

A Efficient Compensation of Carrier Frequency Offset with Fast Processing Matrix

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Abstract— Inter-carrier interference (ICI) that is generated by carrier frequency offset (CFO) still a problem in based on orthogonal frequency division multiplexing (OFDM) system. To solve that problem, some compensation methods in time domain or frequency domain were studied using matrix. However, most studies need operating inverse matrix, so it spends much run time during equalizing process. And that make it difficult to process data in near real time. Therefore, we need one that processes data in near real time without operating inverse matrix. In this paper, first, we analyze problems of CFO in based on OFDM. CFO makes phase rotation and ICI but we focus more on ICI. Second, we propose a matrix that doesn't need operating inverse matrix. The matrix is generated in frequency domain and is multiplied by received signals. Finally, we confirm BER performance using an operating method that fits proposed matrix. And we show that the proposed matrix is efficient as a method reducing ICI through simulation.

Keywords—Matrix; Inter-carrier interference; OFDM; CFO;

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) system uses frequency band efficiently and has the advantage that is more efficient than signal carrier in frequency selective fading channel. So it is studied in many fields [1-2]. And it is used as standards such as digital audio broadcasting (DAB [3]), digital video broadcasting (DVB [4]), digital multimedia broadcasting (DMB [5]), and wireless local area network (802.11a). And also it is used in 4 generation in mobile communication such as LET-Advanced and mobile WiMAX.

It is imperative that mobility be considered in 4 generation system and broadcasting standards. Phase rotation and ICI are generated by Doppler effect and mismatch between transmitter and receiver based on OFDM system. Phase rotation is compensated easily by pilot, but ICI needs complicated processing for compensation.

There have been many works on CFO [6-8], such as some method using coding to reduce ICI [9] and using maximum likelihood estimation (MLE) [10]. Also, ICI self-cancellation method can decrease ICI easily [11]. However, ICI self-cancellation method sends two consecutive data, so data efficiency is decreased. As another compensation method, matrix has been studied. This method makes matrix in frequency domain [12] and time domain [13] according to

each domain channel characteristic. Particularly, the matrix that is generated in frequency domain has been studied. The diagonal matrix represents phase rotation and the rests represent ICI. So to reduce ICI the rest parts still have been studied. However, Most of them need operating inverse matrix. Inverse matrix actually needs much arithmetic operation. Therefore, equalizer of receiver is to be complicated. In mobile communication, real time processing is important and particularly in 4 generation, due to increasing data speed it will be more important.

In this paper, first, we analyze CFO. When a matrix is made in frequency domain, its diagonal matrix is that one subcarrier affects the subcarrier itself. And the rests are that one subcarrier affects the other subcarriers. And phase rotation affects both diagonal matrix part and the rests part. Second, using this characteristic of CFO, we make matrix that compensates CFO efficiently. The proposed matrix doesn't need operating inverse matrix. It means that the system is able to adapt fast speed processing system. Finally, we do operation for proposed matrix. The proposed matrix needs specific operation to get improved performance efficiently. And through simulation, we confirm the BER performance.

II. SYSTEM MODEL

OFDM system uses IFFT and FFT calculation to obtain orthogonality of signals. Equation (1) represents IFFT calculation where k is subcarrier's index in frequency domain and n is sample's index in time domain. Time domain signals $x(n)$ are transmitted.

$$x(n) = \sum_{k=0}^{N-1} X(k) \cdot e^{i2\pi \frac{k}{N}n} \quad (1)$$

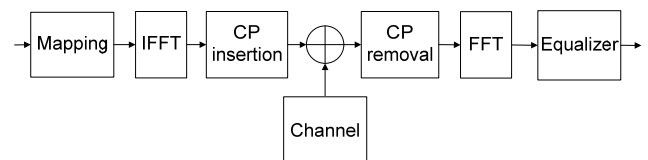


Fig.1. System Model.

Transmitted signals $x(n)$ are calculated by FFT in receiver. Equation (2) is FFT calculation.

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$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-i2\pi \frac{n}{N} k} \quad (2)$$

Figure 1 is system mode.

In this system, CFO affects the transmitted signals and those are compensated using proposed matrix in equalizer. And we assume that CFO is constant and estimated exactly.

III. OFFSET ANALYSIS AND COMPENSATION

A. Carrier Frequency Offset

CFO is represented in frequency domain as shift. It is represented phase rotation in time domain.

$$x_{\text{Doppler}}(p) = x(n) \cdot e^{i2\pi \frac{\varepsilon n}{N}} \quad (3)$$

Equation (3) represents phase rotation in time domain due to CFO where n is sample index. The signal $x(n)$ that has n samples affects different phase rotation. So we present the signal effected by frequency offset like $x_{\text{Doppler}}(p)$ where the number of n and p is same because the signal is only affected by phase rotation in time domain.

