— It is known that, energy detector for spectrum sensing in cognitive system is simple in implementation, fast, and standards independent. However, it is known that energy detector suffers from its dependency over the noise level of detected signal. Different techniques have been proposed in order to combat that dependency. Among those techniques: antenna diversity, collaborative sensing, and double threshold schemes. In this paper we propose a technique that uses an m-antenna diversity and n-sensors collaboration (m, n) in order to improve the performance of a double threshold energy detector in fading channel. Close formulas for Receiver Operating Characteristics (ROC) and its complement were used in our analysis.

Simulation results in both AWGN and fading channels show an agreement with obtained results in [2] in the case of Gaussian channel and confirm the behaviour of double detector on the sense of improving probability of collision at the expense of spectrum utilization

Keywords— cognitive system, collaborative sensors, spectrum sensing.

I. INTRODUCTION

Recent years have witnessed an increase on spectrum demanding. Although, many licensed spectrums are experiencing low utilization rate as stated in the FCC report published in 2002 [1]. Due to that Cognitive radio system became the centre of interest for telecomm industry and researchers. In order for cognitive user to operate over a licensed spectrum it has first to detect any vacant (white) spectrum slot, later a communication session for a cognitive user could be initiated. Many techniques for spectrum sensing have been proposed, mainly: energy detector and feature detector. Energy detection technique is known to be the least complicated technique and unfortunately the worst in performance due to its dependency to noise variance. Although, recent research have shown that energy detection under the fading channel is effective in meeting the detectability requirements via collaborative sensing [2] as does feature detector. In [3] it is found that the maximum of the matched filter is the same as in the energy detection except that it is independent of the dwell time. The standards independently of energy detection and its fast sensing features are attracting researchers to designate approaches to enhance

its performance. Finally, multiple threshold schema, adaptive threshold technique, and adaptive multiple threshold schema are presented in the literature in order to improve its performance [4][5][10].

II. PROBLEM FORMULATION

Energy detection is the simplest method in spectrum sensing. No prior knowledge regarding the transmitted signal structure is needed. Energy detector has the advantage of simplicity in implementation where it consists of pre-filter which is a band-pass filter to determine the centre frequency and the band of interest, square device to measure the received energy, integrator to determine the observation interval, and a decision device with threshold to compare the output of the integrator with the threshold and decide if a signal is present or absent.

On the other hand, the energy detector suffers from its dependency on the noise variance to determine its threshold. This makes the performance of the energy detector poor compared to other methods of detection [6].

A. AWGN channel

For an AWGN channel, the received signal $r(t)$ takes the form:

$$r(t) = h s(t) + n(t) \quad (1)$$

Where $h = 0$ or 1 under hypotheses H_0 (No signal) or H_1 (signal present), respectively. This signal forms the input to the energy detector. The output from the integrator Y will be the test statistic to test the two hypotheses H_0 and H_1 . Y is known to be [7]:

$$Y = \sum_{i=1}^{2u} (n'_i)^2 \quad (2)$$

Here n_i represents the noise amplitude at certain time. In the case where the received signal is only noise component, Y follows chi-square (χ^2) distribution with $2u$ degree of freedom at hypothesis H_0 . On the other hand, when primary signal present, the decision statistic Y will have a noncentral χ^2

distribution with $2u$ degree of freedom and non centrality parameter 2γ [7]. The decision statistic can be written as:

$$Y = \begin{cases} \chi_{2u}^2 \\ \chi_{2u}^2(2\gamma) \end{cases} \quad (3)$$

And the probability density function (PDF) can be written as [7]:

$$f_r(y) = \begin{cases} \frac{y^{u-1} e^{-y/2}}{2^u \Gamma(u)} & H_0 \\ 0.5 \left(\frac{y}{2\gamma}\right)^{(u-1)/2} e^{\frac{-(2\gamma+y)}{2}} I_{u-1}(\sqrt{2\gamma y}) & H_1 \end{cases} \quad (4)$$

Where Γ is the gamma function, I_v is the v th-order modified Bessel function.

