

Effect of a Pre-filter on Timing Jitter

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Abstract—The jitter variance in a digital transmission system has three components: SS, SN, and NN. The timing circuit is assumed to consist of a square-law device, a pre-filter, a PLL, and a ZCD. When the pre-filter is assumed to be a second-order Butterworth BPF, the SS component of jitter variance, which is often referred to as self-noise, decreases by more than 10 dB with a roll-off factor of 0.3 for 16QAM and SS becomes proportional to the cube of the PLL bandwidth. The Q value of the pre-filter is determined based on the PLL bandwidth. In addition, the SN and NN are proportional to the PLL bandwidth. The mechanism of these phenomena is clarified in terms of the jitter spectral characteristics and the extractor (a pre-filter and a PLL) transfer function characteristics.

Keywords—Jitter variance, PLL bandwidth, PAM, ASK, QAM

I. INTRODUCTION

A square-law timing detection scheme is a well-known technique for linear modulation formats. It has been shown that self-noise could be eliminated in a noiseless baseband PAM system if a pre-filter was used at the input of a square-law device to shape the frequency spectrum of pulses with even symmetry centered at one-half the clock frequency [1]. The authors previously considered that this spectral shaping method might also be effective for a timing circuit in a noisy system and carrier transmission systems. Therefore, the timing jitter characteristics were analyzed for a timing circuit using a pre-filter and a square-law device followed by a narrow-band band-pass filter for PAM, BPSK, and QPSK schemes with an additive white Gaussian noise (AWGN) [2]. In [3], the conditions were imposed on a pre-filter before a square-law device in order to minimize self-noise and optimize the performance against AWGN. A new pre-filter transfer function was derived. [4] extended the spectral shaping method to the case of a digital synchronizer to improve the performance of Gardner's feedback symbol timing estimator algorithms [5]. [6] derives a closed form expression for the power of self-noise in a blind (nondata-aided) feedforward timing estimator which is a modified version of [7] and a pre-filter was proposed to achieve nearly jitter free timing estimator when the similar condition presented in [1] was fulfilled. On the other hand, a modified algorithm of a Gardner detector [5] for symbol timing recovery was proposed and simulation results revealed that self-noise reduction amounted to 10.7 dB for QPSK signals with high SNRs and a roll-off factor of 0.25 [8]. We consider that the design method of a pre-filter might be complex to realize physically.

The present paper shows the reduction effect of the SS

component of jitter variance, which is referred to as self-noise or pattern jitter [9], assuming a timing circuit in which a square-law device and a pre-filter followed by a PLL are arranged in tandem. Using this timing circuit, the design method of the pre-filter becomes simpler and self-noise reduction might be expected. Incidentally, a closed-form representation of the jitter power spectral density (jitter PSD) for PAM, ASK, and QAM was derived for an arbitrary SNR and an arbitrary band-limiting scheme with AWGN applying a linear model of a PLL [10]. Jitter PSD consists of SS, SN, and NN components. Each component of jitter variance can be calculated by integrating the corresponding component of the jitter PSD over a finite frequency domain. As a typical application of the closed-form representation, the jitter variance was calculated for a cosine roll-off band-limiting scheme [11][12]. Thus far, the pre-filter transfer function has been assumed to be unity for simplicity. In the present paper, the pre-filter is assumed to be a second-order Butterworth band-pass filter (BPF). Thus, we deal with the effect of the pre-filter on jitter variance.

The SS component of jitter variance can be reduced by 10 dB with a roll-off factor of 0.3 and a PLL normalized noise bandwidth of 0.01 as compared to the case in which the pre-filter transfer function is assumed to be unity. The SN and NN components of jitter variance are proportional to the PLL normalized noise bandwidth, b_L , whereas the SS component, i.e., the self-noise, is proportional to the cube of b_L . This result is rather general and is applicable to any transmission system and any roll-off factor considered in the present paper. The mechanism of these phenomena is clarified through a detailed examination of the relationships between the jitter source PSD, the extractor transfer function characteristics, and the jitter PSD.

II. ASSUMPTION

A. Receiver Configuration and Band-limiting Scheme

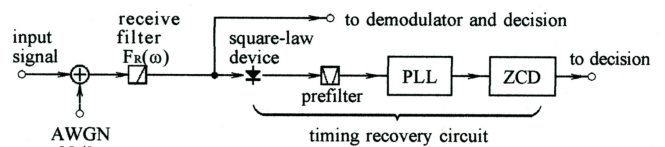


Figure 1. Receiver configuration.