Received signal $x_{\text{Doppler}}(p)$ is affected by a normalized offset ε .

$$\varepsilon = \frac{f_{\text{Doppler}}}{\text{Carrier Spacing}}, \quad f_{\text{Doppler}} = \frac{v \cdot f_{\text{carrier}}}{c} \quad (4)$$

In (4), f_{carrier} is carrier frequency and c is the speed of light and v is mobility of receiver. CFO due to Doppler can represent normalized value ε and can have random value because of mismatch between transmitter and receiver. In this paper, we assume the CFO is constant and calculate the offset effect with the normalized value.

Equation (5) is received signals with CFO.

$$\begin{aligned} X_{\text{received}}(k') &= \sum_{p=0}^{N-1} x_{\text{Doppler}}(p) \cdot e^{-i2\pi \frac{p}{N} k'} \\ &= \sum_{n=0}^{N-1} x(n) \cdot e^{i2\pi \frac{\varepsilon n}{N}} \cdot e^{-i2\pi \frac{n}{N} k'} \end{aligned} \quad (5)$$

After the signal is affected by CFO, the received signal passes through FFT block. Original signal is different from received signal in receiver, so those are represented like transmitted signal is k and received signals is k' . if there are no CFO, those k' and k have same value. p is sample of signal that is affected by Doppler.

$$\begin{aligned} X_{\text{received}}(k') &= \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} X(k) \cdot e^{i2\pi \frac{k}{N} n} \cdot e^{i2\pi \frac{\varepsilon n}{N}} \cdot e^{-i2\pi \frac{k'}{N} n} \\ &= \sum_{n=0}^{N-1} \sum_{k=k'}^{N-1} X(k) \cdot e^{i2\pi \frac{(k-k')n}{N}} \cdot e^{i2\pi \frac{\varepsilon n}{N}} \\ &\quad + \sum_{n=0}^{N-1} \sum_{k \neq k'}^{N-1} X(k) \cdot e^{i2\pi \frac{(k-k')n}{N}} \cdot e^{i2\pi \frac{\varepsilon n}{N}} \end{aligned} \quad (6)$$

Received signal is represented in (6) like ICI that one subcarrier affect other subcarriers $\sum_{n=0}^{N-1} \sum_{k \neq k'}^{N-1} X(k) \cdot e^{i2\pi \frac{(k-k')n}{N}}$ and diagonal matrix that one subcarrier affects itself $\sum_{n=0}^{N-1} \sum_{k=k'}^{N-1} X(k) \cdot e^{i2\pi \frac{(k-k')n}{N}}$. In figure 3, ICI is $L(m)$, $L(-m)$ and diagonal matrix is $L(0)$. Those both parts have same phase rotation like $\sum_{n=0}^{N-1} e^{i2\pi \frac{\varepsilon n}{N}}$.

B. Proposed Matrix Creation

If orthogonality collapses like figure 2 due to frequency offset, one subcarrier affects others like Δb . For example, like figure 2, if index of a subcarrier is p , $p+1$ subcarrier is affected by p like Δb_1 and $p-1$ subcarrier is affected by p like Δb_{-1} .

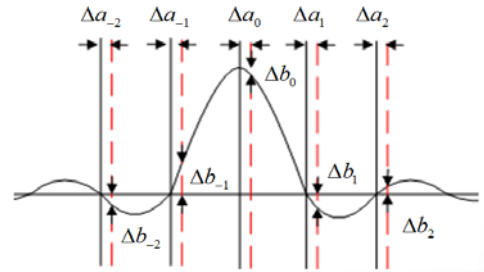


Fig. 2. When Frequency offset occurs in Sinc function.

The matrix without operating inverse matrix can be made using the characteristic of figure 2.

$$S_{\text{sinc}}(q) = \frac{\sin \pi q}{\pi q} \quad (7)$$

Equation (7) is sinc function. If offset is generated by CFO like figure 2, we can represent (8) where the size of Δa is same the size of normalized CFO ε .

$$\begin{aligned} S_{\text{sinc}}(q + \varepsilon) &= \frac{\sin \pi(q + \varepsilon)}{\pi(q + \varepsilon)} \\ &= L(m), \quad m : N+1 \sim N-1, \text{ integer} \end{aligned} \quad (8)$$

In this system, we assume the normalized CFO ε are estimated exactly. Therefore normalized CFO ε is inserted into (8) without additional processing and calculate $L(m)$. The CFO is generated in time domain as $-\varepsilon$ and is compensated in frequency domain as ε because FFT operation doesn't change characteristic between positive and negative. The number of m is $2*N-1$ and N is the number of subcarriers.

$$M_{\text{matrix}} = \begin{bmatrix} L(0) & L(1) & \cdots & L(N-1) \\ L(-1) & L(0) & \cdots & L(N-2) \\ \vdots & \vdots & \ddots & \vdots \\ L(-N+1) & L(-N+2) & \cdots & L(0) \end{bmatrix} \quad (9)$$

The proposed matrix is made as (9) using (8). The matrix (9)'s dimension is $N*N$. Row of the matrix represents that one

subcarrier affects itself and other subcarriers because the row is similar sinc function with offset. On other way, column of the matrix represents that N subcarriers affect one subcarrier.

