

Convex Analysis Based Beamforming of Decode-and-Forward Cooperation for Improving Wireless Physical Layer Security

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Abstract—Physical-layer-based security aims at ensuring the reliability of communication and preventing eavesdropping by taking advantage of the physical layer's characteristics rather than the data encryption in upper layer. Cooperation is a way to achieve this goal with many benefits for wireless communication. In particular, the cooperation scheme called decode-and-forward (DF) is discussed in this paper and our objective is to design each cooperating node's beamforming factor for maximum achievable secrecy rate. Considering that individual power constraint is more reasonable than total power constraint and to set noise power levels at the destination and the eavesdropper different is more practical than the same, we get the whole optimization problem which is unconvex. With the help of perfect global channel state information (CSI), the problem is solved through a way where convex optimization and one-dimensional search are combined together. And strict proofs are presented for this method. Numerical results of the obtained maximum achievable secrecy rate show that the proposed design can significantly improve the security performance of wireless systems compared to directly transmission.

Keywords—physical layer security, maximum achievable secrecy rate, cooperating relays, beamforming, convex analysis.

I. INTRODUCTION

Secure data transmission plays an important role in wireless communication system. However, the open nature of wireless communication makes it vulnerable to wiretapping. At physical layer, this problem was first studied by Wyner [1] from an information-theoretic perspective. He gave out the definition of secrecy rate and secrecy capacity and showed that if the source-eavesdropper channel is a degraded version of the source-destination channel, secure communication is possible without relying on secret keys. Then, Wyner's result was generalized to broadcast channel [2], Gaussian channel [3] and fading channel [4], [5].

To overcome the problem that the traditional single antenna system based PHY layer security approaches are infeasible when Wyner's condition is not met [1], [3], some recent works have been proposed by using multi-antenna system [6],[7],[8]. Additionally, cooperation via relaying where the source to destination transmission is helped by relays is more practical compared to multi-antenna system. Totally, there are three cooperative schemes i.e. decode-and-forward (DF), amplify-and-forward (AF) and cooperative jamming (CJ). And in particular, the security performance of DF based

cooperation system is studied in recent years [9], [10], [11]. In [9] and [10], a DF based cooperative protocol was considered and beamforming vector of relays was designed for the achievable secrecy rate maximization or transmit power minimization. However these works were conducted with a total power constraint which was their limitation. Different from the antennas of MISO system, the relays in cooperative system are distributed and independent. So the individual relay power constraints are more reasonable than the total power constraint. Junwei Zhang [11] considered the maximization of the secrecy rate of DF model with individual relay power constraints, but some necessary proofs were lacked.

In this paper, we set different noise power at different nodes to make the system model more practical than that in [11]. And a new algorithm is proposed by combining the convex optimization and the one-dimensional search together to obtain the maximum achievable secrecy rate with sufficient proofs.

This paper is organized as follows. In Section II, we will introduce the system model and the DF-based cooperative protocol. In Section III, we will propose and prove our algorithm for the maximum secrecy rate and the corresponding beamforming vector. Simulations are described in Section IV, and conclusions are given in Section V.

II. SYSTEM MODEL AND COOPERATIVE PROTOCOL

In this paper, we consider a scenario in which there is only one source node S , one eavesdropper node E , one destination node D and N relay nodes labelled as $\{R_0, \dots, R_{N-1}\}$. As Figure 1 illustrates, the source and relays are in the same cluster, while the destination and eavesdropper are located far away from this cluster. Each network node is equipped with only a single omni-directional antenna. All channels are flat fading, quasistatic and memoryless. And the global CSI is available for system design. And thermal noise at all nodes is zero-mean white complex Gaussian. Besides, the number of relays willing to help the source is assumed known before optimization.

