

GLRT-based Detection in SIRV Clutter with Steering Vector Mismatches

Xin Dai*, Guolong Cui**, Lingjiang Kong**

* College of Automation, Chongqing University, Chongqing, China

**School of Electronic Engineering, University of Electric Science and Technology of China, Chengdu, China

Corresponding Author: cuiguolong@uestc.edu.cn

Abstract—This paper mainly deals with the problem of detecting a target in SIRV clutter with steering vector mismatches. Assuming that the mismatch of the steering vector is satisfied the conic constraint, the generalized likelihood ratio test (GLRT) is devised, and the geometry explain is presented for the derived solution. Meanwhile, fixed point estimate (FPE) of the covariance structure is adopted using the secondary data. Finally, we assess the performance of the derived test by Monte Carlo techniques, and several numerical results show that the derived test is consistent with the famous normalized matched filter (NMF), and has the ability to reject high mismatched signals.

Keywords—the SIRV clutter, steering vector mismatches, GLRT, normalized matched filter (NMF)

I. INTRODUCTION

Recently, many works are concerned on the adaptive detection problem with steering vector mismatches against Gaussian clutter. In [1], the adaptive robust detection was devised in Gaussian clutter with unknown covariance matrix in presence of steering vector mismatches, and the work was extended by considering of the interference in [2]. In [3], the distributed target was investigated in presence of mismatches, and in [4], the test was design in partially homogeneous environment. The above results show that the acceptance/rejection performance is improved greatly by considering the steering vector mismatches at the design stage.

However, in practical application, clutter is more consistent to some high tails non-Gaussian distributions. The Spherically Invariant Random Vector (SIRV) model has been proved to describe the real radar measurements well, especially in high resolution radars or/and at low grazing angles. It is described as the product of a “slowly varying” texture component, times a more “rapidly varying” speckle process, Gaussian distributed due to the local validity of Central Limit Theorem [5-8].

Motivated by extending the target detection problem with steering vector mismatches in Gaussian background to that against SIRV clutter, we consider a general binary hypotheses target detection model as in [1], the difference is that the test is devised in SIRV clutter. Since the optimum Neyman-Pearson receiver is not available, the suboptimum GLRT-based receiver is devised by assuming that the actual steering vector is constraint within a cone.

To make the derived GLRT-based receiver fully adaptive, we will resort to the secondary data, which is free of signals and shares the same covariance matrix with the primary data, to estimate the exact covariance matrix. The fixed-point estimate (FPE) is adopted [9-11], since in SIRVs, it is unbiased, consistent and ensures the CFAR property with respect to both the structure of the covariance matrix and the clutter power levels. The sampled covariance matrix performs well in Gaussian background, however, in SIRVs, it does not work well.

Finally, several numerical simulations are given using Monte Carlo techniques, and the results show that the GLRT-based receiver is consistent with NMF, meanwhile, it can reject high mismatched signals.

II. PROBLEM FORMULATION AND DETECTOR DESIGN

We assume that the signals are collected form N sensors, and the problem of detecting a target can be formulated in terms of the following binary hypotheses test:

$$\begin{cases} H_0: & \mathbf{z} = \mathbf{c} \\ H_1: & \mathbf{z} = \mathbf{u} + \mathbf{c} \end{cases} \quad (1)$$

where $\mathbf{z}$, $\mathbf{u}$ and $\mathbf{c}$ are the N -dimensional complex vectors sampled from the sensors' based-band equivalent of the received signals, the transmitted signals and the clutter, respectively. In general, the clutter vector is modeled as SIRV [8], which can be written as

$$\mathbf{c} = \tau \mathbf{g}$$

where the texture component τ is a non-negative random variable with probability density function (pdf) $f_\tau(\cdot)$ and unite root mean square value. The speckle component $\mathbf{g}$ is a zero-mean complex Gaussian vector with covariance matrix $\mathbf{M}$, and statistically independent with texture τ .

