

# Outage Performance Analysis of Dual-hop AF Relaying System with Underlay Spectrum Sharing

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**Abstract**—In this paper, we investigate the outage performance of underlay cognitive dual-hop relaying systems under the effect of using statistical instead of instantaneous information of the interference links (from the secondary source and relay to the primary user) and the first relaying link (from the secondary source to relay) to calculate transmit power and amplifying gain for secondary network. We derive the closed-form of outage probability expression for some considered cases and validate them through simulation results. It is numerically demonstrated that using instantaneous information of the first relaying link for calculating the amplifying gain plays a vital role while using interference link statistical information does not strongly affect the system performance. Furthermore, this work shows that the fixed gain specified by averaging the individual random components of the variable gain is not applicable in underlay cognitive dual-hop relaying systems.

**Index Terms**—Amplify-and-forward, fixed gain relaying, outage probability, underlay spectrum sharing, cognitive radio.

## I. INTRODUCTION

Dual-hop relaying system, a special case of multi-hop relaying transmission, has been considered as an efficient technique to extend network coverage and improve the system throughput without using large power at transmitters [1]–[4]. By using a relay between a source and a destination, this technique can reduce the overall path-loss and can be used in the case of a deep fade occurrence on the direct link from the source to the destination. Based on the signal processing at the relay, relaying protocols can be grouped technically into amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) [1], [5]. Among them, AF is the simplest protocol, where the received signals at the relay are amplified and retransmitted towards the destination. The advantage of this protocol is its simplicity and low cost implementation. In AF protocol, the amplifying gain applied at the relay can be specified by using the instantaneous channel information state (CSI) of the first hop, this is denoted as CSI-assisted relay. With this method, the relay produces a variable gain and as a result the power of the retransmitted signal is fixed. Another method to reduce relay complexity by avoiding the continuous channel estimation is semi-blind relaying [6], [7], where the relay can use the channel statistics of the first hop instead of instantaneous CSI. The relay in this method produces a fixed gain and consequently results in a signal with variable power at the relay output. However, the performance

of semi-blind relay systems is slightly degraded compared to that of CSI-assisted relay systems [6].

Besides, cognitive radio (CR) has been considered as another promising technique to improve spectrum utilization by allowing the cognitive radio/secondary users (SUs) to use the spectrum licensed to the primary users (PUs). In designing cognitive wireless networks, there are three approaches including underlay, overlay and interweave approaches [8]. As a comparison between underlay and overlay method, the work in [9] showed that the access opportunities of the SUs in the underlay cognitive radio system can be enhanced considerably, and thus the overall throughput of the secondary users can be improved significantly. In this paper, we adopt to use underlay approach.

In underlay spectrum sharing approach, the SUs and PUs can be permitted to simultaneously realize their transmissions [10]. However, in order to constrain the interference not to exceed a certain level that PUs can tolerate in the underlay mode, SUs must adaptively limit their transmit power, significantly reducing their transmission range. Relaying techniques which take advantage of shorter range communication for lower path loss can complement and overcome the above shortage of underlay cognitive networks. Indeed, a secondary source, instead of directly communicating a distant secondary destination with the high transmit power for reliable reception, can ask another SU in between them to relay its information. As such, the shorter communication range between the secondary source (SS) and the secondary relay (SR), and between the secondary relay and the secondary destination (SD) apparently offers lower path loss for point-to-point communication, requiring lower transmit power (i.e., lower interference level) while still guaranteeing the same accurate data transmission. Recently, the combination of dual-hop relaying and underlay cognitive radio has been received much attention in the research community, i.e. see [11]–[18]. In particular, the works in [11]–[14] investigated the performance of cognitive dual-hop relaying system using DF protocol, while AF protocol is studied in [13], [15], [16], [18]. Other recent works considering the presence of the secondary direct link (between the secondary source to destination), known as cognitive dual-hop cooperative relaying, can be found in [14], [17], [19], [20].

