

Pre-Equalizer Systems with Transmit Equalization Code for Three and Four Transmit Antennas

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Abstract— We introduce space-time block coding (STBC) technique with pre-equalization and new block code design denoted by transmit equalization code (TEC), especially for four transmit antennas case. Proposed code is designed to eliminate interferences between transmit antennas and a receive antenna with great transmission rate. Transmit symbol which is encoded by the new code is equalized at transmitter. The weights for equalizer that consists of only one tap are calculated from fading channel information. Thus, pre-equalization technique can be implemented simply. Effect from the variance between transmitted and received symbol is minimized due to pre-equalization process (PEP) with proposed code design, performance of PEP-TEC applied system has excellent improvement than previous techniques at the same bandwidth efficiency. We prove outstanding simulation result as the bit error rate (BER) measurements to verify performance of the proposed method.

Keywords— Transmit Diversity, Pre-Equalization

I. INTRODUCTION

STBC has been one of the important techniques to improve system performance. In [1], Alamouti proposed STBC for two transmit antennas. Even if it is restricted only two transmit antennas, Alamouti's scheme yields optimal performance for any condition so that it is popular until now. Previous STBC techniques for more than three transmission antennas [2]-[5] usually require additional procedure or redundancy to obtain desired performance, and have restriction for constellation usage. To extend more than two transmit antenna, in [2]-[3], Tarokh et al proposed orthogonal STBC (OSTBC). He provided generalized STBC guideline which has at least half rate and proved that there is no complex-orthogonal STBC design existence with full-rate for more than three antennas through decision metric (DM). Also, he provided some special OSTBC for three and four transmit antenna with rate 3/4. Special-case OSTBC codes are well known as optimal scheme without any restrictions. Another technology for system performance enhancement, PEP applied systems are proposed in [6]-[7]. In the conventional PEP applied systems focused on equalizer design adaptively. Usually adaptively designed systems provide enhanced system performance; however, they require multiple-trial samples for obtaining statistical information as well complex structure.

In this paper, we propose PEP applied multiple transmission system with effective TEC matrix for three and four transmit antennas. TEC matrix is designed to eliminate

all interference components between transmit antennas and a receive antenna for PEP applied systems. Through DM with the TEC and PEP applied environment, symbol can be decoded optimally at the receiver with simple algorithm. Decoding algorithm can be derived from minimization of DM described as squared-error between transmit and received signal. Proposed method has no restriction for constellation selection. It means that the TEC is able to keep good bandwidth efficiency. Especially, proposed scheme has full-rate at four transmit antennas. Also, proposed scheme contributes improvement of BER performance.

This paper is organized as follows. In section II, we simply review optimal condition for STBC technique from decision metric. In section III, we explain proposed system structure and new encoding and decoding algorithm for four transmission antennas. In section IV, we prove proposed algorithm by computer simulation result. Finally we conclude in section V.

II. DECISION METRIC

We consider the configuration of conventional a multiple transmit wireless communication systems for N_T transmit antennas and single receive antenna. We assume that N_R is number of received symbols during one code block decoding period, then dimension of STBC encoding matrix S is $N_R \times N_T$. Received signal y_i at i -th transmission, $i = 1, 2, \dots, N_R$ can be written by

$$y_i = \sum_{j=1}^{N_T} h_j s_{ij} + n_i \quad (1)$$

where s_{ij} is an intersecting component of S at i -th row and j -th column, h_j is flat fading channel coefficient between j -th transmit and receive antenna, n_i is additive white Gaussian noise channel coefficient with zero mean and variance σ_n .

Defining DM is the first step to obtain optimal encoding / decoding algorithm. Based on (1), DM will be simply defined by absolute-squared error (ASE) between transmit and receive signal. To obtain optimal solution, ASE should be minimized as follow.

$$m \hat{u} \sum_{i=1}^{N_R} \left| y_i - \sum_{j=1}^{N_T} h_j s_{ij} \right|^2 \quad (2)$$

After expanding absolute-squared term in (2), DM is decomposed as five components, such as,

$$m \hat{n} \sum_{i=1}^{N_R} \left\{ |y_i|^2 - y_i^* \sum_{j=1}^{N_T} h_j s_{ij} - y_i \sum_{j=1}^{N_T} h_j^* s_{ij}^* + \sum_{j=1}^{N_T} |h_j s_{ij}|^2 + \sum_{j=1}^{N_T} \sum_{\substack{k=1 \\ k \neq j}}^{N_T} h_j h_k^* s_{ij} s_{ik}^* \right\} \quad (3)$$