Figure 1 shows the receiver configuration. The timing circuit consists of a square-law device, a pre-filter, a PLL and a zero crossing detector (ZCD) in tandem. AWGN with PSD of $N_0/2$ is applied at the receive filter input. The pre-filter is

assumed to be a second-order Butterworth BPF tuned to the repetition rate of the center frequency. The pre-filter bandwidth, denoted as $b/(2\pi)$, is assumed to be k times the PLL bandwidth. Consequently, Q is given as

$$Q = \omega_r / b = 1/(2k \cdot b_L). \quad (1)$$

Here, the value of k is assumed to be 5, and b_L is the PLL normalized noise bandwidth and can be defined as follows:

$$b_L = B_L T = 2\pi B_L / \omega_r, \quad (2)$$

where B_L is the one-sided PLL noise bandwidth, which is a measure of the PLL bandwidth, T is the symbol period, and ω_r is the clock radian frequency. The value of b_L is assumed to range from 10^{-4} to 10^{-2} . The PLL is assumed to be of second order, and the loop filter consists of an active filter. A typical value of the PLL damping factor, ζ , is $1/\sqrt{2}$. ZCD generates a timing pulse at the zero crossing time of a sinusoidal wave at the PLL output. Typical transmission systems include PAM, ASK, and QAM. The root roll-off band limitation is applied, and the roll-off factor, α , ranges from 0.1 to 1.0. The SNR, ρ , at the receive filter output ranges from -5 to 40 dB. The transmit data sequence, a_n , in PAM and ASK and the data sequences, a_n and b_n , in QAM take i.i.d. random variables: $-(M-1), -(M-3), \dots, -3, -1, 1, 3, \dots, M-3, M-1$, where M is an even number and is referred to as the alphabet size.

B. Extractor Transfer Function Characteristics

A pre-filter and a PLL are referred to collectively as an extractor.

1) Pre-filter: The pre-filter is assumed to be a second-order Butterworth BPF. The equivalent low-pass transfer function $B(\omega)$ of the BPF can be expressed as follows:

$$B(\omega) = \frac{1}{1 - (2Q \frac{\omega}{\omega_r})^2 + j\sqrt{2}(2Q \frac{\omega}{\omega_r})}, \quad (3)$$

where ω_r is the clock radian frequency, and Q is as given in (1). The pre-filter transfer function characteristics $|B(\omega)|^2$ can be obtained as follows:

$$|B(\omega)|^2 = \frac{1}{1 + (2Q \frac{\omega}{\omega_r})^4}. \quad (4)$$

2) Second-order PLL: The closed-loop transfer function of PLL can be expressed as follows:

$$H(\omega) = \frac{\omega_n^2 + j2\zeta \omega_n \omega}{\omega_n^2 - \omega^2 + j2\zeta \omega_n \omega}, \quad (5)$$

where ω_n is a characteristic frequency, and ζ is a damping

factor. $|H(\omega)|^2$ is obtained as follows:

$$|H(\omega)|^2 = \frac{1 + 4\zeta^2 \omega^2 \omega_n^{-2}}{(1 - \omega^2 \omega_n^{-2})^2 + 4\zeta^2 \omega^2 \omega_n^{-2}}. \quad (6)$$

A one-sided PLL noise bandwidth B_L can be obtained as follows:

$$B_L = \frac{1}{2\pi} \int_0^\infty |H(\omega)|^2 d\omega = \frac{\omega_n}{2} \left(\zeta + \frac{1}{4\zeta} \right). \quad (7)$$

The damping factor ζ is typically assumed to be $1/\sqrt{2}$. Thus, substituting (2) and (7) into (6) yields $|H(\omega)|^2$ as follows:

$$|H(\omega)|^2 = \frac{1 + (9/4)\pi^2 b_L^{-2} (\omega/\omega_r)^2}{1 + (9/8)\pi^4 b_L^{-4} (\omega/\omega_r)^4}. \quad (8)$$

3) Extractor Transfer Function: The extractor transfer function characteristics $H_k(\omega)$ in [10] can be given as follows:

$$H_k(\omega) = |B(\omega) \cdot H(\omega)|^2, \quad (9)$$

where the parameters C_T , θ_T , and D_2 as given in [10] become $C_T = |B(0)| = 1$, $\theta_T = \angle B(0) = 0$, and $D_2 = \exp(j2\theta_T) = 1$.

