

Study of PAPR Reduction Methods in OFDM Systems

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Abstract— High PAPR is the main drawback of OFDM systems. DSI method, PTS method and SLM method are three of the most promising techniques to reduce PAPR. This Work studies these methods and there is a comparison between their performances.

Keywords— DSI, IDFT, SLM, PAPR, PTS

I. INTRODUCTION

Multicarrier modulation systems, often also called as orthogonal frequency division multiplexing (OFDM), are competing well with single carrier systems. One of the advantages of OFDM system over the single carrier system is the better performance in fading phenomena of multipath environment. The OFDM system has a serious drawback which is denoted as high Peak to Average Power Ratio (PAPR) [1], [2]. The nature of modulation causes these high peaks. When multiple sinusoids are added together to form the multicarrier signal, these peaks are generated. When all the peaks are added together, the peak appears. For example the peak magnitude would have a value of N , where the average might be quite low due to the destructive interference between the sinusoids. So the ratio between peak and average will be high which is usually undesirable for amplification stage and it usually strains the analog circuitry [3]. When the signal is introduced to the high power amplifier (HPA), high peaks could cause problems. Because HPA is a non-linear component and high PAPR signals would require high range of dynamic linearity from the analog circuits. So power amplifiers with higher linearity are more expensive and consume more power and normally have less efficiency. Even the best power amplifier in case of linearity and efficiency has to operate with back-off to maintain this problem [3].

In this paper a study on three major PAPR reduction methods, Dummy Sequence Insertion (DSI) [11], Partial Transmit Sequence (PTS) [14], and Selected Mapping (SLM) [2] Method is presented. Also the simulation results of these methods using Matlab software are given.

II. DEFINITION OF PAPR

Basically peak-to-average power ratio (PAPR) is the most popular parameter used to evaluate the dynamic range of the time-domain OFDM signal or signal envelop variation or the

crest factor (CF) where $PAPR=CF^2$. Crest factor is another parameter which is widely used in the literature, and defined as the square root of the PAPR.

For example, the data sequence in (1) with length of N during the time interval of $[0, T]$ is assumed as a modulated data for the multicarrier system.

$$A = (A_0, A_1, \dots, A_{(N-1)}) \quad (1)$$

Where A_i is a symbol from a signal constellation and T is the OFDM symbol duration. Then the complex envelope of the baseband OFDM signal for N carriers is given by (2).

$$s(t) = \sum_{n=0}^{N-1} A_n e^{j\omega_n t} \quad (2)$$

Where, $\omega_0 = 2\pi/T$ and $j = \sqrt{-1}$. In practical systems, a guard interval (cyclic prefix) is inserted by the transmitter in order to remove Inter-Symbol Interference (ISI), and Inter-Channel Interference (ICI) in the multipath environment. However, it can be ignored since it does not affect the PAPR [4]. The PAPR of the transmit signal $s(t)$, defined above, is the ratio of the maximum instantaneous power and the average power, given by

$$PAPR(A) = \frac{\max |s(t)|^2}{E \{ |s(t)|^2 \}} \quad (3)$$

Where, E . denotes the expectation operator. Usually, the continuous time PAPR of $s(t)$ is approximated using the discrete time PAPR, which is obtained from the samples of the OFDM signal [4].

There have been many researches devoted to development of Crest Factor Reduction (CFR) or PAPR reduction algorithms. Some review of them can be found in [5]-[13]. The review shows that SLM and DSI methods have significant improvement in PAPR reduction but they have limitations in terms of efficiency and hardware implementation. In this paper, combination of these two methods is proposed.

III. THE PERFORMANCE OF THE DSI AND SLM METHODS

The SLM method was first proposed in [2] as distortionless PAPR reduction method. In SLM, M phase rotation ($B(1)$, $B(2)$, ..., $B(U)$), is multiplied to the copies of the original signal (S) to produce candidate signals. After the Inverse Discrete Fourier Transform (IDFT), the PAPR of all candidates $\{S(1), S(2), \dots, S(U)\}$ is measured and the sequence with the lowest PAPR is selected to transmit. When U is high, the reduction is better but a large number of IDFT blocks are required to generate candidate signals. This results in increased computational complexity. The block diagram of SLM method is shown in Figure 1.