Applying whitening and rotating transform on the received signal, the detection problem becomes

$$\begin{cases} H_0: & \mathbf{r} = \mathbf{n} \\ H_1: & \mathbf{r} = \mathbf{p} + \mathbf{n} \end{cases} \quad (2)$$

where $\mathbf{r} = \mathbf{U}\mathbf{M}^{-1/2}\mathbf{z}$, $\mathbf{n} = \mathbf{U}\mathbf{M}^{-1/2}\mathbf{c} = \tau\mathbf{U}\mathbf{M}^{-1/2}\mathbf{g}$, and $\mathbf{U}$ is a unitary transformation such that $\mathbf{p} = \mathbf{U}\mathbf{M}^{-1/2}\mathbf{u} = \alpha\mathbf{e}_N$, $\mathbf{e}_N = [0, 0, \dots, 1]^T$ (^{T} denotes the transpose) in perfect matching case with $\mathbf{u} = \mathbf{s}$, and α is a complex unknown parameter. However, in practical applications, mismatches usually occur

and cause deviations from the nominal direction $\mathbf{e}_N$. Moreover, assume that $\mathbf{p}$ is belong to the cone set Γ defined as follows [1].

Definition: Let $\mathbf{x} = [\mathbf{x}_{N-1}^T, x_N]^T$ is an N -dimensional complex vector with last component x_N , and the cone set

$$\Gamma = \left\{ \mathbf{x} = [\mathbf{x}_{N-1}^T, x_N]^T \in \mathbb{C}^N : \|\mathbf{x}_{N-1}\| \leq \gamma |x_N| \right\} \quad (3)$$

$$= \left\{ \mathbf{x} \in \mathbb{C}^N : \mathbf{x}^\dagger \mathbf{\Sigma} \mathbf{x} \leq 0 \right\}$$

where $\mathbb{C}^N$ is the N -dimensional complex vector space, $\|\cdot\|$ is the Euclidean norm, $|\cdot|$ is the modulus of a complex number, and $\mathbf{\Sigma} = \text{diag}(1, \dots, 1, -\gamma^2)$ is an N -dimensional diagonal matrix with $\tan \theta_c = \gamma$, θ_c is the acute angle of the cone, and $(\cdot)^\dagger$ is the conjugate transpose.

In general, the Neyman-Pearson test is not available, thus, the sub-optimal GLRT-based receiver is derived which is equivalent to replacing the unknown parameters with their maximum likelihood estimates. Specifically, the GLRT for the problem (1) can be written as

$$\frac{\max_{\tau, \mathbf{p} \in \Gamma} f(\mathbf{z} | H_1, \tau, \mathbf{p})}{\max_{\tau} f(\mathbf{z} | H_0, \tau)} \underset{H_0}{\overset{H_1}{\gtrless}} G \quad (4)$$

where G is the detection threshold, $f(\mathbf{r} | H_1, \tau, \mathbf{p})$ and $f(\mathbf{r} | H_0, \tau)$ are pdfs of the complex receive signals under H_1 and H_0 hypotheses, further more, they are

$$f(\mathbf{z} | H_1, \tau, \mathbf{p}) = \frac{1}{\pi^N \det(\tau \mathbf{M})} \exp \left\{ -\frac{(\mathbf{z} - \mathbf{u})^\dagger \mathbf{M}^{-1} (\mathbf{z} - \mathbf{u})}{\tau} \right\}$$

$$= \frac{1}{\pi^N \det(\tau \mathbf{M})} \exp \left\{ -\frac{(\mathbf{r} - \mathbf{p})^\dagger (\mathbf{r} - \mathbf{p})}{\tau} \right\}$$

$$f(\mathbf{z} | H_0, \tau, \mathbf{p}) = \frac{1}{\pi^N \det(\tau \mathbf{M})} \exp \left\{ -\frac{\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{z}}{\tau} \right\}$$

$$= \frac{1}{\pi^N \det(\tau \mathbf{M})} \exp \left\{ -\frac{\mathbf{r}^\dagger \mathbf{r}}{\tau} \right\}$$

Thus, by substituting the maximum likelihood estimates (MLEs) of the texture component τ under H_0 and H_1 hypotheses, respectively, after some algebraic manipulations, the GLRT-based test (4) can be written as,

$$\max_{\mathbf{p} \in \Gamma} \frac{\mathbf{r}^\dagger \mathbf{r}}{(\mathbf{r} - \mathbf{p})^\dagger (\mathbf{r} - \mathbf{p})} = \frac{\mathbf{r}^\dagger \mathbf{r}}{\min_{\mathbf{p} \in \Gamma} (\mathbf{r} - \mathbf{p})^\dagger (\mathbf{r} - \mathbf{p})} \underset{H_0}{\overset{H_1}{\gtrless}} G \quad (5)$$

Performed as in [2], it is not difficult to obtain that

$$\min_{\mathbf{p} \in \Gamma} (\mathbf{r} - \mathbf{p})^\dagger (\mathbf{r} - \mathbf{p}) = \frac{1}{1 + \gamma^2} (\|\mathbf{r}_{N-1}\| - \gamma |r_N|)^2 u(\mathbf{r}^\dagger \mathbf{\Sigma} \mathbf{r}) \quad (6)$$

where $u(\cdot)$ is the unit step function.