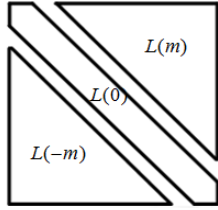


Fig. 3. Structure of Matrix.

Figure 3 shows how the matrix consists of (6). $L(0)$

$\sum_{n=0}^{N-1} \sum_{k=k'}^{N-1} X(k) \cdot e^{i2\pi \frac{(k-k')n}{N}}$ represent that one subcarrier affects

itself. $L(m)$, $L(-m)$ $\sum_{n=0}^{N-1} \sum_{k=k'}^{N-1} X(k) \cdot e^{i2\pi \frac{(k-k')n}{N}}$ represent one subcarrier affect other subcarriers except for itself. Therefore $L(m)$, $L(-m)$ are ICI.

Whole components of figure 3 are rotated like $\sum_{n=0}^{N-1} e^{i2\pi \frac{\epsilon}{N} n}$.

So the phase rotation that is generated by CFO can be compensated using (10). In other words, $L(m)$, $L(-m)$ as well as $L(0)$ have phase problem.

$$M_{Matrix} = M_{Matrix} \times e^{i2\pi \cdot \text{theta}} \quad (10)$$

In this system, because comb type pilot is used, the phase rotation is estimated per symbol. And theta is found using (11) as angle.

$$\text{theta} = \tan^{-1} \left(\frac{\text{imag}(P)}{\text{real}(P)} \right) \quad (11)$$

P is average of pilots.

C. CFO Compensation using the proposed Matrix

The proposed matrix M is represented like (12). If $b_m = |L(m)|$, the matrix of (9) is same the matrix of (12).

$$M_{matrix} = \begin{Bmatrix} b_0 & -b_1 & \cdots & -b_{N-1} \\ +b_{-1} & b_0 & \cdots & +b_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ +b_{-N+1} & -b_{-N+2} & \cdots & b_0 \end{Bmatrix} \quad (12)$$

The subcarrier of mth affects the subcarrier of $m-1$ th as size of $-b_{m-1}$ and affects the subcarrier of $m+1$ th as size of $+b_{m+1}$. b is the absolute value that is generated by CFO. Column of the matrix M represent that one subcarrier is affected from other subcarriers. Therefore we can compensate received signals by multiplying the M_{matrix}^T . However, in (8), we don't consider effect of random data. The transmitted signal is random signal and if there is CFO, the received signal has different value comparing transmitted signal. Because the random c we can't estimate how many any subcarrier affects other subcarriers exactly. So after compensation using the M matrix, the ICI is not compensated.

$$X_{received}(k') = X_{transmitted}(k) + c, \quad k = k' \quad (13)$$

If CFO is generated in random signal, the received signal is different from transmitted signal like (13). Because the error c is affected by random signal of transmitter, estimation of c is difficult.

So we propose a method (14) that can improve performance.

$$X_{inversed}(k') = X_{received\ data}(k') \times (-1)^K, \quad K = 0 \sim N-1: \text{Subcarrier index number} \quad (14)$$

Equation (14) is a method that turns the phase of subcarrier that has odd index number. In figure 4, if we think the subcarrier of mth a_m is center, left subcarriers by the a_m is affected as '-' from a_m and right subcarriers by the a_m is affected as '+' from a_m .

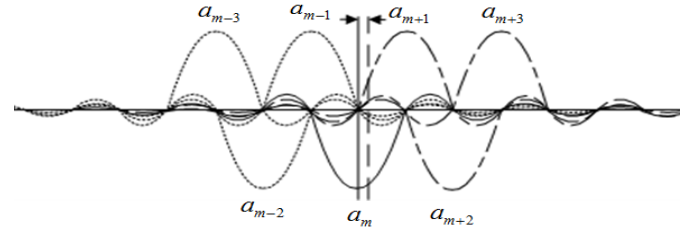


Fig. 4. Modified received signals.