Final term in (3) has important meaning. The final term means correlations of transmit signals from different wireless channels. Each correlated signal interrupts finding minimized solution for DM so that it must be eliminated. The correlated signal is well known as interference. The optimum solution will be obtained when all interferences are cancelled, therefore DM minimizing condition can be written as follow.

$$\sum_{i=1}^{N_R} \sum_{j=1}^{N_T} \sum_{\substack{k=1 \\ k \neq j}}^{N_T} h_j h_k^* s_{ij} s_{ik}^* = 0 \quad (4)$$

Calculation result for the optimal condition is dependent on the value of elements in encoding matrix which is able to be adjustable at transmitter. Thus, (4) will be useful to verify newly designed encoding matrix mathematically.

After satisfying (4), decoding algorithm is derived straightforwardly from either perfect-square transformation method or differentiation toward desired signal component basically, even though decoding algorithm is dependent on its encoding matrix design. Since two mathematical tools show us the same results, however, decoding algorithm can be designed through convenient way.

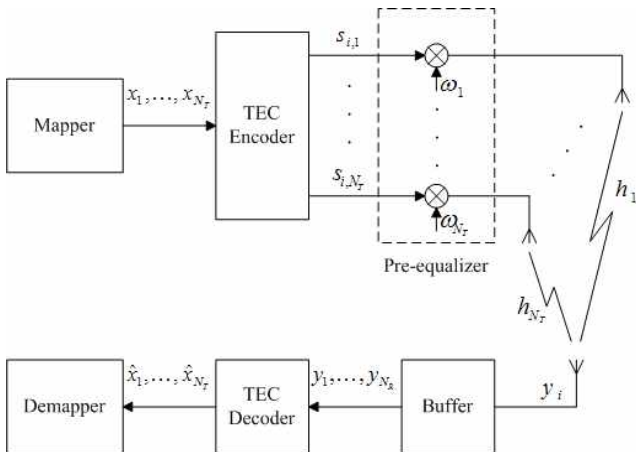


Figure 1. Block diagram for PEP-TEC systems

III. PEP SYSTEM STRUCTURE ON TEC CONFIGURATIONS

In this section, we take into account PEP applied systems with TEC configuration. Encoding and decoding algorithm design for multiple transmission systems are the main objectives, in this paper, especially three and four transmit antennas usage cases.

Block diagram of PEP applied multiple transmission system with TEC configuration is simply shown in Figure 1. Transmitter consists of bit to symbol mapper, TEC encoder, pre-equalizer blocks. Bit to symbol mapper converts bit stream to transmit symbols $x_1, \dots, x_{N_T}$ by baseband modulation such as M-ary PSK or QAM. Transmit symbols are transformed encoded symbols $s_{i,1}, \dots, s_{i,N_T}$ by TEC encoder block. PEP is simply implemented by multiplying encoded symbols of i -th transmission period and its one-tap weights $\omega_1, \dots, \omega_{N_T}$ in pre-equalizer block. Signal passed fading and Gaussian channel is arrived at receiver. Each j -th received signal y_j is stored in buffer until N_R receptions. Received signals in buffer $y_1, \dots, y_{N_R}$ are decoded in TEC decoder which detects estimated symbols $\hat{x}_1, \dots, \hat{x}_{N_T}$. Finally, estimated symbols are converted bit sequence by demapping block.

To optimize TEC system in Figure 1, condition explained as (4) must be satisfied. There are two considerations for optimum system configuration. The first consideration is weight design in PEP block. PEP block helps to make loose condition comparing with (4). To analyze given condition, (4) can be rearranged as

$$\sum_{j=1}^{N_T} \sum_{\substack{k=1 \\ k \neq j}}^{N_T} h_j h_k^* \left(\sum_{i=1}^{N_R} s_{ij} s_{ik}^* \right) = 0 \quad (5)$$

(5) implies that N_R signal components are available to eliminate interference from different j and k -th path. On the other hand, let us assume that the PEP block is able to eliminate h_j and h_k perfectly. Then the equation can be rewritten as

$$\sum_{j=1}^{N_T} \sum_{\substack{k=1 \\ k \neq j}}^{N_T} \sum_{i=1}^{N_R} s_{ij} s_{ik}^* = 0 \quad (6)$$

To satisfy condition of optimal solution, $N_R \times N_T \times (N_T - 1)$ signal components are available in (6). Basically N_T is assumed more than one, so $N_R \times N_T \times (N_T - 1) > N_R$ obviously. It means that (6) has various signal combinations more than conventional condition. Thus, signal design for multiple transmit systems is more convenient due to PEP block. In this paper, we assume that the ideally estimated channel information is available at transmitter. Then, the tap coefficient in PEP block can be implemented simply by given channel information such as

fading channel coefficient. Then the m -th tap coefficient ω_m is defined by $h_m^*/|h_m|^2$ for fading channel coefficient h_m between m -th transmit and receive antenna. Then the interference terms in (4) will be eliminated and simplified as (6).