In the following, we present the geometry explain of the above solution, and the constraint cone is shown in Figure 1.

In general, if $\mathbf{r}^\dagger \mathbf{\Sigma} \mathbf{r} < 0$, or to say that the vector $\mathbf{r}$ lies with in the cone, it is obvious that the minimum value is achieved at $\mathbf{r} = \mathbf{p}$, such that

$$\min_{\mathbf{p} \in \Gamma} (\mathbf{r} - \mathbf{p})^\dagger (\mathbf{r} - \mathbf{p}) = 0$$

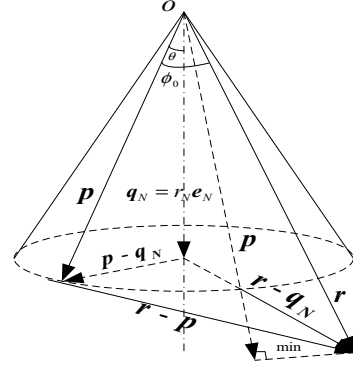


Figure 1 The geometry of the constraint cone

However, in the case of $\mathbf{r}^\dagger \mathbf{\Sigma} \mathbf{r} \geq 0$, the norm triangle inequality is satisfied, i.e.,

$$\|\mathbf{r} - \mathbf{p}\| = \|(\mathbf{r} - \mathbf{q}_N) - (\mathbf{p} - \mathbf{q}_N)\| \geq \|\mathbf{r} - \mathbf{q}_N\| - \|\mathbf{p} - \mathbf{q}_N\| \quad (7)$$

$$\geq \|\mathbf{r} - \mathbf{q}_N\| - \|\mathbf{p}_c - \mathbf{q}_N\|$$

where $\mathbf{q}_N$ is the projection of $\mathbf{r}$ onto $\mathbf{e}_N$, $\mathbf{p}$ is the vector with in the cone, and $\mathbf{p}_c$ is the vector on the surface of cone, such that $\mathbf{p}_c^\dagger \mathbf{\Sigma} \mathbf{p}_c = 0$. Thus, the possible $\mathbf{p}_c$ lies in the same plane with $\mathbf{r}$ and $\mathbf{q}_N$, then, the minimum value of $\|\mathbf{r} - \mathbf{p}\|$ is obtained, i.e.,

$$\min_{\mathbf{p}} \|\mathbf{r} - \mathbf{p}\|^2 = (\|\mathbf{r}_{N-1}\| - \gamma |r_N|)^2 \cos^2 \theta_c = \frac{1}{1 + \gamma^2} (\|\mathbf{r}_{N-1}\| - \gamma |r_N|)^2 \quad (8)$$

Concerning on the condition $\mathbf{r}^\dagger \mathbf{\Sigma} \mathbf{r} > 0$, the solution is the same as (6).

Thus, the GLRT-based test can be expressed in terms of the original data vector, i.e.,

$$\frac{\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{z}}{\left(\sqrt{\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{z} - \frac{|\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{s}|^2}{\mathbf{s}^\dagger \mathbf{M}^{-1} \mathbf{s}}} - \gamma \sqrt{\frac{|\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{s}|^2}{\mathbf{s}^\dagger \mathbf{M}^{-1} \mathbf{s}}} \right)^2} u \left(\mathbf{z}^\dagger \mathbf{M}^{-1/2} \mathbf{\Sigma} \mathbf{M}^{-1/2} \mathbf{z} \right) \underset{H_0}{\overset{H_1}{\gtrless}} G \quad (9)$$

Define that the normalized matched filter (NMF) is

$$l_{\text{NMF}} = \frac{|\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{s}|^2}{(\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{z})(\mathbf{s}^\dagger \mathbf{M}^{-1} \mathbf{s})} \quad (10)$$

Thus, the GLRT-based test (7) can be recast as

$$\frac{1}{(\sqrt{1 - l_{\text{NMF}}} - \gamma \sqrt{l_{\text{NMF}}})^2} u \left(\mathbf{z}^\dagger \mathbf{M}^{-1/2} \mathbf{\Sigma} \mathbf{M}^{-1/2} \mathbf{z} \right) \underset{H_0}{\overset{H_1}{\gtrless}} G \quad (11)$$

where G is the suitable modification of the original threshold.

