

Design of Optical Orthogonal Codes with Variable Chip Rate for Flexible Optical CDMA Systems

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Abstract— Optical orthogonal codes (OOCs) have been studied by many researchers to increase its flexibility such as variable code length and variable code weight. Recently, the authors have proposed OOCs with variable chip rates (VC-OOC), which can keep the orthogonal property of the strict OOC even when different chip rate codes are used in an optical CDMA network for signal multiplexing. In this paper, the VC-OOCs are reviewed and some approaches of code design using the balanced incomplete block design (BIBD) sequences are proposed. The BIBD parameters suitable for the variable chip rate codes are obtained and discussed.

Keywords— Optical CDMA systems, optical orthogonal codes, variable chip rate, optical access networks

I. INTRODUCTION

Optical code division multiple access (CDMA) systems have powerful capabilities in feature optical access networks with advantages of asynchronous transmission/multiplexing, low latency, full optical processing and so on [1]. Optical orthogonal codes (OOCs) are used for encoding/decoding processes. Each user is assigned different codeword of an OOC, and an OOK-modulated optical pulse is time-spread into several pulses like unique sparse sequence. At the decoding process, an optical correlator is de-spreading the received signal and expected user's signal can be detected in full optical processing.

Many researchers have studied OOCs having not only fixed code length/weight but also variable code length/weight to improve its flexibility for multimedia data transmission [2-13]. Recently, the authors have proposed the variable chip rate OOC (VC-OOC), where the orthogonal property of the strict OOC are achieved even when the codewords' optical pulse widths are different [14]. The VC-OOC is suitable for gradual development of optical access networks, unequal error protection, security enhancement, and so on. In the previous works, the random code search, and the theoretical upper bound on code size and the BER performance have been discussed. In this paper, the VC-OOCs are reviewed and some approaches of code design using the balanced incomplete block design (BIBD) sequences are proposed. The design is based on distribution of intervals of any pair of optical pulses of all codewords in an OOC. The efficient parameters of the BIBD sequence for the VC-OOCs are analysed by computer

programs, and code construction methods and its features are discussed.

II. OPTICAL ORTHOGONAL CODES WITH VARIABLE CHIP RATE

A. Optical CDMA Systems

The OOCs are used as time-spreading codes in the incoherent optical CDMA systems where intensity modulation and direct detection (IM/DD) are employed. The OOCs are designed by the two correlation conditions; the side-lobe of auto-correlation of each codeword is limited to λ_a and the cross-correlation between any pair of codewords is limited to λ_c . Most powerful OOCs having $\lambda_a = \lambda_c = 1$ are called as *strict* OOCs. In this paper, the strict OOCs are assumed.

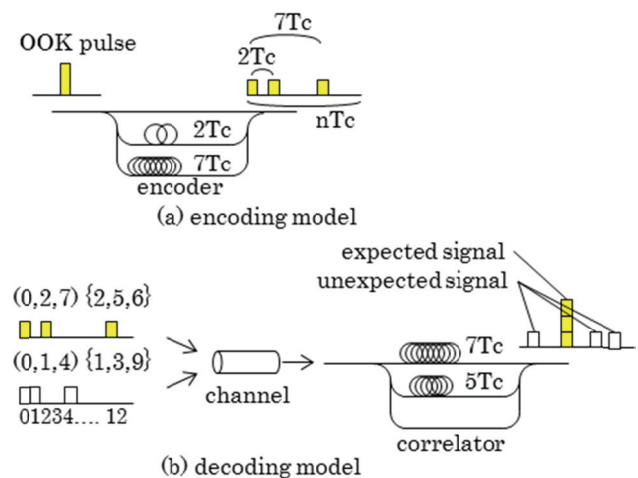


Figure 1. An example of an OOC in an optical CDMA system.

