

Low Complexity ADRG-PTS Scheme for PAPR Reduction in OFDM Systems

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Abstract—In this paper a novel technique for reducing the peak to average power ratio (PAPR) reduction based on the addition of random signals with a complex Gaussian distribution (ADRG) and combining it with partial transmit sequence (PTS) is proposed. Unlike the conventional PTS (C-PTS) which needs several inverse fast Fourier transform (IFFT) operations, the proposed ADRG-PTS technique requires only half IFFT operations. Simulation and results are examined with 16 QAM OFDM signal.

Keywords— ADRG, IFFT, OFDM, PTS, PAPR

I. INTRODUCTION

As the bandwidth demand in communication systems increases, new transmission formats like orthogonal frequency division multiplexing (OFDM) has been applied in modern communication systems such as WiMAX, LTE and DVB. Despite the advantages of OFDM signals like high spectral efficiency and robustness against ISI, the OFDM signals suffer from disadvantages in which the main one is high PAPR [1], [2]. The reason of high PAPR is that in time domain the OFDM signal is actually the sum of many narrowband signals. At some instances, the sum of these narrowband signals may be large and at the other times may be small, which means that the peak of the signal can be much larger than the average value. This high PAPR signal when transmitted through a nonlinear power amplifier creates spectral broadening and also an increase in the dynamic range of the digital to analog converter (DAC). The result will be an increase in the cost of the system and reduction in efficiency. To overcome this drawback, several techniques for reducing the PAPR have been proposed. Some of the most important techniques are selected mapping (SLM) [3] which is in frequency domain, and PTS [2], [5], [6], [12] which is in time domain. C-PTS technique use iterative routine similar to the trial-and-error method for finding the optimum phase factors that leads to lower PAPR. C-PTS is distortionless but is time-consuming, and needs large number of computations [2]. Here first we propose a new technique called ADRG [4] which on its own can reduce PAPR, and then for improving the PAPR

reduction and decreasing the complexity of C-PTS, a technique called ADRG-PTS is proposed. The technique has been called based on the order of the implementation in the system. In this paper by applying the ADRG-PTS technique not only the PAPR is reduced but also the complexity is decreased compared with C-PTS.

II. CONVENTIONAL OFDM

In OFDM systems, a fixed number of successive input data samples are modulated first (e.g. PSK or QAM), and then jointly correlated together using IFFT at the transmitter side. IFFT is used to produce orthogonal data subcarriers. Mathematically, IFFT combines all the input signals (superposition process) to produce each element (signal) of the output OFDM symbol. The time domain complex baseband OFDM signal can be represented as [1]:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{n}{N} k}, \quad n = 0, 1, 2, \dots, N-1 \quad (1)$$

Where x_n is the n -th signal component in OFDM output symbol, X_k is the k -th data modulated symbol in OFDM frequency domain, and N is the number of subcarriers.

The PAPR (in dB) of the transmitted OFDM signal can be expressed as [2]:

$$PAPR = \frac{(\max(|x_n|^2))}{E[|x_n|^2]} \quad (2)$$

Where $E[.]$ is the expected value operator. The theoretical maximum of the PAPR for N number of subcarriers is as follows:

$$PAPR_{\max} = 10 \log(N) \text{ dB} \quad (3)$$

PAPR is a random variable because it is a function of the input data, and the input data are random variable. Therefore PAPR can be analyzed by using level crossing rate theorem which calculates the mean number of times that the envelope of a stationary signal crosses a given level. Knowing the amplitude distribution of the OFDM output signals, it is easy to compute the probability that the instantaneous amplitude will be above a given threshold and the same for power. This is performed by calculating the complementary cumulative distribution function (CCDF) for different PAPR values as

follows:

$$CCDF = \Pr(PAPR > PAPR_0) \quad (4)$$

Here the effect of additive white Gaussian noise (AWGN) on OFDM performance is described. As OFDM systems use standard digital modulation formats to modulate the subcarriers, PSK and QAM are usually used due to their excellent error resilient performance. The main important block in OFDM is IFFT. IFFT change the distribution of the signal without alteration of its average power. Since the AWGN is in time domain, channel corresponds to AWGN of the same average power is in frequency domain, hence the bit error rate (BER) performance of the OFDM system is identical to that of the single carrier systems using same modulation parameters. The BER or bit error P_{be} in an AWGN channel is given by:

$$P_{be, MQAM} \approx \frac{4(\sqrt{M}-1)}{k\sqrt{M}} Q\left(\sqrt{\frac{3k}{(M-1)} \cdot \frac{E_b}{N_o}}\right) \quad (5)$$

where M is the modulation order, $k = \log_2(M)$ refers to the number of bits per symbol, and $Q(\cdot)$ is the Gaussian Q function defined as:

$$Q(y) = \text{erfc}\left(\frac{y}{\sqrt{2}}\right) \quad (6)$$

In this paper the performance of BER versus energy bit signal to noise ratio (SNR) (E_b/N_o) is analyzed.