III. JITTER PSD and JITTER VARIANCE

A. Jitter PSD

Since the band-limiting scheme is assumed to be root roll-off here, jitter PSDs $K_{SS}(\omega)$, $K_{SN}(\omega)$, and $K_{NN}(\omega)$ at the PLL output are expressed as follows:

$$K_{SS}(\omega) = |B(\omega) \cdot H(\omega)|^2 G_{SS}(\omega), \quad (10)$$

$$K_{SN}(\omega) = |B(\omega) \cdot H(\omega)|^2 G_{SN}(\omega), \quad (11)$$

$$K_{NN}(\omega) = |B(\omega) \cdot H(\omega)|^2 G_{NN}(\omega), \quad (12)$$

$$G_{SS}(\omega) = \alpha^2 T^{-1} \{ C_{SS}^{(1)} G_1(\omega) + C_{SS}^{(2)} G_2(\omega) \}, \quad (13)$$

$$G_{SN}(\omega) = \alpha^2 T^{-1} C_{SN} \rho^{-1} G_3(\omega), \quad (14)$$

$$G_{NN}(\omega) = \alpha^2 T^{-1} C_{NN} \rho^{-2} G_4(\omega), \quad (15)$$

where $G_{SS}(\omega)$, $G_{SN}(\omega)$, and $G_{NN}(\omega)$ are the SS, SN, and NN components of jitter source PSD, respectively, at the output of square-law device. $G_1(\omega)$, $G_2(\omega)$, $G_3(\omega)$, and $G_4(\omega)$ are expressed by the integral representations as described in the Appendix A. ρ is the SNR at the receive filter output. $C_{SS}^{(1)}$, $C_{SS}^{(2)}$, C_{SN} , and C_{NN} are the coefficients as determined by the communication scheme. In the case of 16QAM, these coefficients are represented as follows:

$$C_{SS}^{(1)} = 1/2, C_{SS}^{(2)} = 0.34, C_{SN} = 1/2, C_{NN} = 1/2. \quad (16)$$

B. Jitter Variance of SS, SN, and NN Components

Three components of jitter variance at the PLL output can be obtained by integrating the corresponding component of jitter PSD. The total jitter variance (σ_ϕ^2) is expressed as the sum of the three components, SS ($\sigma_{\phi SS}^2$), SN ($\sigma_{\phi SN}^2$), and NN ($\sigma_{\phi NN}^2$), which are obtained as follows:

$$\sigma_\phi^2 = \sigma_{\phi SS}^2 + \sigma_{\phi SN}^2 + \sigma_{\phi NN}^2, \quad (17)$$

$$\sigma_{\phi SS}^2 = \frac{1}{\alpha^2 \omega_r} \int_{-\infty}^{\infty} K_{SS}(\omega) d\omega, \quad (18)$$

$$\sigma_{\phi SN}^2 = \frac{1}{\alpha^2 \omega_r} \int_{-\infty}^{\infty} K_{SN}(\omega) d\omega, \quad (19)$$

$$\sigma_{\phi NN}^2 = \frac{1}{\alpha^2 \omega_r} \int_{-\infty}^{\infty} K_{NN}(\omega) d\omega. \quad (20)$$

Jitter variance at the ZCD output can be given by the same expressions from (17) to (20) as described in [10].

IV. CALCULATED JITTER VARIANCE

The dependency of the SS, SN, and NN components of jitter variance on the communication scheme can be determined by the coefficients $C_{SS}^{(1)}$, $C_{SS}^{(2)}$, C_{SN} , and C_{NN} [10]. Since the values of jitter variance in PAM and ASK can be obtained from those in QAM [11], the calculated examples in QAM are presented in this section.

A. Dependency on SNR ρ

Figure 2 shows a numerical example of jitter variance and its three components as a function of the SNR, ρ . The SS component is constant in terms of SNR and increases with increasing M . When M is greater than 4, the SS component increases only slightly and, for practical purposes, can be regarded as a constant value. The SS component characteristics for the case where the pre-filter transfer function is assumed to be unity are denoted as B and those for the case where the pre-filter is assumed to be a second-order Butterworth BPF are denoted as A. Thus, the pre-filter of the BPF reduces the SS component of jitter variance denoted as B to that denoted as A by 10 dB. The SN and NN components are not related to the alphabet size, M , and the pre-filter transfer function. Therefore, the total jitter variance exhibited by the dotted line can be reduced by 10 dB at SNRs of greater than 30 dB, because the SS component is dominant.