Figure 1 shows an example of an OOC, having two codewords, in an optical CDMA system. The optical CDMA system includes only optical devices like optical encoder and correlator for detection of expected user's signal with low latency. An encoder model is depicted in Figure 1(a). An OOK modulated optical pulse is input to the optical encoder with three fiber delay lines of 0, $2T_c$, $7T_c$, where T_c denotes chip duration time. Encoded signal shape corresponds to the codeword of (0, 2, 7), where the numbers enclosed within parentheses indicate pulse positions. The notation of {2, 5, 6}

shows neighbouring pulse intervals of the codeword. Here, code length, n , is 13. Figure 1(b) shows decoding model. The users have different unique codewords. In this example, the codeword (0, 2, 7) is assigned to an expected user, and the codeword (0, 1, 4) is assigned to unexpected user. The received optical signal is de-spread by the correlator based on the reverse codeword, and then the optical signal from the expected user can be detected with pulse height of 3 (code weight) at the output of the correlator; whereas the unexpected user's signal is spread again by the correlator. Larger code weight provides better performance against interferences.

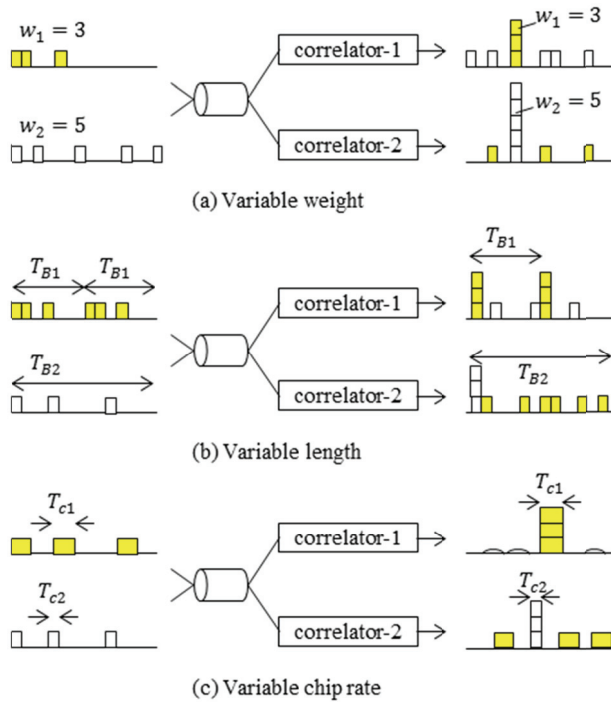


Figure 2. Optical CDMA system.

B. Optical Orthogonal Codes with Variable Chip Rates

Figure 2 shows how the idea came to the VC-OOCs. An optical CDMA using a variable weight OOC is shown in Figure 2(a). The codeword with larger code weight, w_2 , of 5 has better performance compared to the codeword with smaller code weight, w_1 , of 3. The codeword assignment should be made based on users' QoS requirements flexibly. Figure 2(b) shows an example of variable code length OOC. Different data rates of $1/T_{B1}$ and $1/T_{B2}$ can be provided to the users. We must design the variable length OOC from shorter codewords to longer codewords, because a longer codeword is suffered from serially concatenated sequence of shorter codewords. Such step-by-step (successive) design is useful in this asymmetric case. Figure 2(c) depicts code extension to VC-OOCs. The strict OOC property can be provided even when different pulse width is mixed in the same optical CDMA system. Since the chip rates of $1/T_{c1}$ and $1/T_{c2}$ are variable, VC-OOCs have quite flexible parameters in both data rates and interference performance. The performance of

faster chip rate users can be improved owing to suppressed interference from slower chip rate signal to faster chip rate circuit, if the optical gate circuit is put in the receiver.

III. DISTRIBUTION OF PULSE INTERVALS

A. Relationship between Strict OOC Property and Distribution of Pulse Intervals

As mentioned before, the OOCs is designed by the two properties; the maximum side-lobe of auto-correlation of each codeword is limited to λ_a and the maximum cross-correlation between any pair of codewords is limited to λ_c . The strict OOCs are most powerful where the condition of $\lambda_a = \lambda_c = 1$ is satisfied. In such the condition, the number of overlapped pulse positions between any pair of codewords is less than or equal to one even when any phase shift takes place. In this paper, we assumed this strict OOC property for our code construction.