In contrast to the conventional dual-hop CSI-assisted AF relaying systems, where the amplifying gain only depends

on the first relaying link (from the secondary source to the relay), the amplifying gain in underlay cognitive dual-hop AF relaying systems with CSI-assisted relay depends on not only the instantaneous CSI of the first relaying link but also the interference links (from secondary source and relay to primary receiver). The asymptotic outage performance analysis of such systems was derived in [13], [15], while the exact analysis has been recently reported in [16]. To reduce the complexity at the relay, the semi-blind relay approach can be used. However, to our best knowledge, the outage performance of such systems has been not investigated so far. Hence, in this work, we will fill this gap. Furthermore, we also consider some other cases, in which the relay can use statistical information of interference links and instantaneous CSI of relaying link to determine the amplifying gain. The case of using statistical information instead of instantaneous CSI of the interference link to determine the transmit power at the secondary source will be also examined.

The remainder of this paper is organized as follows. In Section II, we introduce the underlay cognitive dual-hop AF relaying system under consideration. In Section III, we derive the closed-form expressions of the end-to-end outage probability for the considered system under many assumptions as introduced. Simulation results and some discussions are presented in Section IV and finally, conclusions drawn in Section V close the paper.

## II. SYSTEM MODEL

We consider the same network as in [16], which is shown in Fig. 1. The secondary network including a SU-Transmitter, a SU-Relay and a SU-Receiver, denoted as  $s$ ,  $r$ , and  $d$ , respectively, coexists with the primary network. Assuming that all nodes operate in half-duplex mode and are equipped with single antenna. Under the underlay approach, the SUs and the PUs will share the same bandwidth and the transmission of SU-nodes should not cause any harmful interference on PUs. With the help of a SU-relay, the communication between SU-Tx and SU-Rx occurs in two time slots. In the first time slot, SU-Tx transmits its signals to SU-Relay. Using AF relaying, the SU-Relay  $r$  amplifies the received signals with a gain  $G$  and forwards the amplified signals to  $d$  in the second time slot.

We denote  $h_{ab}$  as the channel coefficient between  $a \in \{s, r\}$  and  $b \in \{r, d, p\}$ . Under Rayleigh fading channels, the instantaneous channel power  $|h_{ab}|^2$  follows exponential distribution with parameter  $\lambda_{ab}$ , i.e.  $\lambda_{ab} = 1/\mathbb{E}\{|h_{ab}|^2\} = 1/\Omega_{ab}$ .

Assuming that  $s$  transmits signal  $x$  to  $r$  with power of  $P_s$ , the received signal at  $r$  is expressed as  $y_r = h_{sr}x + n_r$ , where  $n_r$  is additive white Gaussian noise (AWGN) at  $r$ . After amplifying the received signals with the gain  $G$ ,  $r$  retransmits the amplified signal to  $d$  with power of  $P_r$ , the signal received at  $d$  is  $y_d = h_{rd}Gh_{sr}x + h_{rd}Gn_r + n_d$ , where  $n_d$  is AWGN<sup>1</sup> at  $d$  and  $G$  is the amplifying gain. The equivalent end-to-end

<sup>1</sup> $n_r$  and  $n_d$  have the same zero mean and variance of  $N_0$ .

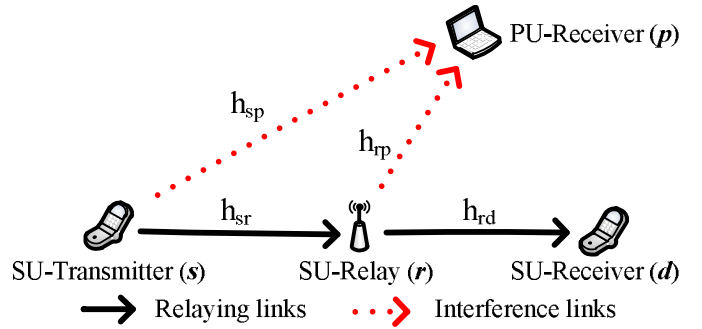


Fig. 1. The dual-hop relaying with underlay spectrum sharing system.