The system works under a DF-based cooperative protocol. The protocol is divided into two stages and can be described as follows. In Stage I, the source transmits a symbol x_s to other nodes within the cluster, and then the relays receive and

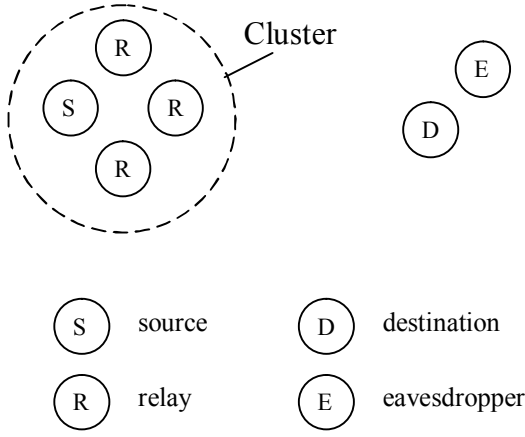


Figure 1. System model

decode it. The power of this symbol is P , i.e. $E(|x_s|^2) = P$ where $E\{\cdot\}$ denotes expectation. The received signal at the relay R_i can be expressed as

$$y_{R,i} = x_s l_i + n_{R,i} \quad (1)$$

where $l_i \in \mathbb{C}$ denotes the channel between R_i and S and $n_{R,i}$ is the noise at R_i with variance $\sigma_{R,i}^2$.

As the distance between the source and the relays are not too long, the relays can decode the received signal properly, and the power of the signal broadcasted by the source would be small so that the faraway destination and eavesdropper can receive none of it.

In Stage II, relay nodes collaboratively transmit a weighted version of the normalized symbol of x_s to the destination and eavesdropper. The signal y_D which is received at D equals

$$y_D = \sum_{i=0}^{N-1} w_i h_i \frac{x_s}{\sqrt{P}} + n_D \quad (2)$$

where w_i ($i = 0, 1, \dots, N-1$) means the beamforming factor at R_i , $h_i \in \mathbb{C}$ is the channel between R_i and D , and n_D is the noise at D with variance σ_D^2 . Then the signal y_E which is the signal at E can be expressed as,

$$y_E = \sum_{i=0}^{N-1} w_i g_i \frac{x_s}{\sqrt{P}} + n_E, \quad (3)$$

where $g_i \in \mathbb{C}$ denotes the channel between R_i and E , and n_E is the noise at E with variance σ_E^2 .

Let's define $\mathbf{w} = [w_0, \dots, w_{N-1}]^T$, $\mathbf{h} = [h_0, \dots, h_{N-1}]^H$, $\mathbf{g} = [g_0, \dots, g_{N-1}]^H$ and $R_h = \mathbf{h}\mathbf{h}^H$, $R_g = \mathbf{g}\mathbf{g}^H$ where superscripts $(\cdot)^T$ and $(\cdot)^H$ represent transpose and conjugate transpose respectively. Then the SNR at D and E can be expressed as $\Gamma_D = |\mathbf{h}^H \mathbf{w}|^2 / \sigma_D^2$ and $\Gamma_E = |\mathbf{g}^H \mathbf{w}|^2 / \sigma_E^2$

respectively. As discussed in [3], for a given $\mathbf{w}$ the secrecy capacity $C_s(\mathbf{w})$ is

$$\begin{aligned} C_s(\mathbf{w}) &= \max\left\{\frac{1}{2}(\log(1 + \Gamma_D) - \log(1 + \Gamma_E)), 0\right\} \\ &= \max\left\{\frac{1}{2}\log\left(\frac{1 + \Gamma_D}{1 + \Gamma_E}\right), 0\right\} \end{aligned} \quad (4)$$

III. DESIGN FOR SECRECY CAPACITY MAXIMIZATION

Aiming at finding out the maximum achievable secrecy rate of this system which works under the protocol we described, it is obvious that we should try to maximize $C_s(\mathbf{w})$ via the design of the beamforming vector as equivalent. Considering individual power constraints is more practical in the relay system studied in this paper, the problem what we are interested in is formulated as follows,

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize:}} \quad & C_s(\mathbf{w}) \\ \text{subject to:} \quad & |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (5)$$

where p_i is the power constraint for R_i , $\forall i = 0, \dots, N-1$.