In order to measure the performance of the energy detector, the probability of false alarm (P_f) and the probability of detection (P_d) are evaluated from (4) such that [7]:

$$P_f = \Pr(Y > \lambda | H_0) = \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \quad (5)$$

$$P_d = \Pr(Y > \lambda | H_1) = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \quad (6)$$

Using equation (5) and (6), the CROC curve for an AWGN channel is obtained for $u=5$ and $\text{SNR}=10$ dB as depicted in figure 1

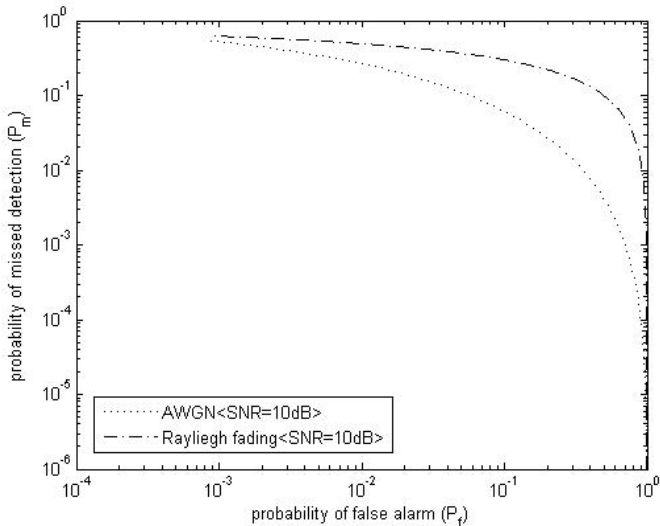


Figure 1. CROC curves in AWGN and Rayleigh channel

B. Rayleigh Fading Channel:

In a more realistic channel where a multipath faded components of the received signal are present. The distribution of the probability density function of the received signal amplitude will follow a Rayleigh distribution of the form [7]:

$$f_\gamma(\gamma) = \frac{1}{\gamma} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \quad (7)$$

Averaging the right hand side of (6) over (7) result in the average probability of detection in the Rayleigh channel as following [7]:

$$\bar{P}_d = e^{-\frac{\lambda}{2}} \sum_{n=0}^{u-1} \left(\frac{1}{n!}\right) \left(\frac{\lambda}{2}\right)^n + \left(\frac{1+\bar{\gamma}}{\bar{\gamma}}\right)^{u-1} \left[e^{-\frac{\lambda}{2(1+\bar{\gamma})}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{u-2} \left(\frac{1}{n!}\right) \left(\frac{\lambda \bar{\gamma}}{2(1+\bar{\gamma})}\right)^n \right] \quad (8)$$

The probability of false alarm is equivalents in both channels due to the fact it does not depend on signal present. Figure 1 shows the performance of the energy detector over the Rayleigh channel.

III. ENERGY DETECTOR PERFORMANCE WITH (m, n) DIVERSITY and COLLABORATIVE SENSORS

One of the most effective techniques in combating fading is diversity, using of multiple independent antenna in the receiver. The improvement caused by diversity can be used to improve the performance of the energy detector. By assuming that each antenna receives a signal that suffers different multipath effect, independent and identically distributed Rayleigh branches, and by using selection combining approach, the probability of detection is found to be [7]:

$$P_d = L \sum_{i=1}^{L-1} \left(\frac{(-1)^i}{1+i} \binom{L-1}{i} \overline{P_{dRay}} \left(\frac{\bar{\gamma}}{i+1} \right) \right) \quad (9)$$

Where L is number of independent branches, and $\overline{P_{dRay}} \left(\frac{\bar{\gamma}}{i+1} \right)$ represents the P_d obtained in equation (8) but with replacement of each γ in (8) with $\left(\frac{\bar{\gamma}}{i+1} \right)$.