SNR at the secondary destination is expressed as

$$\gamma_{eq} = \frac{G^2|h_{rd}|^2|h_{sr}|^2P_s}{G^2|h_{rd}|^2N_0 + N_0}. \quad (1)$$

To enhance performance of SU-networks, the transmit power of  $s$  and  $r$  are set at  $P_s = \frac{I_p}{|h_{sp}|^2}$  and  $P_r = \frac{I_p}{|h_{rp}|^2}$ , respectively, where  $I_p$  is the maximum tolerable interference power at the PU-Receiver  $p$ . Since  $P_r = G^2(|h_{sr}|^2P_s + N_0)$ , the amplifying gain,  $G$ , can be written as

$$G^2 = \frac{I_p}{N_0} \frac{1}{|h_{rp}|^2} \left( \frac{1}{\frac{I_p}{N_0} \frac{|h_{sr}|^2}{|h_{sp}|^2} + 1} \right). \quad (2)$$

## III. OUTAGE PERFORMANCE ANALYSIS

We start this section by revisiting the end-to-end SNR (1). According to [16], we have

$$\gamma_{eq0} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1}, \quad (3)$$

where  $\gamma_1 = \frac{I_p}{N_0} \frac{|h_{sr}|^2}{|h_{sp}|^2}$  and  $\gamma_2 = \frac{I_p}{N_0} \frac{|h_{rd}|^2}{|h_{rp}|^2}$ . The outage probability of such the system is written as  $P_{out}^0(\gamma_{th}) = F_{\gamma_{eq0}}(\gamma_{th})$ , where  $F_{\gamma_{eq0}}(\gamma)$  is the exact cumulative distribution function (CDF) of  $\gamma_{eq0}$ , which has been recently reported in [16].

Observing (2), we straightforwardly see that instantaneous CSI of  $s \rightarrow p$ ,  $s \rightarrow r$ , and  $r \rightarrow p$  links, i.e.  $h_{sp}$ ,  $h_{sr}$  and  $h_{rp}$ , are needed to calculate the amplifying gain,  $G$ . However, in practice, it is not easy to obtain the instantaneous CSI of such links [8], [10]. A possible solution is to use the average CSI instead of instantaneous CSI. Depending how much CSI we have at the source and the relay, in this work, we investigate four cases, which are described in detail as follows:

1) *Case 1:* The relay uses average channel power of all its related links, i.e.  $s \rightarrow r$  and  $r \rightarrow p$ , to determine the amplifying gain. This case is equivalent to semi-blind relaying in conventional dual-hop relaying systems (i.e fixed gain). Generally, the fixed gain can be obtained by averaging either the overall CSI-assisted variable relay gain [6], [21] or individual random components in the amplifying gain [7], [22]. In this work, we consider the latter. Mathematically, the

fixed gain can be expressed as

$$G_1^2 = \frac{I_p}{N_0} \frac{1}{\Omega_{rp}} \left( \frac{1}{\frac{I_p}{N_0} \frac{\Omega_{sr}}{\Omega_{sp}} + 1} \right). \quad (4)$$

2) *Case 2:* Along with the conditions as in case 1, the source in case 2 uses the average channel power of the link  $s \rightarrow p$  for determining the transmit power, i.e.  $P_s = \frac{I_p}{\mathbb{E}\{|h_{sp}|^2\}} = \frac{I_p}{\Omega_{sp}}$ . Without the need of instantaneous SNRs, the complexity of the source and relay in this case is therefore significantly reduced.

3) *Case 3:* In this case, only average channel powers of interference links, i.e.  $s \rightarrow p$  and  $r \rightarrow p$ , are provided. Such the case is applicable when the primary network does not want to provide CSI of links from secondary transmitters to its receiver or the channel reciprocity property can not be applied [10]. However, instantaneous CSI of the data link ( $s \rightarrow r$ ) is still available at the relay. With these assumptions, we can evaluate the effect of the instantaneous CSI knowledge of the interference/relaying links on the system performance. The amplifying gain in this case can be written as

$$G_3^2 = \frac{I_p}{N_0} \frac{1}{\Omega_{rp}} \left( \frac{1}{\frac{I_p}{N_0} \frac{|h_{sr}|^2}{\Omega_{sp}} + 1} \right). \quad (5)$$

4) *Case 4:* Similar to case 3 and the transmit power at the source is determined as  $P_s = \frac{I_p}{\Omega_{sp}}$  which is identical to the case 2.

