

The Performance of ML Estimation of OFDM Symbol Time with and without Correlation Coefficient

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Abstract— In this paper, we show the results of the maximum likelihood (ML) estimation of orthogonal frequency-division multiplexing (OFDM) symbol time as to whether there is a correlation coefficient (CC) or not. The CC is related to signal to noise ratio (SNR), and SNR/(SNR+1), and approximately 1 at high SNR. However, because it is difficult to calculate a SNR in a receiver it is desirable to delete the CC. Thus, we investigate the performance of ML estimation with and without a CC at low SNR under various channel conditions such as additive white Gaussian noise (AWGN), frequency offset and multipath delays. In addition, we simulate the frame error rate (FER) with the same conditions. The results of simulation show that ML estimation with CC is superior to that without it with respect to mean square error (MSE) of OFDM symbol time and FER under all channel environments. However, the former is inferior to the latter in terms of the complexity of an OFDM receiver. Thus, there is trade-off between the performance and simplicity of it.

Keywords— ML estimation, correlation coefficient, mean square error, FER

I. INTRODUCTION

OFDM systems have been deployed in many systems such as wireless communication, as well as European digital video broadcasting [1]-[3]. A synchronization of OFDM symbol time is the first task in OFDM systems and there have been many studies of it [4]-[7]. In general, in order to facilitate the synchronization of OFDM symbol time and preserve orthogonality of OFDM and avoid the ISI, the last tens of samples are copied to the front of that OFDM symbol as a cyclic prefix (CP). Thus, the resulting OFDM symbol has a periodicity of the length of CP and there are many methods for an estimation of OFDM symbol time using that property [8], [9]. If OFDM symbol time is estimated within the length of CP, there is no ISI and no FER. Otherwise, there is ISI and FER. Even if there is no ISI, there is an estimation variation of OFDM symbol time within the length of CP. ML estimation of OFDM symbol time is one of them. However, such an approach needs a CC which requires a calculation of SNR in a OFDM receiver. It is preferable to delete the CC for simplicity of a receiver.

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Thus, we investigate how much the ML estimation degrades at low SNR under some channel conditions as to whether there is CC or not. In other words, we simulate the MSE of ML estimation and FER. The results of simulation show that the MSE and FER of ML estimation with CC is superior to that without it. However, there is trade-off between the performance and complexity of a receiver.

II. ML ESTIMATION OF OFDM SYMBOL TIME

First, we introduce the simulation model of OFDM system and channel. Next, we explain the ML estimation with and without CC. Last, we show the results of simulation.

A. OFDM System

Figure 1 shows the OFDM system we simulate.

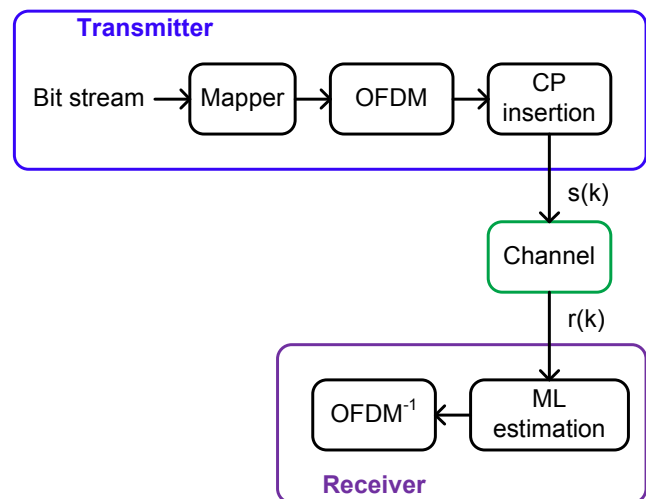


Figure 1. A block diagram of a OFDM transceiver.

As shown in Figure 1, the transmitter in OFDM system consists of mapper, OFDM and CP insertion. Bit stream is mapped into symbols by mapper. OFDM is inverse fast Fourier transform (IFFT) with N subcarriers. Because IFFT modulates symbols of the mapper output, the output of it is N samples and has a complex value. CP insertion copies the last

L samples of IFFT output and appends them to it. Thus, one OFDM symbol has the length of N plus L shown in Figure 2 and has the periodicity of CP length.