It is visible for multiple sensors to cooperate in improving the energy detector performance by sharing their decisions [8]. The expressions of the P_f and P_d are found to be [8]:

$$Q_d = 1 - (1 - P_d)^n \quad (10)$$

$$Q_f = 1 - (1 - P_f)^n \quad (11)$$

Here P_d is either as stated in (8), the case of no diversity, or as in (9), the case of selection combining diversity. P_f is still as in (5) for the same reason mentioned before.

Figure 2 shows the CROC curves of system consists of two, $n=2$, and four, $n=4$, sensors, respectively, and for $u = 5$ and $\text{SNR} = 10\text{dB}$. In this figure, the diversity also included with different number of branches (m) in order to show the improvement in the performance.

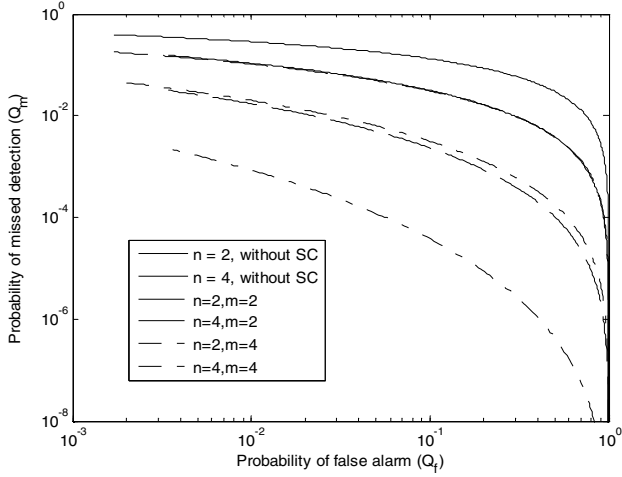


Figure 2. CROC curves for (n) collaborative sensors with (m) diversity branches

IV. DOUBLE THRESHOLD DETECTOR PERFORMANCE IN FADING ENVIRONMENT WITH DIVERSITY AND SENSORS COLLABORATION

It is found in [4] that double threshold scheme ($\lambda_0 < \lambda < \lambda_1$) in an AWGN channel would improve the performance of an energy detector in the sense of probability of collision, at the same time the performance will be degraded in the sense of spectrum unavailable probability but in a less factor. It is important to confirm such a conclusion in practical wireless environment where the channel suffers from multipath phenomenon. The probability of collision and spectrum unavailable are defined in [4] as following:

$$P_{coll.} = \Pr(Y < \lambda_0 | H_1) \quad (12)$$

$$P_{unav.} = \Pr(Y > \lambda_0 | H_0) \quad (13)$$

We can see in (12) that the probability of collision is the complement of probability of detection defined in (6) but with threshold value λ_0 :

$$\begin{aligned} P_{coll.} &= 1 - \Pr(Y > \lambda_0 | H_1) \\ &= 1 - P_d = 1 - Q_m(\sqrt{2\gamma}, \sqrt{\lambda_0}) \end{aligned} \quad (14)$$

Also, the probability of unavailable spectrum appears to be equivalent to probability of false alarm defined in (5) as follows:

$$\begin{aligned} P_{unav.} &= \Pr(Y > \lambda_0 | H_0) = P_f \\ &= \frac{\Gamma(u, \lambda_0 / 2)}{\Gamma(u)} \end{aligned} \quad (15)$$

The improvement in the probability of collision and unavailable spectrum in an AWGN channel for a detector using double threshold over a detector with a single threshold, is shown in figure 3. Moreover, figure 4 shows the improvement of a double threshold detector over a single threshold detector in the case of a Rayleigh fading channel with selection combining diversity and collaborative sensors.

Depending on these results it is confirm now that the double threshold scheme is indeed improving the probability of collision in a more practical wireless cognitive channel modelled by a Rayleigh PDF.