These dependencies of jitter variance on SNR hold for any communication scheme, any alphabet size, and any roll-off factor considered in the present paper.

B. Dependency on PLL Normalized Noise Bandwidth

Figure 3 shows a numerical example for three components of jitter variance as a function of PLL normalized noise bandwidth b_L . The characteristics denoted as A indicate that the SS component of jitter variance is proportional to the cube of b_L . The characteristics denoted as B indicate that the SS

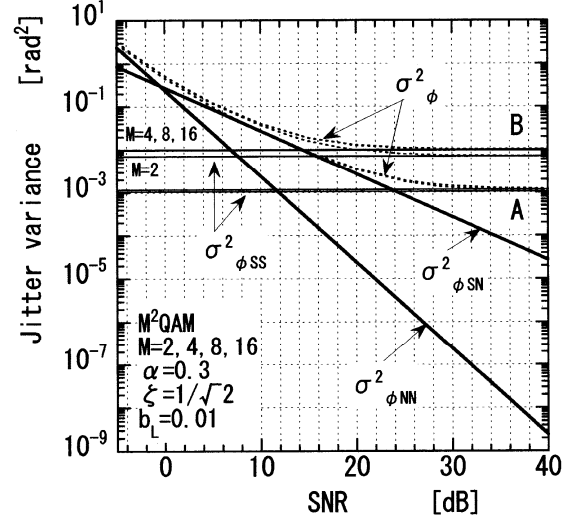


Figure 2. Jitter variance and the three components of jitter variance as functions of the SNR.

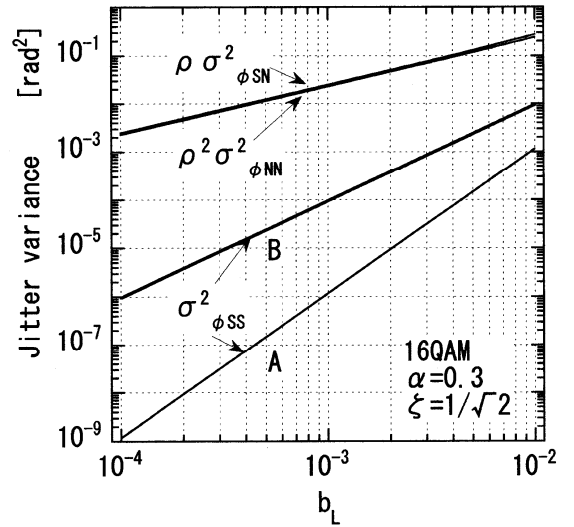


Figure 3. SS, SN, and NN components of jitter variance as functions of b_L .

component of jitter variance is proportional to the square of b_L . The pre-filter of the BPF reduces the SS component by 10 dB at b_L of 10^{-2} , by 20 dB at b_L of 10^{-3} , and by 30 dB at b_L of 10^{-4} . The SN and NN components are proportional to b_L and are not related to the pre-filter transfer function.

The tradeoff exists between a PLL bandwidth and its tracking performance of symbol timing. This subject will be discussed in future.

V. CALCULATED SPECTRAL CHARACTERISTICS and DISCUSSION

The SS, SN, and NN components of jitter variance can be obtained by integrating the corresponding component of jitter PSD over a finite frequency domain. The jitter PSD can be given as the product of the jitter source PSD and the extractor transfer function characteristics.

In this section, the mechanism of the reduction and the proportional characteristics for three components of jitter variance are discussed in terms of the jitter spectral characteristics and extractor transfer function characteristics.