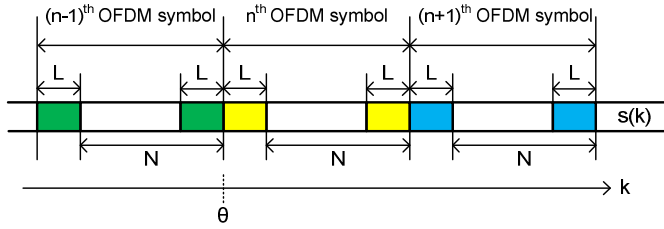


Figure 2. The output of a OFDM transmitter.

The OFDM symbols are transmitted over channel and a receiver comprises ML estimation and OFDM⁻¹. ML estimation detects a OFDM symbol time and OFDM⁻¹ is FFT and demodulates the received signal.

B. The channel model

Figure 2 shows the block diagram of channel we apply.

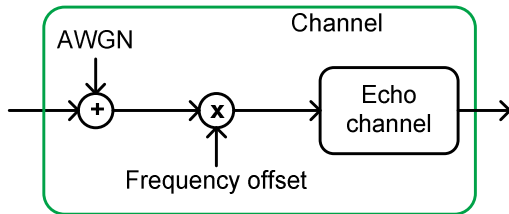


Figure 3. The block diagram of channel for simulation.

As shown in Figure 3, the channel consists of AWGN, frequency offset (FO) and echo channels. We adopt the channel of digital video broadcasting for cable systems (DVB-C2) as echo channel, and there are two cases of echo channel: echo channel 1 (EC1) and echo channel 2 (EC2) [2], [3].

Figure 4 and Figure 5 shows the power of delay paths and frequency response of EC1.

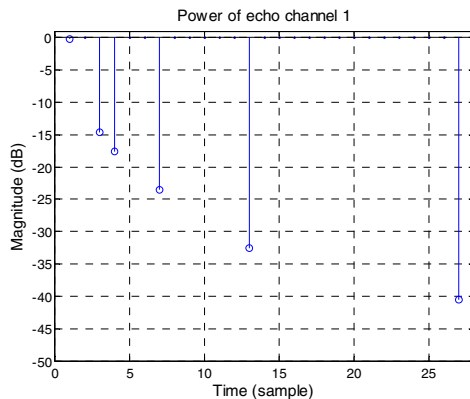


Figure 4. Power delay profile of EC1.

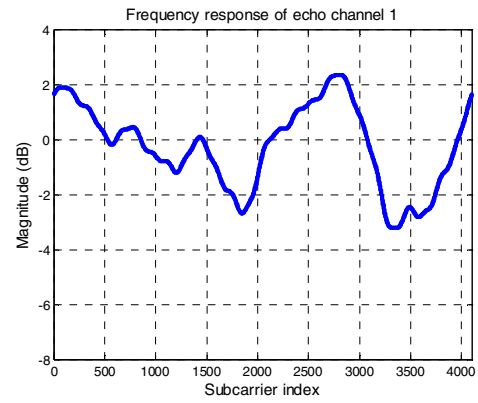


Figure 5. The frequency response of EC1.

Figure 6 and Figure 7 shows the power of delay paths and frequency response of echo channel 2 respectively.

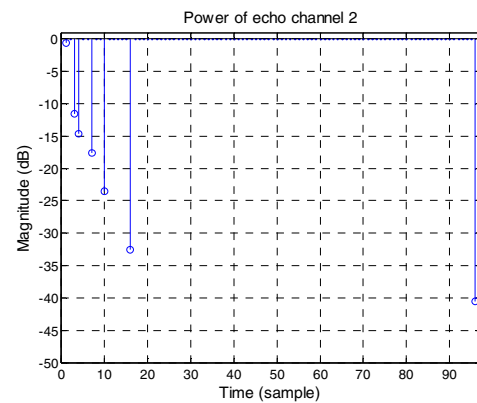


Figure 6. Power delay profile of EC2.

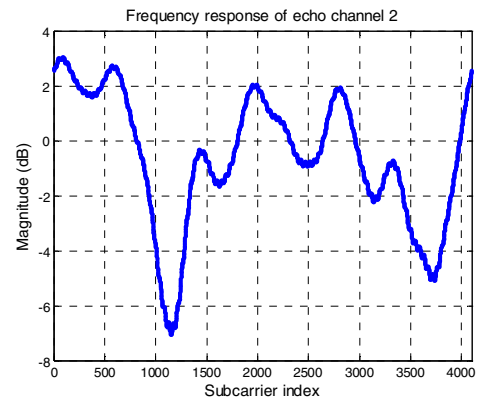


Figure 7. The frequency response of EC2.