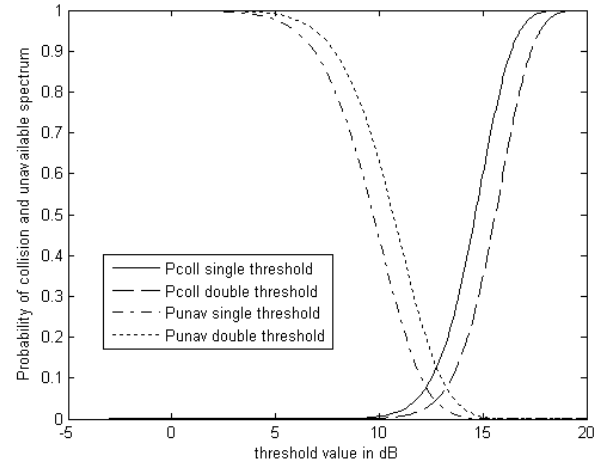


Figure 3. Probability of collision and probability of unavailable spectrum for AWGN channel for single and double threshold

In order to improve the unavailable spectrum probability, we recommend that, the used threshold is λ_1 instead of λ_0 . In this case, P_{unav} will be improved, without diversity and collaboration, as seen in figure 5 without any effect over P_d .

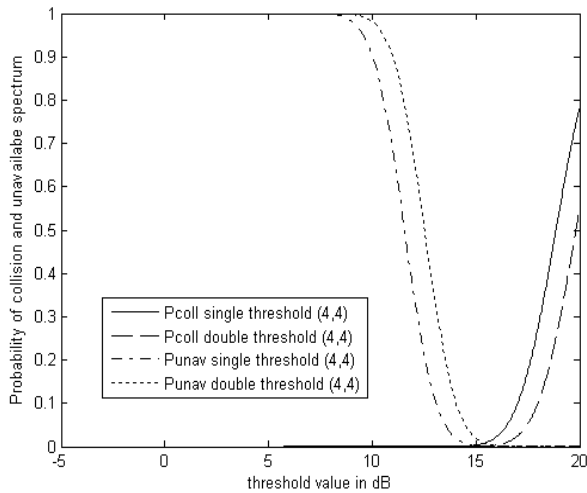


Figure 4. Probability of collision and unavailable spectrum in Rayleigh fading channel with diversity and collaborative sensors for single and double threshold

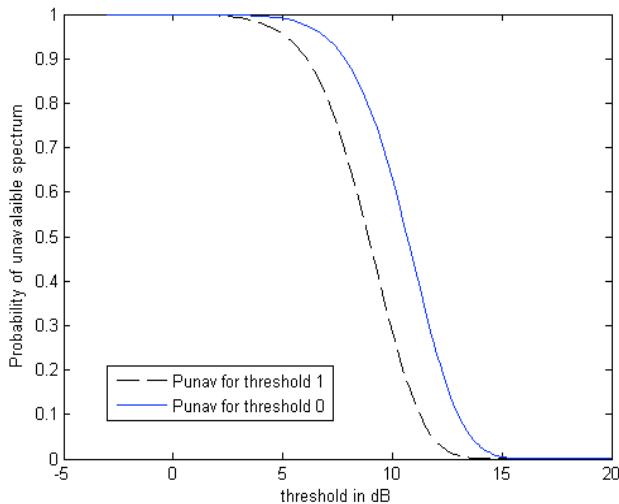


Figure 5. Probability of unavailable spectrum using λ_0 and λ_1

VI. CONCLUSION

In this paper, we studied the performance of a double threshold energy detector in fading channel. It is found that the probability of collection has improved over the expenses of spectrum utilization. In addition, we have improved the performance of a double threshold energy detector in fading channel by proposing antenna diversity with selection combining and collaborative sensing. A dramatic improvement in the detector's performance has been found which qualified energy detector as a strong candidate for spectrum sensing in cognitive networks.

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