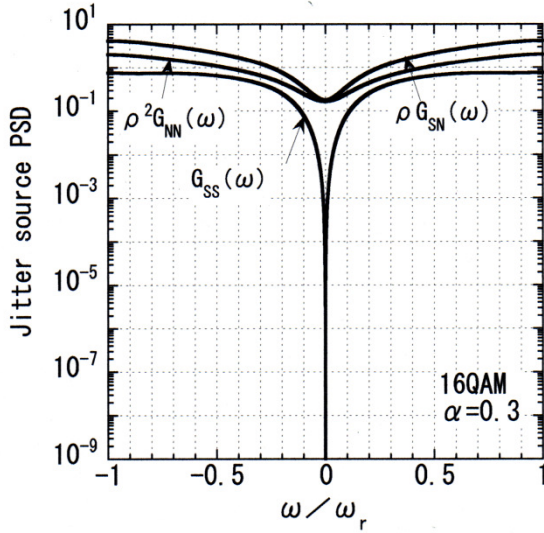


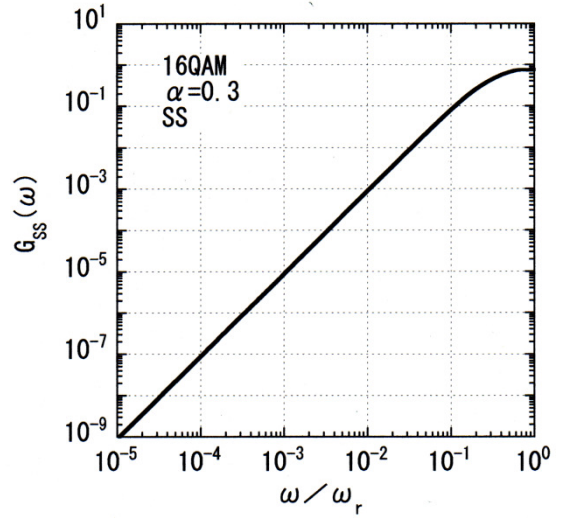
Figure 4. SS, SN, and NN components of the jitter source PSD.

A. Jitter Source Spectral Characteristics

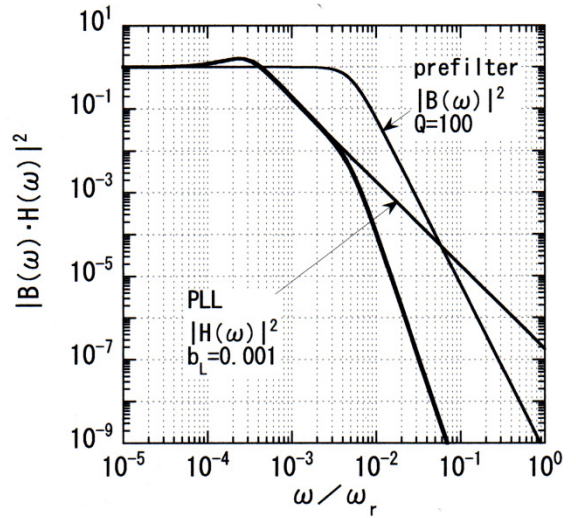
Figure 4 shows a numerical example of the SS, SN, and NN components of the jitter source PSD at the output of the square-law device. The SS component exhibits a sharp dip in the vicinity of zero frequency. Although the SN and NN components are depressed slightly around zero frequency, they exhibit comparatively flat characteristics as compared to the SS component. These characteristics are symmetric with respect to zero frequency.

B. SS Component

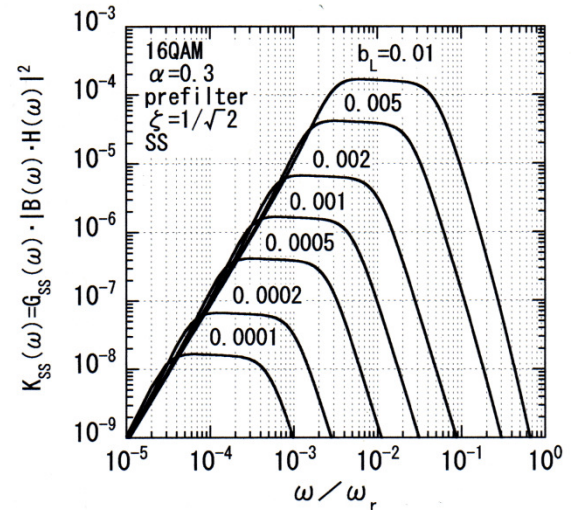
Figures 5(a), 5(b), and 5(c) show calculated examples of the SS component of the jitter source PSD, the extractor transfer function characteristics, and the jitter PSD, respectively. The SS component of the jitter source PSD is proportional to the square of the frequency below $\omega/\omega_r = 10^{-1}$, as shown in Figure 5(a). In Figure 5(b), $|B(\omega)|^2$ represents the pre-filter transfer function characteristics with a Q value of 100, and $|H(\omega)|^2$ represents the second-order PLL transfer function characteristics with a b_L value of 0.001. The thick solid line



(a) SS component of the jitter source PSD.