As the property of function $\max\{\bullet, 0\}$ and $\log(\bullet)$, in order to solve (5), we could solve the following problem first,

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize:}} \quad & \frac{1 + \frac{\mathbf{w}^H R_h \mathbf{w}}{\sigma_D^2}}{1 + \frac{\mathbf{w}^H R_g \mathbf{w}}{\sigma_E^2}} \\ \text{subject to:} \quad & |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (6)$$

which can be re-expressed as

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize:}} \quad & \frac{\sigma_E^2(\sigma_D^2 + \mathbf{w}^H R_h \mathbf{w})}{\sigma_D^2(\sigma_E^2 + \mathbf{w}^H R_g \mathbf{w})} \\ \text{subject to:} \quad & |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (7)$$

Because σ_E^2 / σ_D^2 is a constant, (7) can be simplified into

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize:}} \quad & \frac{\sigma_D^2 + \mathbf{w}^H R_h \mathbf{w}}{\sigma_E^2 + \mathbf{w}^H R_g \mathbf{w}} \\ \text{subject to:} \quad & |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (8)$$

However, solving (8) is challenging owing to its non-convex character. Motivated by [12] and [13], our method is to first study a subproblem with the denominator of its objective function fixed, and then use one dimension search to find the solution. Moreover, the strict proof of this method is presented.

A. Subproblem With Fixed $\sigma_E^2 + \mathbf{w}^H R_g \mathbf{w}$

Fixing $\sigma_E^2 + \mathbf{w}^H R_g \mathbf{w}$ in (8) to a scalar t , then our problem transforms into

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize:}} \quad & \mathbf{w}^H R_h \mathbf{w} \\ \text{subject to:} \quad & \sigma_E^2 + \mathbf{w}^H R_g \mathbf{w} = t \\ & |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (9)$$

We denote the optimal objective value of (9) as $f(t)$, optimal solution as $\mathbf{w}^*(t)$. Define $R(t) = (f(t) + \sigma_D^2) / t$. Then we just need to concentrate on finding out t^* which is the maximum point of $R(t)$, because $R(t^*)$ equals to the optimal value of (8) and $\mathbf{w}^*(t^*)$ is also the optimal point of it. However, (9) is also difficult to tackle because of the existence of equality constraint. In order to overcome this, we changes (9) into the following optimization problem,

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize:}} \quad & \mathbf{w}^H R_h \mathbf{w} \\ \text{subject to:} \quad & \sigma_E^2 + \mathbf{w}^H R_g \mathbf{w} \leq t \\ & |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (10)$$

Let's define the optimal value of (10) as $f_1(t)$ and the corresponding optimal point as $\mathbf{w}_1^*(t)$. Let $R_1(t) = \varphi(t) / t$ where $\varphi(t) = f_1(t) + \sigma_D^2$ and denote $R_1(t)$'s maximum point as t_1 . Then we will have the conclusion stated in theorem 1 as follows.

Theorem 1: $\mathbf{w}_1^*(t_1)$ is optimal point of (8), and $R_1(t_1)$ is its optimal value.

Proof: When $t = t^*$, $\mathbf{w}^*(t^*)$ is the optimal point of (9) and also is the feasible point of (10). So $f_1(t^*) \geq f(t^*)$. Then we have the relation below,

$$\max_t R_1(t) = R_1(t_1) \geq R_1(t^*) \geq R(t^*) = \max_t R(t). \quad (11)$$

When $t = t_1$, assume that $[\mathbf{w}_1^*(t_1)]^H R_g \mathbf{w}_1^*(t_1) + \sigma_E^2 = t_2 < t_1$. Then we have $f_1(t_1) = f_1(t_2)$. Because $t_2 < t_1$, so $R_1(t_2) > R_1(t_1)$, which contradicts with the fact that t_1 is the maximum point of $R_1(t)$. So we have

$$[\mathbf{w}_1^*(t_1)]^H R_g \mathbf{w}_1^*(t_1) + \sigma_E^2 = t_1. \quad (12)$$

Then in order to obtain $f_1(t_1)$ and $\mathbf{w}_1^*(t_1)$ we could focus on the following problem,

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize:}} \quad & \mathbf{w}^H R_h \mathbf{w} \\ \text{subject to:} \quad & \sigma_E^2 + \mathbf{w}^H R_g \mathbf{w} = t_1 \\ & |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (13)$$