It is known that the distribution of pulse intervals is one of most powerful tools for construction of the strict OOCs. Figure 3 shows an example of distribution of pulse intervals in case of equal code weight and equal code length. When pulse intervals between any pair of pulse positions in all codewords in the OOC are distinct, the strict OOC's property of $\lambda_a = \lambda_c = 1$ is satisfied. In the example shown in Figure 3, the set of pulse intervals, $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, includes unique numbers (pulse intervals), then we can confirm that it's a strict OOC. The pulse intervals are within $n - 1$, where n is the code length. There are $w(w - 1)$ combinations of pulse pairs in a codeword, where w is the code weight. To construct a strict OOC, its code length of n must be larger than $rw(w - 1)$, thus $n - 1 \geq rw(w - 1)$, where r is the number of codewords in the OOC. The OOC shown in Figure 3 is optimum, because $n - 1 = rw(w - 1)$ is satisfied with $n = 13$, $w = 3$ and $r = 2$.

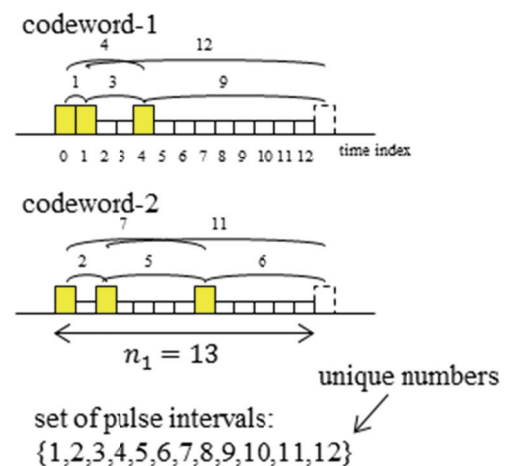


Figure 3. An example of distribution of pulse intervals in case of equal weight/length code.

B. Distribution of Pulse Intervals of VC-OOCs

The distribution of pulse intervals is a powerful tool to confirm whether an OOC is orthogonal or not, even when codewords with different chip rates are mixed in the code. Note that each chip (pulse) in the slower codewords is extended to several contiguous pulses from the view point of the faster codewords. Thus, a pulse at width of T_{c1} is extended to k_2 pulses at width of T_{c2} , when $T_{c1} = k_2 T_{c2}$. The distribution of pulse intervals can be constructed under such the extended situation. The code design of the faster chip rate codewords is difficult compared to that of slower codewords, because of increased restrictions. We suggest that we must begin with the design of the slower chip rate codewords.

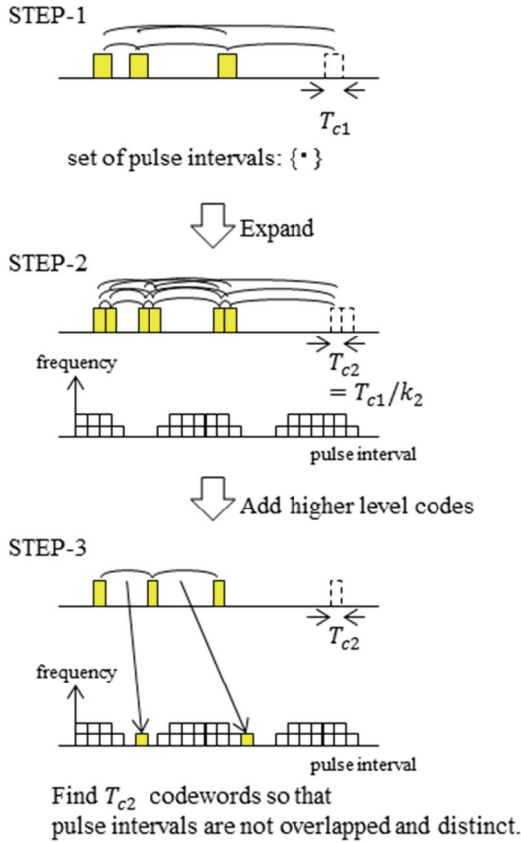
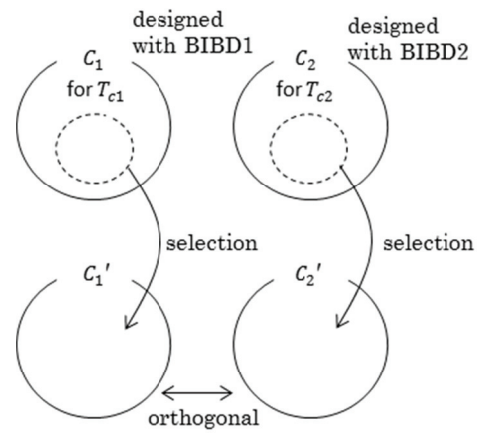


Figure 4. Code construction for OOCs with variable chip rates.