III. ADAPTIVE SCHEMES

Assume the secondary data $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K] \in \mathbb{C}^{N \times K}$ are available to estimate the covariance matrix, which makes the derived test fully adaptive. The fixed-point estimate (FPE) is adopted, which is given by

$$\hat{\mathbf{M}}_{\text{FEP}} = \frac{N}{K} \sum_{k=1}^K \frac{\mathbf{x}_k \mathbf{x}_k^\dagger}{\mathbf{x}_k^\dagger \hat{\mathbf{M}}_{\text{FEP}}^{-1} \mathbf{x}_k} \quad (12)$$

The FPE performs well in SIRVs, and is unbiased, consistent and ensures the CFAR property with respect to both the structure of the covariance matrix and the clutter power levels. To solve the FPE of (12), it can be resorted to the recursive operation, that is,

$$\hat{\mathbf{M}}_{\text{FEP}}(\ell+1) = \frac{N}{K} \sum_{k=1}^K \frac{\mathbf{x}_k \mathbf{x}_k^\dagger}{\mathbf{x}_k^\dagger \hat{\mathbf{M}}_{\text{FEP}}^{-1}(\ell) \mathbf{x}_k}, \ell = 0, 1, \dots, L_{\text{FEP}} \quad (13)$$

and the initial estimate $\hat{\mathbf{M}}_{\text{FEP}}(0)$ is given by

$$\hat{\mathbf{M}}_{\text{FEP}}(0) = \frac{N}{K} \sum_{k=1}^K \frac{\mathbf{x}_k \mathbf{x}_k^\dagger}{\mathbf{x}_k^\dagger \mathbf{x}_k} \quad (14)$$

The recursive error is defined as

$$\text{ER}(\ell) = \frac{\|\hat{\mathbf{M}}_{\text{FEP}}(\ell+1) - \hat{\mathbf{M}}_{\text{FEP}}(\ell)\|_F}{\|\hat{\mathbf{M}}_{\text{FEP}}(\ell+1)\|_F} \quad (15)$$

where $\|\cdot\|_F$ denotes the Frobenius norm and L_{FEP} denotes the number of the recursion. In general, the L_{FEP} is less than 5 for $\text{ER} < 5\%$. By the way, the FPE is also called recursive ML-estimator.

Therefore, the two-step fully adaptive GLRT-based tests are given, i.e.,

$$\frac{1}{\left(\sqrt{1-\hat{I}_{\text{NMF}}} - \gamma\sqrt{\hat{I}_{\text{NMF}}}\right)^2} u\left(\mathbf{z}^\dagger \hat{\mathbf{M}}^{-1/2} \hat{\Sigma} \hat{\mathbf{M}}^{-1/2} \mathbf{z}\right) \underset{H_0}{\overset{H_1}{>}} G_1 \quad (16)$$

where

$$\hat{I}_{\text{NMF}} = \frac{|\mathbf{z}^\dagger \hat{\mathbf{M}}_{\text{FEP}}^{-1} \mathbf{s}|^2}{(\mathbf{z}^\dagger \hat{\mathbf{M}}_{\text{FEP}}^{-1} \mathbf{z})(\mathbf{s}^\dagger \hat{\mathbf{M}}_{\text{FEP}}^{-1} \mathbf{s})} \quad (17)$$

and G_1 is the suitable modification of the original threshold in (11).