As shown in Figure 4, Figure 5, Figure 6 and Figure 7, echo channel 2 has the longer path than echo channel 1 and the deeper spectral null than echo channel 1.

C. ML estimation with and without CC

Basically, ML estimation uses the periodicity of OFDM symbol caused by CP, and shown in Figure 8.

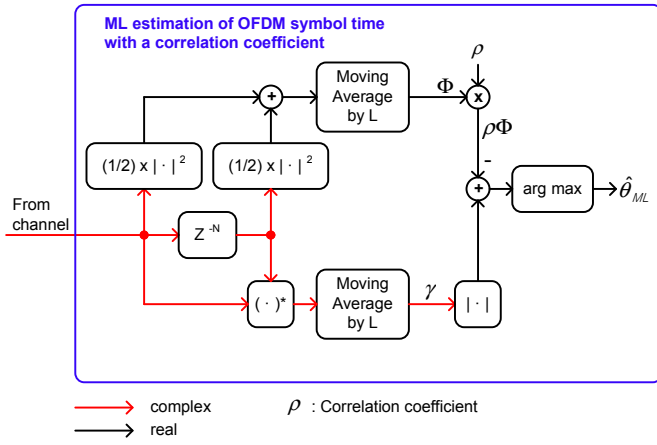


Figure 8. The block diagram of ML estimation of OFDM symbol time with CC.

As seen in Figure 8, $\hat{\theta}_{ML}$ is the estimated OFDM symbol time and becomes

$$\hat{\theta}_{ML} = \arg \max \{ |\gamma(\theta)| - \rho \Phi(\theta) \} \quad (1)$$

$$\gamma(\theta) = \sum_{k=\theta}^{\theta+L-1} r(k) \times r^*(k+N) \quad (2)$$

$$\Phi(\theta) = \left(\frac{1}{2} \right) \sum_{k=\theta}^{\theta+L-1} |r(k)|^2 \times |r(k+N)|^2 \quad (3)$$

$$\rho = \frac{SNR}{SNR+1} \quad (4)$$

where k and θ is sample index, L is the length of CP, N is the number of subcarriers, and ρ is the correlation coefficient (CC) and related to SNR.

The argument of equation (1) has peaks at each $(L+N)$ time instant of each OFDM symbol.

However, it is difficult to estimate the exact SNR and it is desirable to delete the SNR for the simplicity of an OFDM receiver. Thus, we assume that the CC is 1. Accordingly, there is degradation due to such an assumption and we investigate the effect of ML estimation under multiple channel environment.

D. Simulation results

We use simulation to evaluate the performance of ML estimation and the parameters are listed in Table 1.

TABLE 1. PARAMETERS OF DOWNSTREAM TRANSMITTER

Subcarriers number (N)	4096
CP length (L)	64 samples
Signal bandwidth	3.4284 MHz
Subcarrier spacing	1.674 kHz
Sampling frequency	6.8586 MHz
Frequency offset	2.0925 kHz
Echo channel	EC1 and EC2

In each E_b/N_0 , simulations are repeated 10,000 times and we evaluate the performance of the ML estimation with and without CC in terms of MSE of OFDM symbol time and FER under various channel conditions. Figure 9 shows the example of probability density function (PDF) and cumulative density function (CDF) of estimated OFDM symbol time when E_b/N_0 is 4 dB and 12 dB individually.

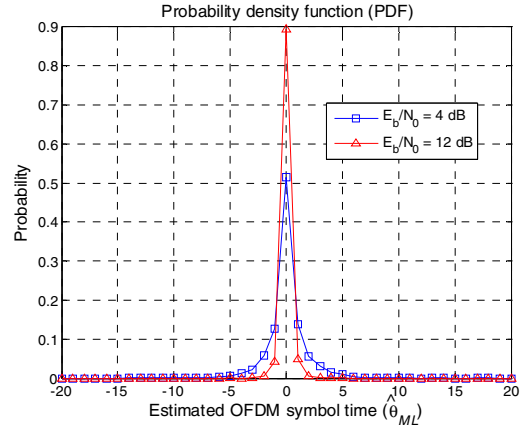


Figure 9. Probability density function of estimated OFDM symbol time.