Next, we will derive the outage probability expressions for four the proposed cases stated above and show the comparisons among them through numerical results.

#### A. Outage expression for case 1

For this case, the relay will amplify the received signals from the source with the scalar gain given in (4). After plugging (4) into (1), the equivalent SNR at the destination can be written as

$$\gamma_{eq1} = \frac{\gamma_1 \gamma_B}{\gamma_B + c}, \quad (6)$$

where  $c = 1/G_1^2$ ,  $\gamma_B = |h_{rd}|^2$ , and  $\gamma_1$  is given in (3). To derive the system outage probability, the CDF of  $\gamma_{eq1}$  should be known. To do so, we perform the transformation for  $F_{\gamma_{eq1}}(\vartheta)$  as follows:

$$\begin{aligned} F_{\gamma_{eq1}}(\vartheta) &= \Pr \left\{ \frac{\gamma_1 \gamma_B}{\gamma_B + c} < \vartheta \right\} \\ &= \int_0^\infty \Pr \left\{ \frac{\gamma_1 y}{y + c} < \vartheta \right\} f_{\gamma_B}(y) dy \\ &= \int_0^\infty F_{\gamma_1} \left( \left[ 1 + \frac{c}{y} \right] \vartheta \right) f_{\gamma_B}(y) dy. \end{aligned} \quad (7)$$

In (7), the CDF of  $\gamma_1$  is easily derived as  $F_{\gamma_1}(\gamma) = \frac{\lambda_1 \gamma}{1 + \lambda_1 \gamma}$  and the PDF of  $\gamma_B$  is expressed as  $f_{\gamma_B}(\gamma) = \lambda_{rd} e^{-\lambda_{rd} \gamma}$ , where  $\lambda_{rd} = 1/\mathbb{E}\{|h_{rd}|^2\} = 1/\Omega_{rd}$ . For simplicity, we define

$F_{\gamma_1}^{(c)}(\gamma) = 1 - F_{\gamma_1}(\gamma) = \frac{1}{1 + \lambda_1 \gamma}$  as the complementary CDF of  $\gamma_1$ . (7) can be further simplified as

$$F_{\gamma_{eq1}}(\vartheta) = 1 - \int_0^\infty F_{\gamma_1}^{(c)} \left( \left[ 1 + \frac{c}{y} \right] \vartheta \right) f_{\gamma_B}(y) dy. \quad (8)$$

After some algebraic manipulations and with the help of [23, eq. 3.352 .4], we arrive at

$$\begin{aligned} F_{\gamma_{eq1}}(\vartheta) &= \frac{\vartheta \lambda_1}{1 + \vartheta \lambda_1} + \frac{c \vartheta \lambda_1 \lambda_{rd}}{(1 + \vartheta \lambda_1)^2} \exp \left( \frac{c \vartheta \lambda_1 \lambda_{rd}}{1 + \vartheta \lambda_1} \right) \\ &\quad \times \Gamma \left( 0, \frac{c \vartheta \lambda_1 \lambda_{rd}}{1 + \vartheta \lambda_1} \right), \end{aligned} \quad (9)$$

where  $\Gamma(\cdot, \cdot)$  is the upper incomplete Gamma function given by [24, eq. 8.4.4]. Hence, the outage probability of this case is determined through the CDF of  $\gamma_{eq1}$  as  $P_{out}^1(\gamma_{th}) = F_{\gamma_{eq1}}(\gamma_{th})$ .