IV. PERFORMANCE ASSESSMENT

A uniform linear array with N omnidirectional elements, and the nominal steering vector $\mathbf{s}$ is given by

$$\mathbf{s} = \frac{1}{\sqrt{N}} [1, 1, \dots, 1]^T$$

and the actual steering vector after whitening and rotating is given

$$\mathbf{p} = A \exp\{j\xi\} \left(\mathbf{e}_N \cos \phi + \frac{\mathbf{e}_N^\perp}{\|\mathbf{e}_N^\perp\|} \sin \phi \right)$$

where A is the amplitude, ξ is the random phase, $\mathbf{e}_N^\perp$ is a complex vector, orthogonal to $\mathbf{e}_N$, with independent identically distributed complex Gaussian random components, and ϕ is the mismatches acute angle between actual steering vector and nominal vector, such that

$$\cos \phi = \frac{|\mathbf{p}^\dagger \mathbf{e}_N|}{\|\mathbf{p}\| \|\mathbf{e}_N\|}$$

We assume a clutter-dominated scenario, which is sampled from K-distribution random process with exponential correction structure covariance matrix, such that

$$\mathbf{M}(k, j) = \rho^{|k-j|}, \quad 1 \leq k, j \leq N$$

where ρ is the one-lag correlation coefficient. The SCR is defined as

$$\text{SCR} = A^2 \mathbf{s}^\dagger \mathbf{M}^{-1} \mathbf{s}$$

The standard Monte Carlo technique is adopted to assessed the performance of derived GLRT-based test and together with NMF.

In Figure 2, the $\cos \phi$ is analyzed, and the results show that the performance is decreased with the small $\cos \phi$, in other words, for large ϕ , the mismatches are more serious. And the test is prone to reject the signals, although the SCR is high. The adaptive derived GLRT is consistent with adaptive NMF in such case, however, in high $\cos \phi$ with the small ϕ , the detection performance of NMF is a litter better, or the GLRT has better rejection performance for mismatched signals.

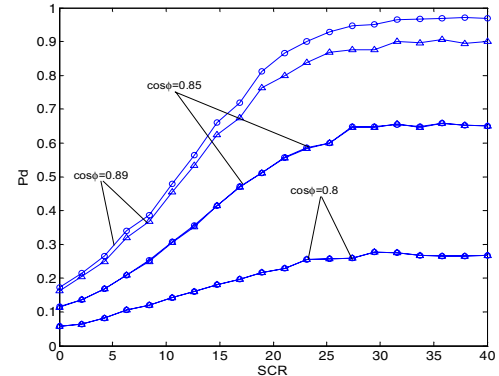


Figure 2 P_d plots v.s. SCR with $\cos \phi = 0.89, 0.85, 0.8$, “o” curves NMF, “Δ” curves GLRT, for $P_{fa} = 10^{-3}$, $N = 8$, $K = 32$, $\nu = 0.5$, $\gamma = 0.5$, $\rho = 0.9$ and non-fluctuating target

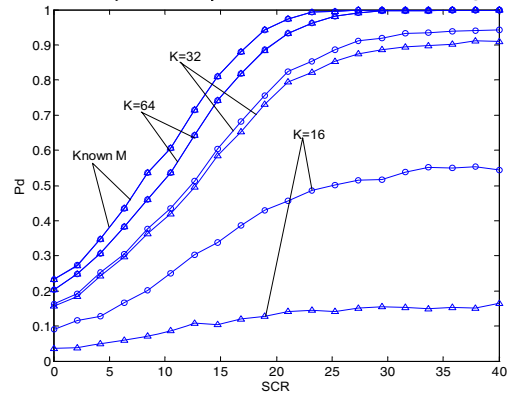


Figure 3 P_d plots v.s. SCR with $K = 16, 32, 64$, “o” curves NMF, “Δ” curves GLRT, for $P_{fa} = 10^{-3}$, $N = 8$, $\nu = 0.5$, $\gamma = 0.5$, $\cos \phi = 0.89$, $\rho = 0.9$ and non-fluctuating target

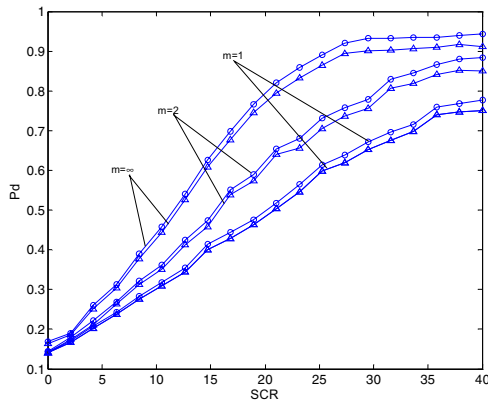


Figure 4 P_d plots v.s. SCR with different fluctuating model, “o” curves NMF, “Δ” curves GLRT, for $P_{fa} = 10^{-3}$, $N = 8$, $\nu = 0.5$, $K = 32$, $\gamma = 0.5$, $\cos \phi = 0.85$, $\rho = 0.9$ and fluctuating target