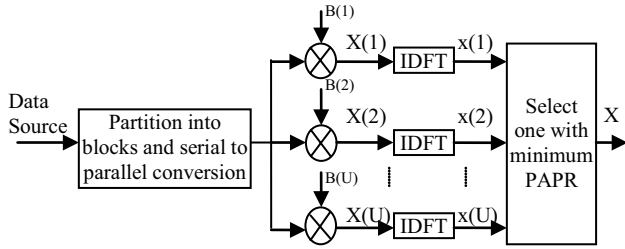


Figure 1. The block diagram of the Selected Mapping (SLM) Method [10]

Many modifications have been done in SLM method but most of them degrade bandwidth efficiency [5]-[8]. In DSI method, dummy sequence is added into the input signal before the IDFT stage. The PAPR of the signal after IDFT is compared with a prescribed threshold level. If the PAPR of the output is lower than the threshold, the data is transmitted. Otherwise another dummy sequence added to reduce the PAPR and the program is repeated until the desired level is reached [9]. The block diagram of DSI method is shown in Figure 2.

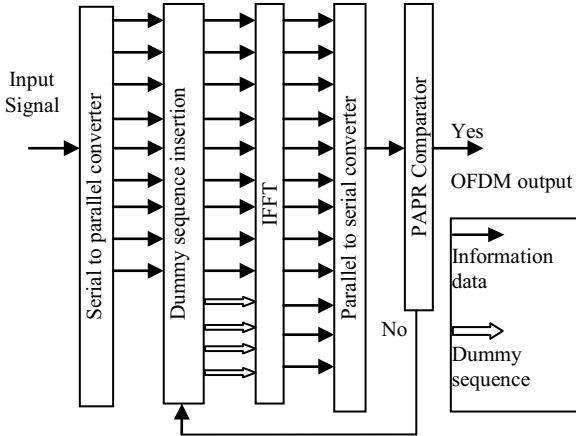


Figure 2. The block diagram of Dummy Sequence Insertion (DSI) method [9]

As it is shown in Figure 2, in DSI method, the zeros of the signal are replaced with dummy signals so the length of data is

increased and so the bandwidth. This will degrade the Transmission Efficiency (TE).

IV. PARTIAL TRANSMIT SEQUENCE SCHEME

The block diagram of the conventional PTS (C-PTS) scheme including DPD and the PA is shown in Figure 3. A frequency domain random input signal with length N is denoted X . X is partitioned into M disjoint subblocks $X_m = [X_{m,0}, X_{m,1}, \dots, X_{m,N-1}]^T$, $m=1, 2, \dots, M$ such that $\sum_{m=1}^M X_m = X$ and then these subblocks are combined to minimize the PAPR in time domain. The subblock partition type applied here is based on interleaving that is best for conventional PTS (C-PTS) technique in terms of computational complexity reduction compared to adjacent and Pseudo-random, however it has the least PAPR reduction among them.

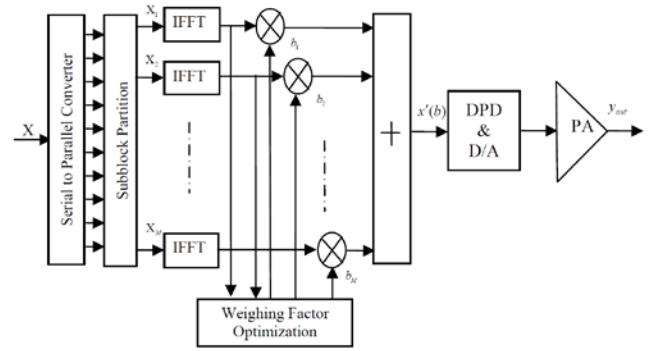


Figure 3. The Block diagram of the C-PTS scheme.

By applying the phase rotation factor $b_m = e^{j\phi_m}$, $m=1, 2, \dots, M$ to the IFFT of the m^{th} subblock X_m , the time domain signal after combining is given by:

$$x'(b) = \sum_{m=1}^M b_m x_m \quad (4)$$

Where $x'(b) = [x'_0(b), x'_1(b), \dots, x'_{N-1}(b)]^T$.

The objective is to find the optimum signal $x'(b)$ with the lowest PAPR. The optimum phase sequence is given by:

$$\tilde{b} = \arg \min \left(\max_{0 \leq k \leq N-1} \left| \sum_{m=1}^M b_m s_{m,k} \right| \right) \quad (5)$$

After finding the optimum $\tilde{b}$ then the optimum signal $s'(b)$ is transmitted. The digital predistortion technique that is applied here is the memory polynomial predistortion (MPP) which has the ability to linearize both with and without memory PA.