III. PROPOSED ADRG-PTS SCHEME

The block diagram of the proposed method for PAPR reduction in OFDM system is shown in Figure 1. The addition of random signals is performed as shown in the block diagram in frequency domain of the OFDM system.

In this approach the complex random signal is first generated and added to the Gaussian random signal. Mathematically the operation represents as addition of two independent and different distributed random variables (uniform and Gaussian distributions). In [7], it is proved the mean and variance of the new random signal that are equal to the sum of the corresponding means and variances of the two random variables as expressed below:

$$X^G = X + R \quad (7)$$

Where $X^G = [X_0^G, X_1^G, \dots, X_{N-1}^G]$ is the sequence of new vector and $X = [X_0, X_1, \dots, X_{N-1}]$ is the vector of the original modulated symbols in complex form and $R = [R_0, R_1, \dots, R_{N-1}]$ is the generated random signal in a complex distribution form. It can be shown that the mean μ_{X^G} and variance $\sigma_{X^G}^2$ of the new

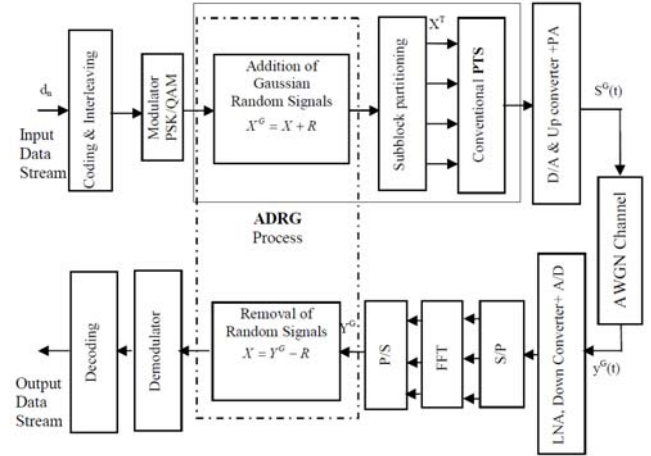


Figure 1. Block diagram of the proposed ADRG-PTS Scheme

signals X^G are calculated by means of addition of two random signals.

$$\mu_{X^G} = \mu_X + \mu_R \quad (8)$$

For large number of data sub-carriers (N) the value of mean and variance of the data symbols at the frequency domain is zero and one, respectively [7]. In the time domain, these parts FFT output have zero mean values as well, therefore the random complex signals should be generated with zero mean. This results in zero mean ($\mu_{X^G} = 0$) of the new vector X . The new vector X depends on the power ratio of the data sub-carriers to the added random signals.

$$SPR = \frac{P_R}{P_D} \quad (9)$$

Where SPR , P_R and P_D are the signal power ratio, the power of the random signals and the power of the original data modulated symbols respectively. For choosing the P_R we predefine a constant value to be used for all OFDM frames without sending any side information to the receiver to recover the original data or assign different P_R values with every individual OFDM frames to make a fixed PAPR value. In the last case, the optimum value of P_R can be obtained by representing on iterative loop. The loop starts from a small P_R , then increases it in steps gradually till the accepted value (equal to or less than) of the PAPR is achieved. The maximum number of loops is determined by the processing time constraint of the system application. The variance associated with the optimum P_R should be coded with higher error protection.

Now the signal X^G is partitioned into M disjoint block $S_m = [S_1, S_2, \dots, S_M]$ and then these subblocks are combined to minimize the PAPR in time domain. After doing the IFFT, the phase factors $b_m = e^{j\theta_m}$, $m = 1, 2, \dots, M$ are used for optimizing the S_m .

In time domain the OFDM signal can be expressed as:

$$s'(b) = \sum_{m=1}^M b_m s_m \quad (10)$$

Where $s'(b) = [s'_0(b), s'_1(b), \dots, s'_{N-1}(b)]^T$. The objective is to find the optimum signal $s'(b)$ with the lowest PAPR. The process is performed by choosing the optimization parameter $\tilde{b}$ with the following condition:

$$\tilde{b} = \arg \min \left(\max_{0 \leq k \leq NF-1} \left| \sum_{m=1}^M b_m s_{m,k} \right| \right) \quad (11)$$

After finding the optimum $\tilde{b}$ then the optimum signal $s'(b)$ is transmitted.

The important advantages of ADRG-PTS method compared to ARDG or PTS, is to reduce the complexity of the OFDM system while maintaining the PAPR performance or even improved it. The complexity of ADRG-PTS can be reduced in terms of the number of subblock partitioning in which the number of IFFT block is halved, and this is because of comparable PAPR reduction to C-PTS when $M=4$ and $W=4$. W is the number of allowed weighting factor which in this paper is assumed from the possible values of $\{1 -1 j -j\}$.