The second consideration is TEC encoder and decoder block design. From (6), we know that the possibility of satisfying optimal condition is increased due to flexible encoding block design. The proposed matrix-formed TEC encoders are defined by $\mathbf{S}_{\text{TEC},3\times 1}$ and $\mathbf{S}_{\text{TEC},4\times 1}$ for three and four transmit antennas respectively as follow.

$$\mathbf{S}_{\text{TEC},3\times 1} = \begin{bmatrix} x_1 & -x_2 & x_3 \\ x_1 & x_2 & -x_3 \\ x_1 & -x_2 & -x_3 \\ x_1 & x_2 & x_3 \end{bmatrix} \quad (7)$$

$$\mathbf{S}_{\text{TEC},4\times 1} = \begin{bmatrix} x_1 & -x_2 & x_3 & -x_4 \\ x_1 & x_2 & -x_3 & -x_4 \\ x_1 & -x_2 & -x_3 & x_4 \\ x_1 & x_2 & x_3 & x_4 \end{bmatrix} \quad (8)$$

where x_i , $i = 1, 2, 3, 4$ is desired transmission symbol, each i -th row and j -th column element in TEC encoding matrix corresponds to s_{ij} , (7) and (8) have 3/4 and full rate respectively.

The optimal decoding algorithm is simply obtainable from DM. PEP applied TEC encoding matrix for (7) which is denoted by $\mathbf{S}_{\text{PEP-TEC},3\times 1}$ is represented by

$$\mathbf{S}_{\text{PEP-TEC},3\times 1} = \begin{bmatrix} \frac{x_1 h_1^*}{|h_1|^2} & -\frac{x_2 h_2^*}{|h_2|^2} & \frac{x_3 h_3^*}{|h_3|^2} \\ \frac{x_1 h_1^*}{|h_1|^2} & \frac{x_2 h_2^*}{|h_2|^2} & -\frac{x_3 h_3^*}{|h_3|^2} \\ \frac{x_1 h_1^*}{|h_1|^2} & -\frac{x_2 h_2^*}{|h_2|^2} & -\frac{x_3 h_3^*}{|h_3|^2} \\ \frac{x_1 h_1^*}{|h_1|^2} & \frac{x_2 h_2^*}{|h_2|^2} & \frac{x_3 h_3^*}{|h_3|^2} \end{bmatrix} \quad (9)$$

By substituting (9) into (3), we have

$$m \hat{n} \left\{ \sum_{i=1}^4 |y_i|^2 + \sum_{j=1}^3 \left(4|x_j|^2 - x_j \Psi_j^* - x_j^* \Psi_j \right) \right\} \quad (10)$$

where the each boosted symbol Ψ_j , $j = 1, 2, 3$, is defined by

$$\begin{aligned} \Psi_1 &= y_1 + y_2 + y_3 + y_4 \\ \Psi_2 &= -y_1 + y_2 - y_3 + y_4 \\ \Psi_3 &= y_1 - y_2 - y_3 + y_4 \end{aligned} \quad (11)$$

Minimized DM can be obtained by partial differentiation toward each transmitted symbol x_j , $j = 1, 2, 3$, so that (10)

is represented by

$$\frac{\partial}{\partial x_j} \left\{ \sum_{i=1}^4 |y_i|^2 + \sum_{j=1}^3 \left(4|x_j|^2 - x_j \Psi_j^* - x_j^* \Psi_j \right) \right\} = 0 \quad (12)$$

Finally, after arranging (12) for x_j , we have estimated symbol $\hat{x}_i$ as follow.

$$\hat{x}_j = \Psi_j / 4 \quad (13)$$

For (8), the estimation result maintains same format such as (13) even though equations from (10) to (12) are changed slightly due to PEP applied TEC encoding matrix from (8). The estimated symbols for (8) are same as (11) for $j = 1, 2, 3$, and $\Psi_j = y_1 + y_2 - y_3 - y_4$ for $j = 4$.