V. COMPUTATIONAL COMPLEXITY

As it was mentioned earlier, the higher number of candidate signals in SLM method leads to a better performance and better PAPR reduction but this means using higher number of IDFT blocks. The implementation of such system needs higher number of multiplications and additions.

The numbers of complex multiplication (n_{mul}) and number of complex addition (n_{add}) of the conventional SLM OFDM scheme are given by $n_{mul}=2^{n-1}nU$ and $n_{add}=2^n nU$, where U is the total number of candidate signals and $N=2^n$ is the number of carriers and $N=256$ for typical WiMAX signal [13].

The complexity of C-PTS can be expressed by [14]:

$$n_{mul} = M \frac{N}{2} \log N + W^{M-1} N (M+1) \quad (6)$$

$$n_{add} = MN \log N + W^{M-1} N (M-1)$$

Where M is the number of subblocks, W is the number of allowed phase factors and N is the IFFT length.

The computational complexity is calculated and presented in Table 1.

TABLE 1. COMPUTATIONAL COMPLEXITY OF SLM

Complex Computation	C-SLM
No. of Multiplication (n_{mul})	2048 ($U=2$)
No. of Addition (n_{add})	4096 ($U=2$)
No. of Multiplication (n_{mul})	4096 ($U=4$)
No. of Addition (n_{add})	8192 ($U=4$)
No. of Multiplication (n_{mul})	8192 ($U=8$)
No. of Addition (n_{add})	16384 ($U=8$)
No. of Multiplication (n_{mul})	16384 ($U=16$)
No. of Addition (n_{add})	32768 ($U=16$)

The Table shows that the number of multiplications and the additions required for implementation of a SLM method is increased when U is higher. Table 2 present the computation complexity of the C-PTS scheme. The number of additions and multiplications are calculated from (6).

TABLE 2. COMPUTATIONAL COMPLEXITY OF PTS

Complex Computation	C-PTS
No. of Multiplication (n_{mul})	3688 ($M=2$)
No. of Addition (n_{add})	2257 ($M=2$)
No. of Multiplication (n_{mul})	83153 ($M=4$)
No. of Addition (n_{add})	51618 ($M=4$)

The calculation is done for $M=2$ and $M=4$. It is presented in Table 2 that by increasing the number of partitions in PTS method, the number of additions and multiplication required for implementing this method is increased. It is obvious that $M=8$ and $M=16$ will increase the complexity more. When $M=4$, the complexity is higher than when SLM is used with $M=16$. In the other hand, SLM method needs to inform receiver about the selected sequence by sending some side information. For example if $U=2$, only one bit ("0" or "1") can show which sequence is selected. When U is higher, higher number of bits is required to be sent to the receiver as side information. This will add more complexity into the system. The side information has no changes in DSI method.

Comparing Table 1 and Table 2 shows that the number of required additions and multiplications in implementing the PTS method is higher than the SLM method.

VI. SIMULATION AND RESULT

The performance of the DSI method is presented in Figure 4. The number of dummy signals are varied and so the result.

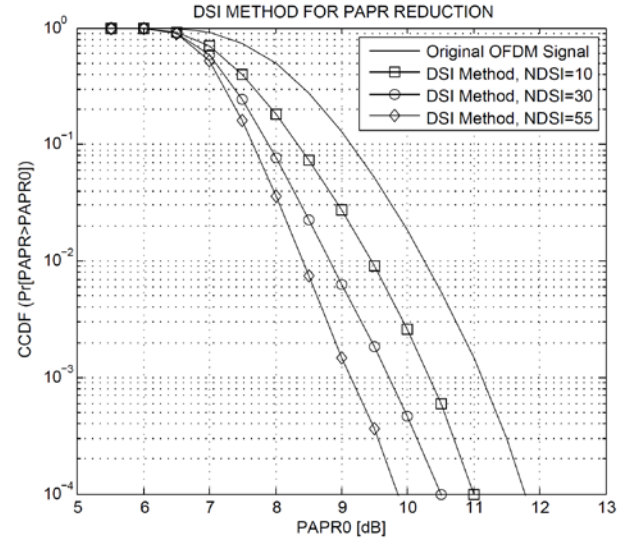


Figure 4. The CCDF of the Dummy Sequence Insertion (DSI) Method

As the figure shows, there is about 11.8dB PAPR at $CCDF=10^{-4}$ for original signal when no PAPR reduction method is used. This number will be reduced by using DSI method. When 10 dummy signals ($NDSI=10$) are inserted in signal zeros, there will be about 0.80dB reduction and PAPR of 11dB at $CCDF=10^{-4}$ can be achieved. When $NDSI=30$, the reduction is more and $PAPR=10.5$ at $CCDF=10^{-4}$. The best reduction is when $NDSI=55$, that $PAPR$ of 9.9dB at $CCDF=10^{-4}$ is achieved.