Comparing (13) with (9), it is obvious that $f_1(t_1) = f(t_1)$ so we can get

$$\max_t R_1(t) = R_1(t_1) = R(t_1) \leq \max_t R(t) = R(t^*). \quad (14)$$

Combine (11) and (14) together, we have

$$\max_t R_1(t) = \max_t R(t). \quad (15)$$

According to (13) and (15), t_1 is also $R(t)$'s maximum point, and $\mathbf{w}_1^*(t_1)$ is also (8)'s optimal point. ■

In the light of Theorem 1, we can find out the maximum point of $R_1(t)$ through solving (10) instead of trying to calculate the complicate problem (8) directly. However (10) is also non-convex. Fortunately, we know a convex problem which can be easily solved has a character that its optimal point is also the optimal point of (10) as follows,

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize:}} \quad & \text{Re}(\mathbf{w}^H \mathbf{h}) \\ \text{subject to:} \quad & \sigma_E^2 + \mathbf{w}^H R_g \mathbf{w} \leq t \\ & |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (16)$$

where $\text{Re}(\mathbf{w}^H \mathbf{h})$ is the real part of $\mathbf{w}^H \mathbf{h}$.

B. Search for the Optimal Solution

In order to obtain $R_1(t)$'s maximum point through which we could find out $\mathbf{w}_1^*(t_1)$, let's state some properties of $f_1(t)$ and $R_1(t)$.

Theorem 2: $\varphi(t)$ is a concave function of t .

Proof: Since $f_1(t)$ can be written as

$$\begin{aligned} \underset{\lambda, \mu}{\text{minimize:}} \quad & \mu t + \sum_{i=0}^{N-1} \lambda_i p_i - \mu \sigma_E^2 \\ \text{subject to:} \quad & \lambda \geq 0 \\ & \mu > 0 \\ & \mu \mathbf{g} \mathbf{g}^H + \text{Diag}(\lambda) - \mathbf{h} \mathbf{h}^H \succeq 0 \end{aligned} \quad (17)$$

where $\text{Diag}\{\mathbf{a}\}$ is a diagonal matrix with the elements of vector $\mathbf{a}$ along its diagonal [13], $\varphi(t)$ can be expressed as

$$\begin{aligned} \underset{\lambda, \mu}{\text{minimize:}} \quad & \mu t + \sum_{i=0}^{N-1} \lambda_i p_i - \mu \sigma_E^2 + \sigma_D^2 \\ \text{subject to:} \quad & \lambda \geq 0 \\ & \mu > 0 \\ & \mu \mathbf{g} \mathbf{g}^H + \text{Diag}(\lambda) - \mathbf{h} \mathbf{h}^H \succeq 0 \end{aligned} \quad (18)$$

It is obvious that $\varphi(t)$ is a point-wise minimum of a family of affine functions. So $\varphi(t)$ is concave [14, p.80]. ■

Theorem 3: $R_1(t)$ is a quasiconcave function of t .

Proof: Suppose $p(x)$ is a concave function and $q(x)$ is a convex function, with $p(x) > 0$ and $q(x) > 0$ on a convex set C . We can easily get $f(x) = p(x) / q(x)$ is quasiconcave on C according to the theorem in [14, p.103]. Then at the base of Theorem 2, it can be concluded that $R_1(t)$ is quasiconcave for $\varphi(t) > 0$ is concave, $t > 0$ is affine (so convex). ■

Theorem 4: There's at most a single interval in $\text{Dom}(R_1(t))$ where $R_1(t)$ is invariant and any t belongs to this interval will be the maximum point of $R_1(t)$.

Proof: First, let's consider the fact that $R_1(t) = C$ in an interval if and only if $\varphi(t) = Ct$. Then we just need to prove that there's only a single interval in $Dom(R_1(t))$ where $\varphi(t)$ is proportional to t .

Part I: Assume $\varphi(t) = Ct$ on two separate interval $[a, b]$ and $[c, d]$ where $a < b < c < d$. Then we can get $\varphi(t)$ is no bigger than Ct on $[b, c]$ from (18). As $\varphi(t)$ is concave, $\varphi(\theta b + (1 - \theta)c) \geq \theta\varphi(b) + (1 - \theta)\varphi(c)$ $\theta \in [0, 1]$ [14 p. 67] which means $\varphi(t)$ is no smaller than Ct on $[b, c]$. Consequently, we have $\varphi(t) = Ct$ on $[b, c]$. Therefore, there is only a single interval where $\varphi(t) = Ct$.