And figure 4 is represented like (15).

$$Y_{inversed_R} = M_{matrix}^T \cdot X_{inversed}$$

$$= \begin{bmatrix} b_0 & +b_1 & -b_{-1} & \cdots & +b_{N-1} \\ -b_{-1} & b_0 & +b_1 & \cdots & -b_{N-2} \\ +b_{-2} & -b_{-1} & b_0 & \cdots & +b_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -b_{-N+1} & +b_{-N+2} & -b_{-N+3} & \cdots & b_0 \end{bmatrix} \cdot \begin{bmatrix} X(0) \\ -X(2) \\ X(3) \\ \vdots \\ -X(N-1) \end{bmatrix} \quad (15)$$

$$= \begin{bmatrix} b_0 & -b_1 & -b_{-1} & \cdots & -b_{N-1} \\ -b_{-1} & -b_0 & +b_1 & \cdots & +b_{N-2} \\ +b_{-2} & +b_{-1} & b_0 & \cdots & +b_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -b_{-N+1} & -b_{-N+2} & -b_{-N+3} & \cdots & -b_0 \end{bmatrix} \cdot \begin{bmatrix} X(0) \\ X(2) \\ X(3) \\ \vdots \\ X(N-1) \end{bmatrix}$$

Equation (12) is presented like (15) with changes of (14). Signs of (15) is changed like figure 4. The signs of left side by Subcarrier of mth have same direction. In other words, because of same signs, the error of the random c in (13) is reduced.

After (15), the compensated signal has better performance because ICI is reduced. However diagonal matrix of (15) is changed.

$$Y_{compensated} = Y_{inversed_R} \times (-1)^K \quad (16)$$

Using (16), we can compensate the data.

IV. SIMULATION RESULTS AND DISCUSSION

TABLE 1. SIMULATION PARAMETERS.

OFDM system	
The number of Subcarrier	64
Bandwidth	960kHz
Symbol Period	66.7 μ s
Subcarrier Spacing	15KHz
CP Length	1.67 μ s
Pilot Type	Comb type Pilot
Modulation	QPSK
Channel	AWGN

Table 1 is simulation environments. This system based on OFDM has 64 subcarrier and subcarrier spacing is 15kHz. In this paper, we assume CFO is time invariant.

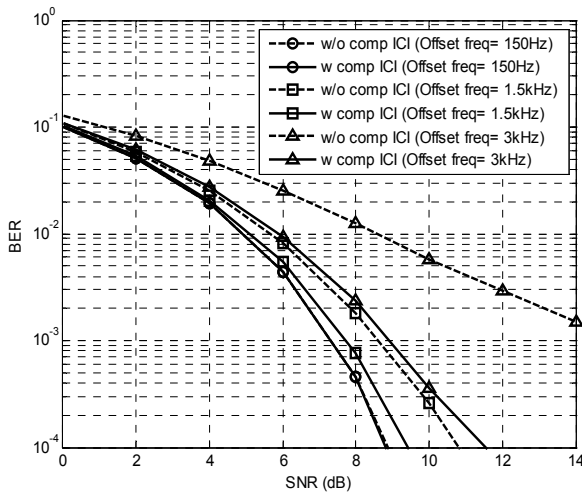


Fig. 5. Comparing BER performance of ICI according to frequency offset changes.

Figure 5 is BER performance after compensation according to scale of frequency offset. The graph shows line and dotted line. First, the line is when phase rotation is only compensated. So it has $L(0)$ but $L(m)$ and $L(-m)$ is zero. Second, the dotted line is when ICI and phase rotation is compensated. So it use $L(0)$, $L(m)$, and $L(-m)$ in figure 3. Second is proposed method and has better performance.

When offset frequency is 150Hz, it has smaller offset. If mismatch between transmitter and receiver is constant, we can think that the receiver has low speed. In this case, the ICI is small, so after compensation with the proposed matrix, the performance is same before compensation. When offset frequency is 1.5kHz, the offset is large because the subcarrier spacing is 15kHz. So BER of about 2dB at 10⁻⁴ is degraded from one when offset is 150Hz. After compensation, we can get 1dB improvement of BER at 10⁻⁴.

When offset frequency is 30kHz. In this case, the receiver has severe CFO. So after compensation of phase rotation, the BER performance is very bad. After compensation of ICI, we can get improved performance among other things. As this result, we can know this system is suitable that have high mobility.

V. CONCLUSION

In this paper, we proposed a matrix that can receive data in high mobility of receiver efficiently. First, we analyzed CFO in frequency domain and made a matrix according to the result of analysis. Through simulation, if CFO is not large, the performance improvement is not large because there is small ICI. However, if offset is increased, ICI degrades performance gradually and the proposed matrix improves performance increasingly. As this result, we can confirm that the proposed method reduces ICI efficiently and this can be used in high speed data system because it doesn't need operating inverse matrix.

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