Figure 4 shows the procedure of code construction for the VC-OOCs. First of all, determine the required number of codewords, $|C_i|$, of subset of the VC-OOC at level- i , C_i . At the STEP-1, accomplish the design of the slowest chip rate codewords of C_1 having chip rate of $1/T_{c1}$. Thus, construct $|C_1|$ codewords to satisfy the condition of no repeated pulse interval, thus distinct pulse intervals, in the whole codewords, and add them to a codeword list. At the STEP-2, go to the next level and design C_2 having chip rate of $1/T_{c2} = k_2/T_{c1}$, where k_2 denotes the chip ratio of level-2 relative to $1/T_{c1}$. At this step, each pulse of the slower codewords is expanded to contiguous k_2 pulses. The distribution of pulse intervals at faster chip rate of $1/T_{c2}$ is also expanded and then we cannot

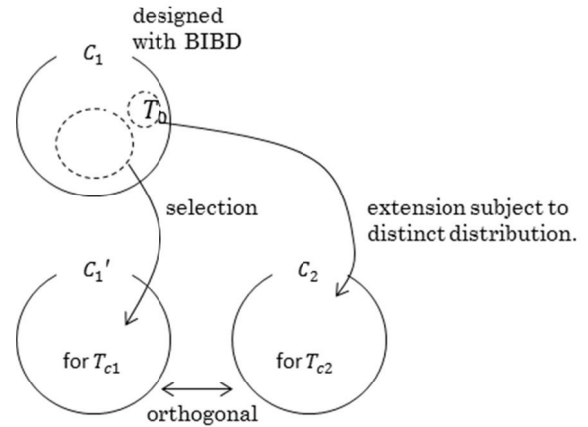
keep that the frequency of usage of pulse intervals is just once anymore. The distribution of pulse intervals includes some peaks having height of 2 or more (just k_2). However, such expanded distribution of previously designed codewords is acceptable. At this step, it is important that pulse intervals of C_2 should be distinct. Thus, the design of the VC-OOCs has asymmetric properties. At the STEP-3, search additional codewords, having chip rate of $1/T_{c2}$, to satisfy pulse intervals are distinct even in the expanded distribution of whole pulse intervals. After required number of codewords of C_2 , $|C_2|$, is found, repeat STEP-2 and STEP-3 to design the next level of code, if necessary.

The VC-OOC constructed by the above procedure, codewords are orthogonal not only inside each level of code, C_i ($i \in \{1, 2, \dots, L\}$), but also even between the different levels of codes, C_i and C_j ($i, j \in \{1, 2, \dots, L\}$, $i \neq j$). Then, the strict design of VC-OOC is accomplished.



Generate BIBD sequences and select orthogonal codewords.

(a) codeword selection approach.



Generate a BIBD sequence and reduce T_0 and surrounding codewords to extend T_0 for C_2 with T_{c2} .

(b) codeword reduction for higher code construction approach.

Figure 5. Two approaches for design of VC-OOCs.

IV. CODE DESIGN USING BIBD SEQUENCES

A. Approach of Code Design

The BIBD sequence is one of powerful techniques for algebraic design of the strict OOCs. The BIBD sequence can provide strict OOCs perfectly not only for equal weight OOCs but also for variable weight OOCs. However, it is difficult to directly use the BIBD for the VC-OOCs. Therefore, we consider two approaches as shown in Figure 5.

Figure 5(a) shows the first approach. Mother codes, C_1 and C_2 , are perfectly designed by the BIBD sequence, individually. Here, the number of code levels, L , is set to 2. The chip rates of C_1 and C_2 , $1/T_{c1}$ and $1/T_{c2}$, are different. After that, select codewords from C_1 and C_2 and construct two sub-sets, C_1' and C_2' , to satisfy the strict OOC property. Such a code consisting of C_1' and C_2' is a VC-OOC. This approach has a disadvantage of high computational complexity in the selection procedure.