IV. SIMULATION RESULTS

In this section, BER performance of PEP-TEC system is investigated by simulation result. In this simulation, multiple transmission systems are considered especially three and four transmit antennas and single receive antenna. M-ary QAM modulations, $M = 4, 16, 64$, are utilized and all of modulated symbols are uncoded. Curves for conventional 3×1 and 4×1 STBC matrices in [2], [4] are considered as reference performances at 3/4 and full rate respectively to demonstrate performance enhancement for proposed PEP-TEC system. It is assumed that independent 1e6 encoded blocks are transmitted; single code block includes 4 symbols; fading channel for each code block is static; SNR range is from 0 to 20 dB, 2 dB step.

Decoding algorithm is shown in (13) for encoding blocks (7) and (8). Structure of decoding algorithm is same for the two encoders, so that performance graphs of PEP-TEC applied systems in figure 2, 3 have the same shapes each other due to unchanged decoding algorithm that keeps its variance. However, they have different bandwidth efficiency. Spectral efficiency for (8) is higher than (7) due to its transmission rate. Figure 2 shows BER curves for three transmit antenna with rate 3/4. At the BER value of 10⁻³, PEP-TEC has approximately 6 dB gains comparing with reference graph at 4/16/64-QAM respectively. Similarly, Figure 3 shows BER curves for four transmit antenna with full rate. PEP-TEC has also approximately 7 dB gains comparing with reference graph at all modulation levels. So the proposed PEP-TEC systems have outstanding diversity gains. Especially, slopes of PEP-TEC are more rapidly changed than reference graphs. Therefore, we confirm that PEP-TEC applied systems achieve excellent performance improvement from simulation results.

V. CONCLUSIONS

In this paper, we presented an encoding and decoding method for multiple transmission system, especially three and four transmission antenna usage case. Proposed PEP-TEC

system mitigates interferences at transmitter using pre-equalization block. Through DM and pre-equalization, we designed TEC encoding matrix and derived decoding algorithm. In particular, proposed TEC encoding matrix provides flexible design criterion as well simple decoding algorithm. It has been verified that outstanding performance improvement can be obtained compared to the conventional methods from simulation results.

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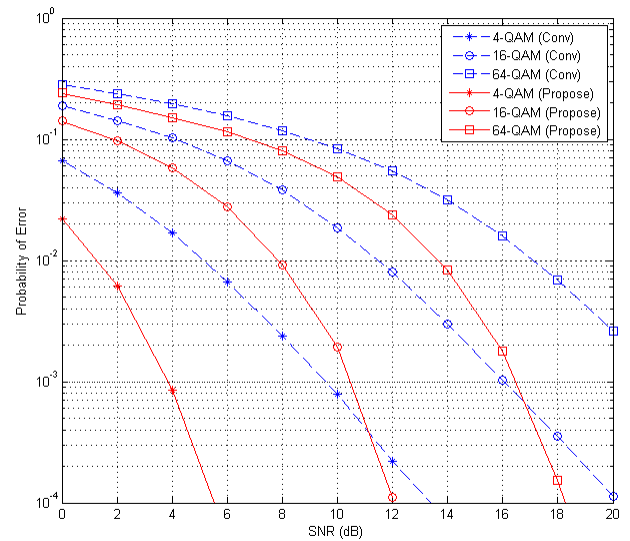


Figure 2. BER performance curves comparison for conventional and proposed multiple transmission system with rate 3/4 for 4/16/64 QAM modulation level

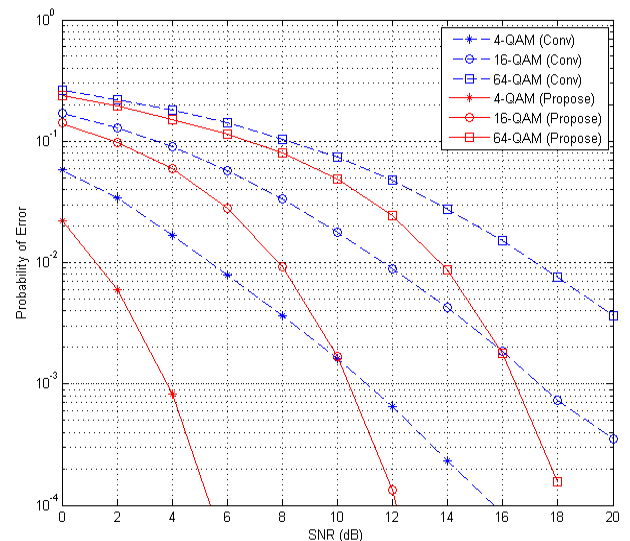


Figure 3. BER performance curves comparison for conventional and proposed multiple transmission system with full rate for 4/16/64 QAM modulation level