When the number of subcarriers is $N=2^n$ and oversampling factor is $L=2^l$, the total complexity including IFFT and phase factor combination, and PAPR calculation for C-PTS can be expressed as below:

$$n_{mul} = 2^{n+l-1} (n+l) M + W^{M-1} N (M+1) \quad (12)$$

$$n_{add} = 2^{n+l} (n+l) M + W^{M-1} N (M-1) \quad (13)$$

where M is the number of subblocks and W is the number of allowed phase factors. These values for the proposed method are:

$$n_{mul} = 2^{n+l-1} (n+l) \frac{M}{2} + W^{\frac{M}{2}-1} N \left(\frac{M}{2} + 1 \right) \quad (14)$$

$$n_{add} = 2^{n+l} (n+l) \frac{M}{2} + W^{\frac{M}{2}-1} N \left(\frac{M}{2} - 1 \right) \quad (15)$$

The computational complexity reduction ratio (CCRR) of ADRG-PTS technique over the C-PTS is defined as:

$$CCRR = \left(1 - \frac{\text{Complexity of the ADRG-PTS}}{\text{Complexity of the C-PTS}} \right) \times 100\% \quad (16)$$

Table 1 gives the CCRR of the proposed ADRG-PTS technique over the conventional PTS and for simplicity we assume oversampling factor $L=1$.

From Table 1, the computational complexity by applying ADRG-PTS technique is reduced significantly. For other values of M , it can be shown that the complexity can be reduced rapidly.

TABLE 1. Computational Complexity Comparison of ADRG-PTS and C-PTS

Complex Computation	No. of Subblocks	C-PTS	ADRG-PTS (M=2)	CCRR
Multiplication	M=4	86016	5120	94%
Addition		57344	5120	91%

With the addition of digital predistortion (DPD) as proposed in [8-11], the efficiency and linearity of the power amplifier will be increased and then together with the current PAPR reduction technique, the amount of the peak of the OFDM signal can be increased more than when there is no DPD and this will result in simpler ADRG-PTS with less power level of random signals (PR).

IV. SIMULATION AND RESULTS

To evaluate and compare the performance of ADRG-PTS with C-PTS and ADRG, Matlab simulation has been performed. We employed OFDM signal with $N=512$ subcarriers and 16 QAM modulation scheme. To obtain the CCDF, 40 000 random OFDM symbols are generated.

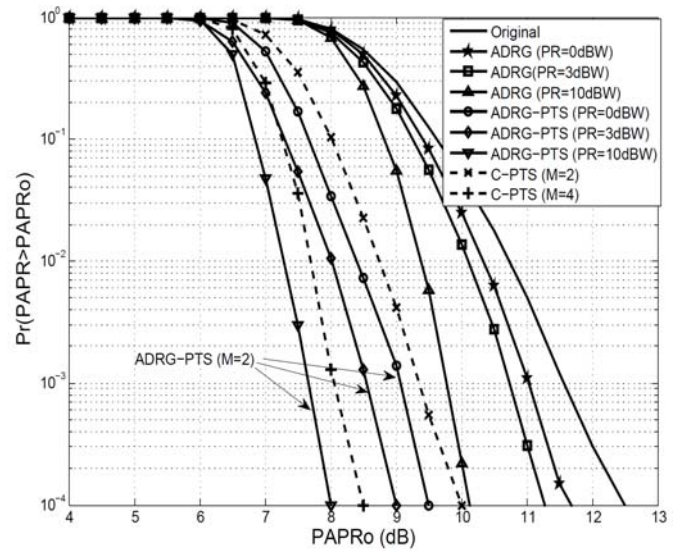


Figure 2. CCDF of the PAPR of conventional PTS and ADRG-PTS technique for 16 QAM modulation

Figure 2 shows the PAPR reduction performance of the proposed ADRG-PTS compared to ADRG alone and C-PTS. From this figure, it is clear that the PAPR performance is improved by applying the ADRG-PTS method. By applying the ADRG-PTS with $M=2$ and for $P_R = 0dBW$ and $3dBW$ the PAPR is less than that of C-PTS with $M=4$ but when $P_R = 10dBW$, the PAPR performance improves significantly.

V. CONCLUSION

In this paper a new method for PAPR reduction in OFDM systems is proposed Based on a combination of ADRG and PTS. By this method the computational complexity is reduced significantly and also the PAPR performance with increasing the power of random signals can be improved at the expense of BER degradation. Implementation of this method will reduce the cost of the system while the performance is maintained. The application of this method covers WiMAX, LTE, 4G and etc.

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