Figure 5(b) shows the second approach. First, the slowest code, C_1 is designed by the BIBD sequence. Next, discard some codewords in C_1 to permit to design faster chip rate codes. After discarding, unused intervals appear in the expanded distribution of pulse intervals of C_2 . This makes possible to insert codewords in C_2 into the codeword list of the VC-OOC. We call this second approach as multi-layered BIBD approach. There are optimum selections for discarding. It should be evaluated by computer simulations. In the next subsection, the optimum selections are used for evaluations and discussions.

B. Code Design Based on BIBD Sequences

The BIBD sequence is based on extension fields [15]. It can provide algebraic design of optimum strict OOCs [10]. Since there is no need for code search, the design can be quickly completed.

In this section, we overview the design of the strict OOC based on the BIBD sequence. Let α be a primitive root, n be code length, r be the required number of codewords. The code length must satisfy $n \geq w(w-1)r+1$. For the optimum strict OOC, $n = w(w-1)r+1$. Note the BIBD sequence requires a prime number of n . The following description consists of two parts in cases of odd and even weights of w .

For odd weight of $w = 2m+1$, let $c = wr$ be a design parameter. Here, m is natural number. Put the $(j+1)$ th optical pulse to the position of $\alpha^{im+jc} \bmod n$ where $j \in \{0,1,\dots,w-2\}$. In addition, put an optical pulse to the position 0 of each codeword. Here, $i \in \{0,1,\dots,r-1\}$ denotes the codeword's index number.

For even weight of $w = 2m$, let $c' = (w-1)r$ be a design parameter. Put the $(j+1)$ th optical pulse to the position of $\alpha^{im+jc'} \bmod n$ where $j \in \{0,1,\dots,w-1\}$.

In the above mentioned construction, arbitrary parameters are not acceptable for strict OOCs. Computer search is needed to find suitable combinations of the BIBD parameters. This parameter search is not so heavy compared to random code search.

C. Optimum Parameters of BIBD Sequences Suitable for VC-OOCs

Optimum parameters of the BIBD sequences are searched by computer simulations. The parameter search was made in various code weights and primitive roots, where m was set to 2 to 8 and α was set to 2 to 30. The results are listed in Tables 1 and 2. Tables 1 and 2 include the result in the case of even and odd code weights, respectively. Although there is some restrictions, such as prime numbers of code length, for the optimum strict OOCs, various combinations of code lengths and code weights are obtained. These BIBD sequence can be candidates of the VC-OOCs.

TABLE 1. BIBD PARAMETERS FOR OPTIMUM OOCs AND MINIMUM REDUCTION RATIO (EVEN WEIGHT).

m	w	α	n	r	minimum reduction ratio
2	4	5	73	6	4 / 6 (66.7%)
2	4	5	97	8	4 / 8 (50.0%)
2	4	3	109	9	4 / 9 (44.4%)
2	4	2	181	15	4 / 15 (26.7%)
2	4	3	229	19	4 / 19 (21.1%)
2	4	7	241	20	4 / 20 (20.0%)
2	4	3	277	23	4 / 23 (17.4%)
2	4	10	337	28	4 / 28 (14.3%)
2	4	2	421	35	4 / 35 (11.4%)
2	4	13	457	38	4 / 38 (10.5%)
2	4	2	541	45	4 / 45 (8.9%)
2	4	2	709	59	4 / 59 (6.8%)
2	4	3	733	61	4 / 61 (6.6%)
2	4	2	757	63	4 / 63 (6.3%)
2	4	2	829	69	4 / 69 (5.8%)
2	4	11	1009	84	4 / 84 (4.8%)
2	4	5	1033	86	4 / 86 (4.7%)
2	4	5	1093	91	4 / 91 (4.4%)
2	4	2	1117	93	4 / 93 (4.3%)
2	4	11	1129	94	4 / 94 (4.3%)
2	4	5	1153	96	4 / 96 (4.2%)
3	6	2	181	6	6 / 6 (100.0%)
3	6	2	211	7	7 / 7 (100.0%)
3	6	7	241	8	8 / 8 (100.0%)
3	6	3	631	21	10 / 21 (47.6%)
3	6	2	691	23	12 / 23 (52.2%)
3	6	2	1531	51	12 / 51 (23.5%)
3	6	2	1831	61	12 / 61 (19.7%)
3	6	2	1861	62	12 / 62 (19.4%)
3	6	6	2791	93	12 / 93 (12.9%)
3	6	2	2851	95	12 / 95 (12.6%)
4	8	11	1009	18	15 / 18 (83.3%)
4	8	3	3137	56	22 / 56 (39.3%)
4	8	5	3697	66	23 / 66 (34.8%)
7	14	5	1093	6	6 / 6 (100.0%)
7	14	2	16381	90	60 / 90 (66.7%)