(b) Extractor transfer function characteristics.



(c) SS component of the jitter PSD.

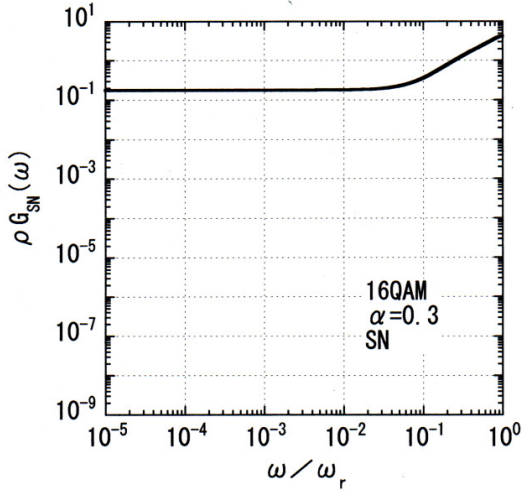
Figure 5. Spectral characteristics of the SS component and the extractor.

shows the extractor transfer function characteristics, i.e., $|B(\omega) \cdot H(\omega)|^2$, in which roll-off characteristics lower than 10^{-2} attenuate sharply due to the pre-filter.

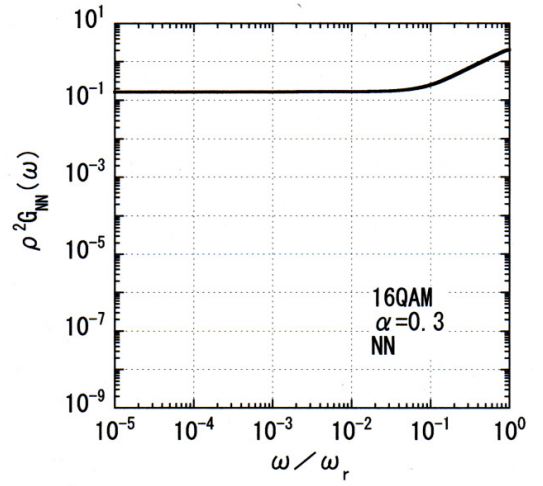
Figure 5(c) shows a calculated example of the SS component of the jitter PSD, $K_{SS}(\omega)$. The jitter PSD is given as the product of the jitter source PSD and the extractor transfer function characteristics, i.e., $G_{SS}(\omega) \cdot |B(\omega) \cdot H(\omega)|^2$, in which the upper band roll-off characteristics exhibit a sharp attenuation due to the pre-filter. The magnitude of jitter PSD distribution for each b_L increases or decreases in proportion to the square of b_L . The bandwidth of jitter PSD distribution for each b_L becomes narrower or broader in proportion to b_L . The jitter variance is given as the frequency integration of the jitter PSD over a finite frequency range. Thus, the SS component of the jitter variance decreases and is proportional to the cube of b_L as compared to the case in which the pre-filter transfer function is assumed to be unity.

C. SN Component

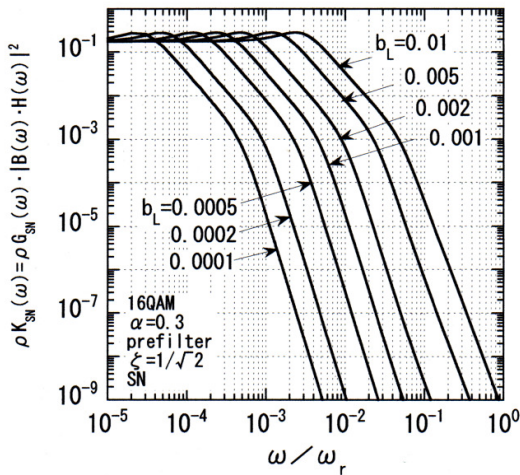
Figures 6(a) and 6(b) show calculated examples of the SN component of the jitter source PSD and the jitter PSD, respectively. Figure 6(a) shows flat characteristics when ω/ω_r is lower than 10^{-1} . The extractor transfer function characteristics for $b_L = 0.001$ are shown in Figure 5(b). Figure 6(b) shows the SN component of the jitter PSD for each b_L . The jitter PSD is given as the product of the jitter source PSD and the extractor transfer function characteristics. The bandwidth of the jitter PSD distribution for each b_L becomes narrower or broader in proportion to b_L . The jitter variance is given as the frequency integration of jitter PSD. Thus, the SN component of the jitter variance is proportional to b_L . $\rho K_{SN}(\omega)$ attenuates sharply due to the pre-filter when $\rho K_{SN}(\omega)$ is lower than 10^{-3} . This may be considered to be small. Thus, from a practical point of view, the pre-filter does not affect the SN component of the jitter variance.