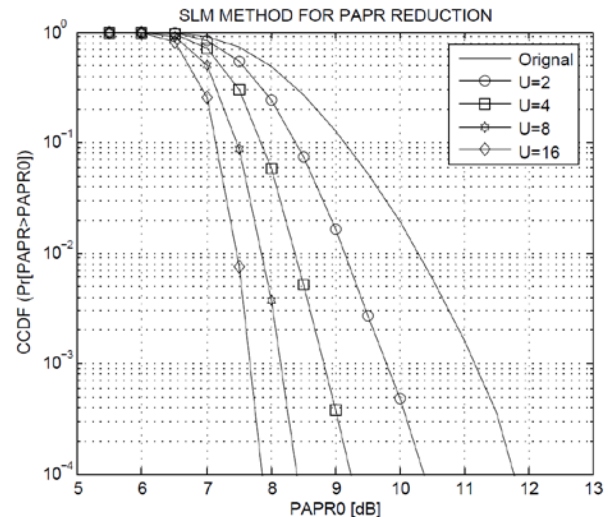


Figure 5. The CCDF of the Selected Mapping (SLM) Method

When the number of dummy signals is higher, the reduction is better, but the bandwidth will be wider and so the Transmission Efficiency (TE) is degraded. This is the reason for many modifications that are proposed in this field. The performance of the SLM method is presented in Figure 5.

As it is presented in Figure 4, the reduction is improved by increasing the number of candidate signals (U). When $U=2$, the PAPR is about 10.4dB at CCDF= 10^{-4} . When $U=4$, PAPR is about 9.2dB at CCDF= 10^{-4} . The PAPR is about 7.9dB when $U=16$ which shows about 4dB reduction compare to the original signal. When $U=2$, implementation of this system needs 2 IDFT blocks and when $U=16$, the system should have 16 IDFT blocks to be implemented. This is very important because the computational complexity is increased when the number of IDFT blocks is higher. So the implementation will be more difficult. Figure 6 shows the CCDF of the PAPR of the C-PTS method for $M=2$ and 4.

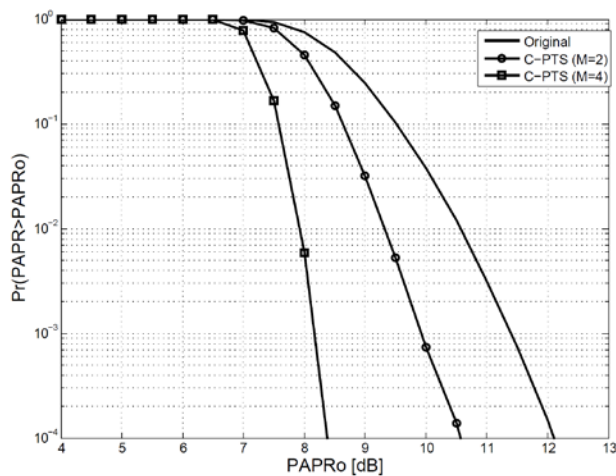


Figure 6. The CCDF of the PAPR of the C-PTS.

It is clear that when the number of subblocks are increased, the amount of reduction in PAPR is also increased. The result is shown for $M=2$ and 4. For higher values the complexity of C-PTS will be increased rapidly hence it is not applicable to use them. From Figure 6, there is about 8.4dB reduction in OFDM signal when PTS method is applied. The reduction when SLM was applied is about 7.8dB from Figure 5. From Figure 4, the reduction value for DSI method is about 9.8dB.

Comparing the CCDF of these three methods shows that SLM method has the better performance.

VII. CONCLUSIONS

The OFDM system was simulated and three of the existing PAPR reduction techniques, Dummy Sequence Insertion (DSI)

Partial Transmit Sequence (PTS) and Selected Mapping (SLM) methods have been applied into the system and the results have been studied. Also some studies about the complexity of implementation of these techniques are presented. The results show better PAPR reduction in SLM method and less complexity in implementation compared with PTS method.

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