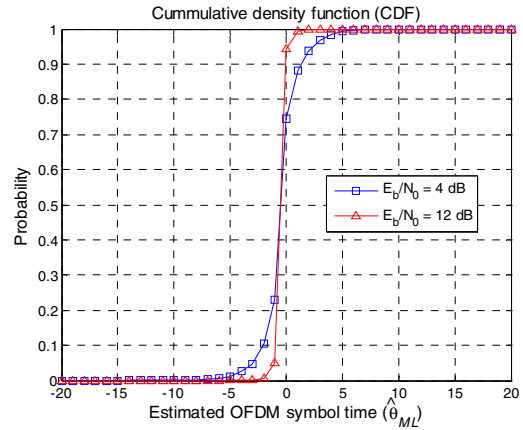


Figure 10. Cumulative density function of estimated OFDM symbol time.

Figure 9 and Figure 10 shows the estimated OFDM symbol time has the symmetric shape and variation centered on mean values which are denoted by 0 and correspond to $L + N + L/2$ time instant. In addition, the MSE of estimated OFDM symbol time is 2.8429 and 0.1564 individually when E_b/N_0 is 4 dB and 12 dB. That means the MSE of estimated OFDM symbol time decreases as E_b/N_0 increases. If the estimation of OFDM symbol time is not made within CP interval, ISI happens. Accordingly, it increases FER.

Figure 11 shows the MSEs of estimated OFDM symbol time with CC under various channel conditions.

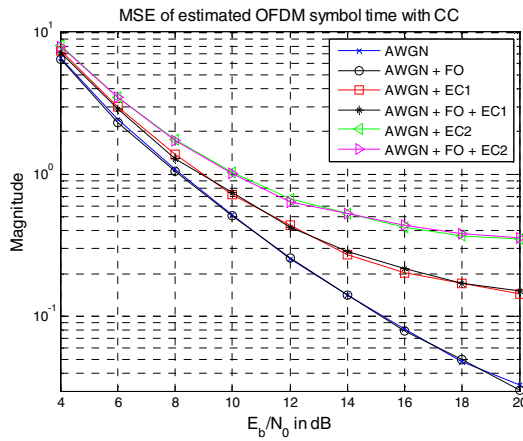


Figure 11. Mean square error of estimated OFDM symbol time with CC.

As shown in Figure 11, the major impact on MSE is EC1 and EC2. The worst case between them is EC2 because EC2 has longer path delays and deeper spectral null than EC1. On the other hand, FO does not have effect on MSE.

Figure 12 shows the MSEs of estimated OFDM symbol time without CC under various channel conditions.

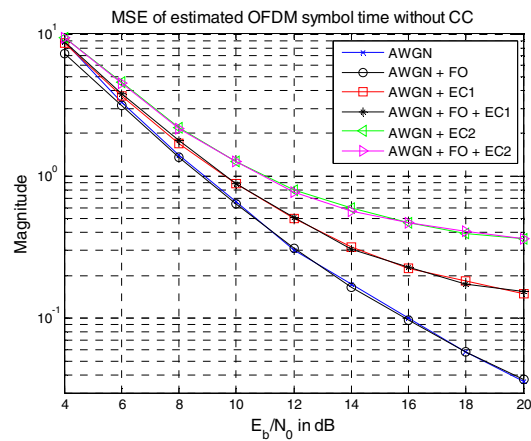


Figure 12. Mean square error of estimated OFDM symbol time without CC.

As shown in Figure 12, the pattern of MSEs is the same as the Figure 11, but worse than Figure 11.

Figure 13 shows the difference of MSEs of estimated OFDM symbol time with and without CC under AWGN, AWGN + EC1 and AWGN + EC2.

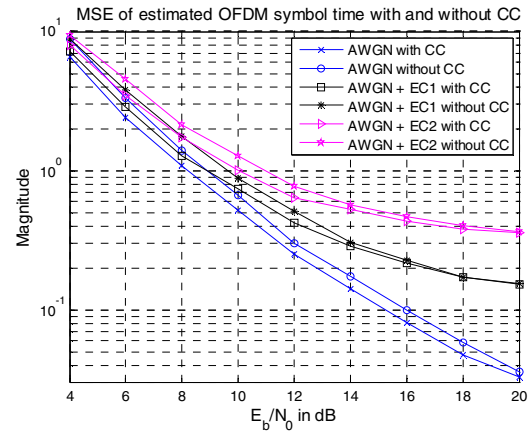


Figure 13. MSE comparison between ML estimation with CC and it without CC.