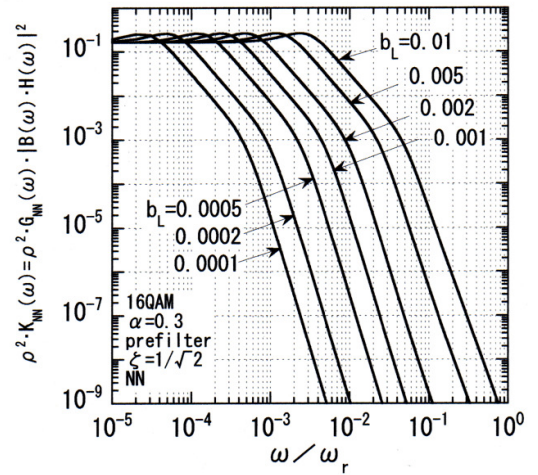
(a) SN component of the jitter source PSD.



(a) NN component of the jitter source PSD.



(b) SN component of the jitter PSD for various values of b_L .



(b) NN component of the jitter PSD for various values of b_L .

Figure 6. Spectral characteristics of the SN component.

Figure 7. Spectral characteristics of the NN component.

D. NN Component

Figures 7(a) and 7(b) shows calculated examples of the NN component of the jitter source PSD and the jitter PSD, respectively. The extractor transfer function characteristics have been shown in Figure 5(b). In this case, the bandwidth of the jitter PSD for each b_L is narrower or broader in proportion to b_L . Thus, the NN component of the jitter variance is proportional to b_L . The pre-filter does not affect the NN component of the jitter variance for the same reason as for the SN component.

The dependencies of the three components of jitter variance on b_L hold for any communication scheme, any alphabet size, and any roll-off factor considered in the present paper.

VI. CONCLUSION

The present paper has demonstrated that the SS component of jitter variance, which is often referred to as self-noise, can be reduced by more than 10 dB with a pre-filter that is assumed to be a second-order Butterworth BPF. Moreover, the SS component has been shown to be proportional to the cube of the PLL normalized noise bandwidth, b_L . The SN and NN components are proportional to b_L and are not related to the pre-filter transfer function. The mechanism of these phenomena is clarified in terms of the jitter spectral characteristics and the extractor (a pre-filter and a PLL) transfer function characteristics. The total jitter variance is reduced by 10 dB with a b_L of 0.01 at a high SNR of greater than 30 dB with a second-order Butterworth BPF as a pre-filter.

We intend to discuss the effect of jitter variance on the symbol error rate in future. In addition, the compromise exists between a PLL normalized noise bandwidth b_L and a PLL tracking performance of symbol timing. This subject will be discussed.

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REFERENCES

- [1] L. E. Franks and J. P. Bubrouski, "Statistical properties of timing jitter in a PAM timing recovery scheme," *IEEE Trans. Commun.*, vol. COM-22, no. 7, pp. 913-920, July 1974.
- [2] I. Wakabayashi and K. Miyauchi, "Jitter characteristics of timing recovery scheme using a prefilter," *IEICE Trans. Commun. (Japanese ed.)*, vol. J86-B, no. 8, pp. 1589-1597, Aug. 2003.
- [3] A. Spalvieri and M. Magarini, "Minimum timing jitter in timing recovery based on pre-filtered square-law timing detection," in *2nd International Conference on Communication Theory, Reliability, and Quality of Services*, pp. 46-51, 2009.
- [4] N. A. D'Andrea and M. Luise, "Design and analysis of a jitter-free clock recovery scheme for QAM systems," *IEEE Trans. on Commun.*, vol.41, no.9, pp.1296-1299, Sept. 1993.
- [5] F. M. Gardner, "A BPSK/QPSK timing-error detector for sampled receivers," *IEEE Trans. on Commun.*, vol.COM-34, no.5, pp.423-429, May 1986.
- [6] K. Shi, Y. Wang, and E. Serpendin, "On the design of a digital blind

feedforward, nearly jitter-free timing-recovery scheme for linear modulation," *IEEE Trans. on Commun.*, vol.52, no.9, pp.1464-1469, Sept. 2004.