Part II: Assume $\varphi(t) = Ct$ on $[a, b]$ and $\varphi(t) = C_1 t$ on $[c, d]$ with $a < b < c < d$ and $C \neq C_1$. Because of (18) we have $\varphi(t) = Ct < C_1 t$ on $[a, b]$. As $t > 0$, $C < C_1$. Similarly, $\varphi(t) = C_1 t < Ct$ on $[c, d]$. As $t > 0$, $C > C_1$. Then contradiction appears. As a result, there is at most a single straight line through the original which partly overlaps with $\varphi(t)$.

Combining the two parts above, we could easily get that there's at most a single interval in $Dom(R_1(t))$ where $R_1(t)$ is constant.

Suppose $\varphi(t) = Ct$ if and only if $t \in [a, b]$, $a < b$. Then from (18) we know $\varphi(t_2) < Ct_2$ for $t_2 > b$. So

$$R_1(t_2) = \frac{\varphi(t_2)}{t_2} < \frac{Ct_2}{t_2}. \quad (19)$$

Through the similar way, for $t_1 < a$, there is

$$R_1(t_1) < \frac{Ct_2}{t_2}. \quad (20)$$

Through (19) and (20), we can conclude $R_1(t)$ achieves its maximum for $\forall t \in [a, b]$ as $R_1(t) = C$ on $[a, b]$. ■

Considering Theorem 2-4, we will find that the optimal point and maximum value of $R_1(t)$ can be efficiently got using Golden Section method which is one of the classic one dimensional search algorithms. Before using this algorithm, we should find an interval including the optimal point of $R_1(t)$.

Denote $[t_{\min}, t_{\max}]$ as this interval. Definitely, $t_{\min}$ would be σ_E^2 , and $t_{\max}$ would be the optimal value of the following problem,

$$\begin{aligned} & \underset{\mathbf{w}}{\text{maximize:}} \quad \sigma_E^2 + \mathbf{w}^H R_g \mathbf{w} \\ & \text{subject to:} \quad |w_i|^2 \leq p_i, \forall i = 0, \dots, N-1 \end{aligned} \quad (21)$$

The complete algorithm is summarized as follows.

Proposed Algorithm

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1: Input:  $\sigma_D^2, \sigma_E^2, p_i, \mathbf{g}, \mathbf{h}$ .
2: begin
3:   initialize  $t_{\min}, t_{\max}, len = t_{\max} - t_{\min}$ .
4:   while  $len > \varepsilon$ , where  $\varepsilon$  is the threshold.
5:      $t_{\text{left}} = t_{\max} - 0.618(t_{\max} - t_{\min})$ .
6:      $t_{\text{right}} = t_{\min} + 0.618(t_{\max} - t_{\min})$ .
7:     calculate  $R_1(t_{\text{left}}), R_1(t_{\text{right}})$ .
8:     if  $R_1(t_{\text{left}}) < R_1(t_{\text{right}})$ .
9:        $t_{\min} = t_{\text{left}}$ .
10:    else if  $R_1(t_{\text{left}}) > R_1(t_{\text{right}})$ 
11:       $t_{\max} = t_{\text{right}}$ 
12:    else
13:       $t_{\min} = t_{\text{left}}$ .
14:       $t_{\max} = t_{\text{right}}$ .
15:    end
16:     $len = t_{\max} - t_{\min}$ 
17:  end
18:   $t_1 = (t_{\max} + t_{\min}) / 2$ 
19:  take  $t = t_1$  into (16) to find out  $\mathbf{w}^*$ .
20:  calculate  $C_s(\mathbf{w}^*) = \max\{\log(\sigma_E^2 R_1(t_1) / \sigma_D^2) / 2, 0\}$ .
21: end.
22: output:  $\mathbf{w}^*, C_s(\mathbf{w}^*)$ .