As shown in Figure 13, the MSEs of estimated OFDM symbol time with CC have better performance than those without CC at low SNR. In addition, Figure 13 shows that their performance converge at high SNR.

Figure 14 shows the FER of ML estimation with CC under multiple channel environments.

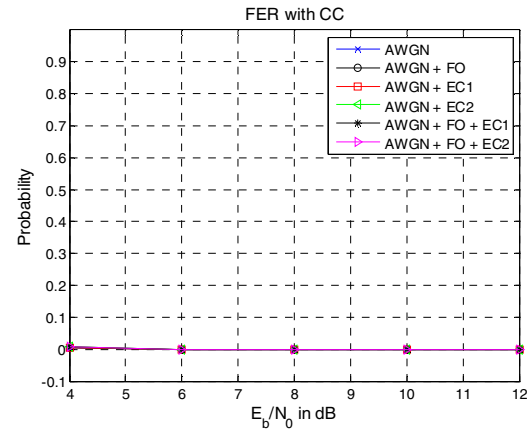


Figure 14. FER of ML estimation with CC.

Figure 14 shows that the FERs are low at low SNR and all channel conditions do not affect the FER. Namely, the estimated OFDM symbol time with CC is made within a CP for almost all cases.

Figure 15 shows the FER of ML estimation without CC under multiple channel environments.

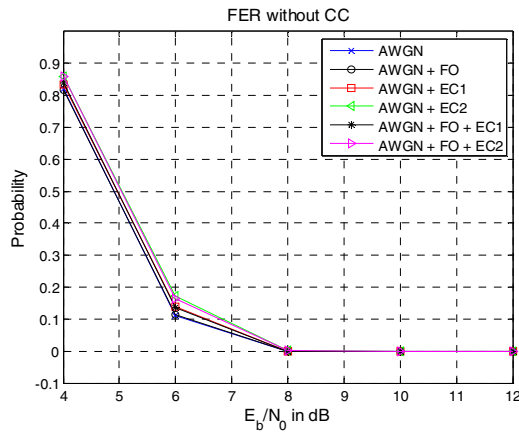


Figure 15. FER of ML estimation without CC.

As shown in Figure 15, the FERs of ML estimation without CC are high at low SNR. In other words, the estimated OFDM symbol time is beyond CP. However, as SNR increases, FER decreases. In addition, FO does not have effect on the MSE. On the other hands, EC1 and EC2 have major impact on the MSE.

Figure 16 shows the difference of FERs of estimated OFDM symbol time with and without CC under AWGN + EC2.

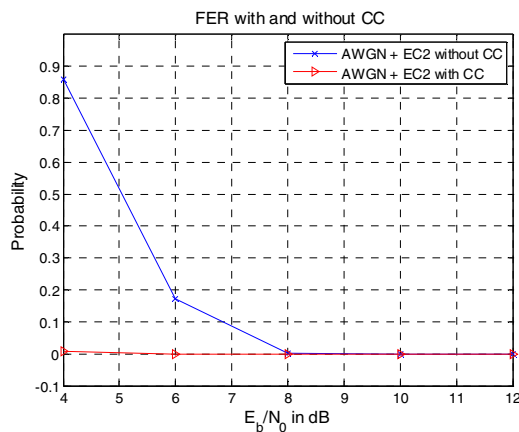


Figure 16. FER comparison between ML estimation with CC and it without CC.

Figure 16 shows that there is no difference of them at high SNR.

III.CONCLUSION

ML estimation using CP needs CC which is related to a received SNR. However, it is hard to calculate the CC exactly and desirable to get rid of it for simplicity of an OFDM receiver.

Thus, we have investigated the performance of ML estimation with and without CC in terms of MSE of estimated OFDM symbol time and FER. In addition, we have presented that MSE and FER of ML estimation with CC has superior to those of without CC at low SNR. However, there is no difference of the performance between them at high SNR.

Accordingly, there is trade-off between the performance and simplicity of an OFDM receiver.

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