D. Evaluation of Proposed Design

In this paper, the second approach is discussed shown in Figure 5(b). We refer it as multi-layered BIBD. In this approach, some partial codewords of C_1 is discarded and unused region in the distribution of pulse intervals is created for adding codewords of C_2 . The discarding codewords of C_1 must be carefully selected, because inefficient discarding causes narrow unused pulse intervals in the distribution. To obtain wide regions of unused pulse intervals, all codewords whose pulse intervals are contiguous to a primary discarding codeword should be discarded together. The number of these codewords should be reduced for high efficiency. So, the minimum reduction ratio to original code size of C_1 is evaluated for various BIBD parameters.

TABLE 2. BIBD PARAMETERS FOR OPTIMUM OOCs AND MINIMUM REDUCTION RATIO (ODD WEIGHT).

m	w	α	n	r	minimum reduction ratio
2	5	2	61	3	3 / 3 (100.0%)
2	5	7	241	12	11 / 12 (91.7%)
2	5	3	281	14	10 / 14 (71.4%)
2	5	2	421	21	11 / 21 (52.4%)
2	5	7	601	30	12 / 30 (40.0%)
2	5	3	641	32	12 / 32 (37.5%)
2	5	2	661	33	11 / 33 (33.3%)
2	5	2	701	35	12 / 35 (34.3%)
2	5	2	821	41	12 / 41 (29.3%)
2	5	3	881	44	12 / 44 (27.3%)
2	5	5	1181	59	12 / 59 (20.3%)
2	5	11	1201	60	12 / 60 (20.0%)
2	5	2	1301	65	12 / 65 (18.5%)
2	5	13	1321	66	12 / 66 (18.2%)
2	5	2	1381	69	12 / 69 (17.4%)
2	5	3	1481	74	12 / 74 (16.2%)
2	5	11	1801	90	12 / 90 (13.3%)
2	5	2	1901	95	12 / 95 (12.6%)
2	7	5	337	8	8 / 8 (100.0%)
2	7	2	421	10	10 / 10 (100.0%)
2	7	2	463	11	11 / 11 (100.0%)
3	7	2	883	21	16 / 21 (76.2%)
3	7	2	1723	41	21 / 41 (51.2%)
3	7	2	3067	73	23 / 73 (31.5%)
3	7	2	3319	79	23 / 79 (29.1%)
3	7	17	3529	84	22 / 84 (26.2%)
3	7	2	3823	91	22 / 91 (24.2%)
3	7	2	3907	93	23 / 93 (24.7%)
4	9	5	1153	16	16 / 16 (100.0%)
4	9	10	1873	26	23 / 26 (88.5%)
4	9	5	2017	28	22 / 28 (78.6%)
4	9	7	6481	90	32 / 90 (35.6%)

We assumed a primary codeword is the first codeword of C_1 , with the codeword's index-1 ($i = 1$). This was the best selection in our consideration by computer search. The minimum reduction ratios are also shown in Tables 1 and 2.

For an example, when $m = 2$, $w = 4$, $\alpha = 5$, $n = 73$ and $r = 6$, 4 codewords of C_1 including the primary codeword should be discarded. Thus, pulse intervals of 3 codewords of C_1 are located at contiguous positions of that of the primary codeword. We call these codewords *neighbouring codewords*. When $m = 3$, $w = 6$ and $\alpha = 2$, it was a bad case because all codewords of C_1 should be discarded in order to discard the prime codeword with its neighbouring codewords. We found some other bad cases, however, overall, C_2 can be designed when 10-60% of C_1 is discarded. Even weight codes with $w = 4, 6, 8, 14$ are efficient compared to odd weight codes.

V. CONCLUSIONS

In this paper, the design of optical orthogonal codes with variable chip rates was discussed. Two approaches are described. In the multi-layered BIBD approach, the minimum reduction ratios for various BIBD parameters were obtained. We could discover the possibility of the design of the VC-OOCs by the multi-layered BIBD approach.

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