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IV. SIMULATION RESULT

In this section, simulations are carried out to investigate the performance of the proposed algorithm. For simplicity, we use a one-dimensional system model where the source, relays, destination and eavesdropper are along a horizontal line. What's more, because the source-relay distance and the distances between relays are very small compared to the source-destination distance and relay-destination distance, the source-destination distance and the distances between different relays and the destination can be considered as the same. So are the source-eavesdropper distance and the distances between the different relays and eavesdropper. To emphasize the effect of distance, a simple line-of-sight channel model which contains the pass loss and a random phase is used. Generally, we can express the channels as $h = d^{-c/2} e^{j\theta}$ where d is the distance, c is the path loss exponent chosen as 3.5 and random phase θ is uniformly distributed over $[0, 2\pi)$. The number of relays is set to 6, i.e. $N = 6$ and the eavesdropper are fixed at 60 m. For individual power constraints, we assume each relay have the same power

budgets and set $p_i = 2.2185\text{mdB}$. The noise power $\sigma_D^2 = -55\text{mdB}$ and $\sigma_E^2 = -65\text{mdB}$.

Firstly, we fix the source/relay location at 0 m and move the destination from 40m to 100 m. The transmit power of direct transmission (DT) is set as 10mdB which is equal to the sum of individual power constraints. The comparison of maximum achievable secrecy rate between this DF based relay system and DT is shown in Figure 2. We can observe that by using DF, compared to DT, there is always a positive secrecy rate during the movement of destination. This result is consistent with our analysis that using relay cooperation and beamforming the security performance of DF based system is better than DT under similar channel conditions and the same transmit power.

Then we fix the position of destination at 50m and move the source/relays from 0 m to 25 m with all other parameters unchanged. Just as expect, Figure 3 illustrate that under the situation that the source-destination channel is worse than the source-eavesdropper channel, the maximum achievable secrecy rate always stays at 0 for DT. And for DF, it increases as the relays are closer to the destination. This is because even through the relay-destination channel and relay-eavesdropper channel both become better when the relays move from 0 to 25, the improving trend of the former is more remarkable.

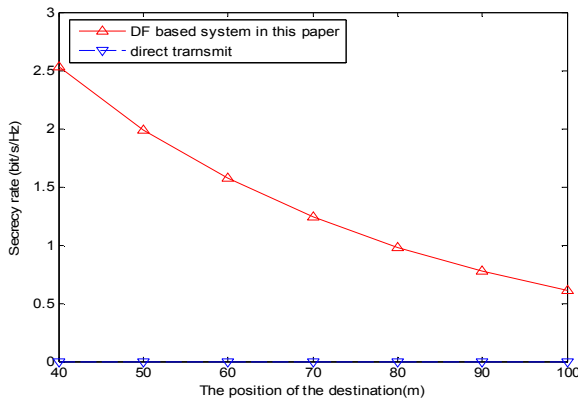


Figure 2. Secrecy rate versus the position of destination.

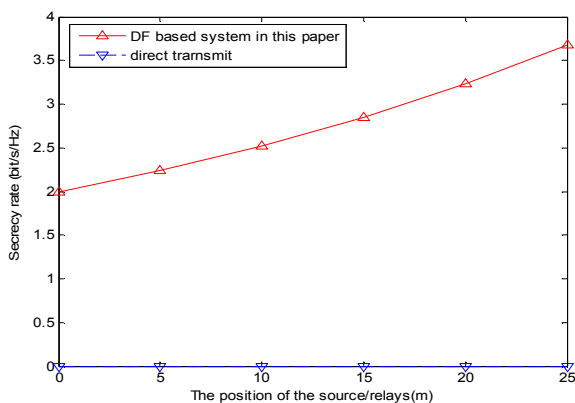


Figure 3. Secrecy rate versus the position of source/relays.

V. CONCLUSIONS

In this paper, we have considered a DF-based cooperative protocol to improve the physical layer security with one eavesdropper. Our attention is focused on the design of beamforming vector to find out the maximum secrecy rate. However our problem formulation is different from others because we assume a more practical scenario where the beamforming vector is subject to individual power constraints and noise power at different node is different. Under the assistance of perfect CSI, we have solved the optimal problem by combining convex optimization and one-dimensional search together and rigorous proof is presented for the correctness of our method.

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