In Figure 3, the size of secondary that effects the detection performance is analyzed, meanwhile, the curves with exact covariance matrix are also given. The curves show that the detection performance is increased by increasing the size K , and the adaptive loss is acceptable. Specifically, the loss is 2dB in the case $P_d = 0.9$ between exact covariance and estimated covariance with $K = 64$. Meanwhile, the adaptive derived GLRT is consistent with adaptive NMF in large size case, however, in the small size K , the detection performance of NMF is better.

The fluctuating target is also analyzed in Figure 4, and the amplitude A has a chi distribution with $2m$ degrees of freedom [1]. The results show that the fluctuation law significantly affects the detection performance, and it is better for higher value of m . It can be explained that for the higher value of m , the fluctuation is more prone to non-fluctuation characteristics.

V. CONCLUSIONS

In this paper, we have addressed the receiver design against SIRV clutter in presence of steering vector mismatches with unknown covariance matrix, and the fully adaptive GLRT-based receiver is devised by resorting to the secondary data. The simulation results show that the derived test is consistent with adaptive NMF in exact covariance case or/and serious mismatched cases. Meanwhile, the fluctuation law of target is

also effects the detection performance greatly. The further work is extending the framework to the polarimetric or/and MIMO case.

REFERENCES

- [1] A. De Maio, “Robust adaptive radar detection in the presence of steering vector mismatches,” *IEEE Trans. on Aerospace and Electronic Systems*, vol. 41, no.4, pp. 1322-1337, Oct. 2005.
- [2] F. Bandiers, A. De Maio, and G. Ricci, “Adaptive CFAR radar detection with conic rejection,” *IEEE Trans. on Signal Processing*, vol. 55, no.6, pp. 2533-2541, Jun. 2007.
- [3] F. Bandiers, D. Orlando, and G. Ricci, “CFAR detection strategies for distributed targets under conic constraints,” *IEEE Trans. on Signal Processing*, vol. 57, no.9, pp. 3305-3316, Sep. 2009.
- [4] O. Besson, “Adaptive detection with bounded steering vector mismatch angle,” *IEEE Trans. on Signal Processing*, vol. 55, no.4, pp. 1560-1564, Apr. 2007.
- [5] E. Conte, and M. Longo, “Modelling and simulation of non-Rayleigh radar clutter,” *IEE Proceedings-F*, vol. 138, no. 2, pp. 121-130, 1991
- [6] K. Sangston, and K. Gerlach, “Coherent detection of radar targets in a non-Gaussian background”, *IEEE Trans. on Aerospace and Electronic Systems*, vol. 30, no.2, pp. 330-340, 1994.
- [7] E. Conte, M. Lops, and G. Ricci, “Asymptotically optimum radar detection in Compound-Gaussian clutter”, *IEEE Trans. on Aerospace and Electronic Systems*, vol. 31, no.2, pp. 611-616, Apr. 1995.
- [8] E. Conte, M. Di Bisceglie, C. Galdi, and G. Ricci, “A procedure for measuring the coherence length of the sea texture”, *IEEE Trans. on Instrumentation and Measurement*, vol.46, no.4, pp. 836-841, 1997.
- [9] F. Gini and M. V. Greco, “Covariance matrix estimation for CFAR detection in correlated heavy tailed clutter, Signal Processing”, *Special Section on Signal Processing with Heavy Tailed Distributions*, vol. 82, no. 12, pp. 1847-1859, Dec 2002.
- [10] F. Pascal, Y. Chitour, J. P. Ovarlez, P. Forster, and P. Larzabal, “Covariance structure maximum-likelihood estimates in compound Gaussian noise: Existence and algorithm analysis”, *IEEE Trans. on Signal Processing*, vol. 56, no. 1, pp. 34-48, Jan 2008.
- [11] F. Pascal, P. Forster, J. P. Ovarlez and P. Larzabal, “Performance analysis of covariance matrix estimates in impulsive noise”, *IEEE Trans. Signal Processing*, vol. 56, no.6, pp. 2206- 2217, Jun. 2008.