- [7] S. J. Lee, "A new non-data-aided feedforward symbol timing estimator using two samples per symbol," *IEEE Commun. Lett.*, vol.6, no.5, pp.205-207, May 2002.
- [8] D. Lim, "A modified Gardner detector for symbol timing recovery of M-PSK signals," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1643-1647, Oct. 2004.
- [9] F. M. Gardner, "Self-noise in synchronizers," *IEEE Trans. Commun.*, vol. COM-28, no. 8, pp. 1159-1163, Aug. 1980.
- [10] K. Miyauchi, I. Wakabayashi, "Analysis of timing jitter in PAM, ASK and QAM," *IEICE Trans. Commun. (Japanese ed.)*, vol. J93-B, no. 8, pp. 1051-1060, Aug. 2010.
- [11] I. Wakabayashi, "Timing jitter in PAM, ASK, and QAM for a bandlimiting scheme," in *12th Proceedings of ICAC2010*, pp. 318-323, Feb. 2010.
- [12] I. Wakabayashi, S. Urano, and M. Sano, "Dependency of jitter variance on PLL normalized noise bandwidth," in *13th Proceedings of ICAC2011*, pp. 1-6, Feb. 2011.

APPENDIX

A. Jitter Source PSD

$G_1(\omega)$, $G_2(\omega)$, $G_3(\omega)$, and $G_4(\omega)$ are the jitter source PSD at the output of a square-law device. $G(\omega)$ is the frequency spectrum of the signal at the receive filter output and is assumed to be a cosine roll-off characteristic here.

$$G_1(\omega) = \frac{1}{4\pi A_T^2} \left[\int_{-\infty}^{\infty} G^2(x) \left(G^2(x - \omega_r) + G^2(x - \omega - \omega_r) \right) dx \right. \\ \left. + \int_{-\infty}^{\infty} G(x) G(x - \omega) \left\{ G(x + \omega_r) G(x - \omega - \omega_r) \right. \right. \\ \left. \left. + G(x - \omega_r) G(x - \omega + \omega_r) \right\} dx \right. \\ \left. - \int_{-\infty}^{\infty} G(x) G(x - \omega) \left\{ G(x + \omega_r) G(x - \omega + \omega_r) \right. \right. \\ \left. \left. + G(x - \omega_r) G(x - \omega - \omega_r) \right\} dx \right. \\ \left. + \int_{-\infty}^{\infty} G(x) G(x + \omega_r) G(x - \omega + \omega_r) G(x - \omega + 2\omega_r) dx \right. \\ \left. + \int_{-\infty}^{\infty} G(x) G(x - \omega_r) G(x - \omega - \omega_r) G(x - \omega - 2\omega_r) dx \right], \quad (\text{A-1})$$

$$G_2(\omega) = -\frac{T}{2A_T^2} \left\{ G_a(\omega - \omega_r) - G_a(\omega + \omega_r) \right\}^2, \quad (\text{A-2})$$

$$G_3(\omega) = \frac{\beta}{2\pi A_T^2} \int_{-\infty}^{\infty} G^2(x) \left\{ G(x - \omega + \omega_r) + G(x - \omega - \omega_r) \right\} dx, \quad (\text{A-3})$$

$$G_4(\omega) = \frac{\beta^2}{2A_T^2} \left\{ G_a(\omega - \omega_r) + G_a(\omega + \omega_r) \right\}, \quad (\text{A-4})$$

where A_T , β , and $G_a(\omega)$ are expressed as follows:

$$A_T = |G_a(\omega_r)| = \alpha / (8T), \quad (\text{A-5})$$

$$\beta = 1 - \alpha / 4, \quad (\text{A-6})$$

$$G_a(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(x) G(\omega - x) dx. \quad (\text{A-7})$$