#### B. Outage expression for case 2

When the amplifying gain is determined as in (4) and  $P_s = I_p/\Omega_{sp}$ , the end-to-end SNR is determined as follows:

$$\gamma_{eq2} = \frac{\gamma_{sr} \gamma_B}{\gamma_B + c}, \quad (10)$$

where  $\gamma_{sr} = \frac{I_p}{N_0} \frac{|h_{sr}|^2}{\Omega_{sp}}$ . Following the same steps as from (7) to (8), the CDF of  $\gamma_{eq2}$  can be expressed as

$$F_{\gamma_{eq2}}(\vartheta) = 1 - \int_0^\infty F_{\gamma_{sr}}^{(c)} \left( \left[ 1 + \frac{c}{y} \right] \vartheta \right) f_{\gamma_B}(y) dy. \quad (11)$$

The CDF of  $\gamma_{sr}$  is of the form  $F_{\gamma_{sr}}(\gamma) = 1 - \exp(-\beta_{sr} \gamma)$ , where  $\beta_{sr} = \frac{N_0}{I_p} \lambda_{sr} \Omega_{sp}$ . Using the relationship of  $F_{\gamma_{sr}}(\gamma) + F_{\gamma_{sr}}^{(c)}(\gamma) = 1$ , the complementary CDF of  $\gamma_{sr}$  is written as  $F_{\gamma_{sr}}^{(c)}(\gamma) = \exp(-\beta_{sr} \gamma)$ . Putting  $F_{\gamma_{sr}}^{(c)}(\gamma)$  and  $f_{\gamma_B}(\gamma)$  into (11), we get the closed-form expression for  $F_{\gamma_{eq2}}(\vartheta)$  after using [23, eq. 3.471 .9] as

$$\begin{aligned} F_{\gamma_{eq2}}(\vartheta) &= 1 - 2\sqrt{c \vartheta \beta_{sr} \lambda_{rd}} \exp(-\beta_{sr} \vartheta) \\ &\quad \times K_1 \left( 2\sqrt{c \vartheta \beta_{sr} \lambda_{rd}} \right), \end{aligned} \quad (12)$$

where  $K_1(\cdot)$  denotes the first order modified Bessel function of the second kind defined in [24, eq. 10.25.3]. Having the closed-form expression of the CDF of  $\gamma_{eq2}$  in hands, we easily obtain the closed-form expression of the system outage probability for this case as  $P_{out}^2(\gamma_{th}) = F_{\gamma_{eq2}}(\gamma_{th})$ .

#### C. Outage expression for case 3

For this case, the amplifying gain is given in (5). Plugging this into (1), we have

$$\gamma_{eq3} = \frac{\frac{I_p}{N_0} \frac{|h_{sr}|^2}{\Omega_{sp}} \frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}}}{\frac{I_p}{N_0} \frac{|h_{sr}|^2}{\Omega_{sp}} + \frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}} + 1}. \quad (13)$$

There are some ways to calculate the CDF of  $\gamma_{eq3}$  in (13) with the note that the terms  $\frac{I_p}{N_0} \frac{|h_{sr}|^2}{\Omega_{sp}}$  and  $\frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}}$  have the common quantity of  $|h_{sr}|^2$ , therefore they will not independent

and the conventional approach that divides (13) into  $\frac{I_p}{N_0} \frac{|h_{sr}|^2}{|h_{sp}|^2}$ ,  $\frac{I_p}{N_0} \frac{|h_{sr}|^2}{\Omega_{sp}}$  and  $\frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}}$  and treat them as independent random variables cannot be applied. Some solutions are averaging them over  $|h_{sr}|^2$  or  $|h_{sp}|^2$ , or dividing (13) into three independent random variables as  $\frac{I_p}{N_0} \frac{|h_{sr}|^2}{\Omega_{sp}}$ ,  $\frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}}$  and  $\frac{\Omega_{sp}}{|h_{sp}|^2}$ . Unfortunately, these ways cannot lead the CDF of (13) to the closed-form expression. Herein, we can obtain the closed-form expression in terms of very tight approximation by doing a simple transformation of (13) as

$$\gamma_{eq3} = \frac{\frac{I_p}{N_0} \frac{|h_{sr}|^2}{|h_{sp}|^2} \frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}}}{\frac{I_p}{N_0} \frac{|h_{sr}|^2}{|h_{sp}|^2} \frac{|h_{sp}|^2}{\Omega_{sp}} + \frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}} + 1}. \quad (14)$$

Denoting  $\gamma_{rd} = \frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}}$ ,  $\gamma_{sp} = \frac{|h_{sp}|^2}{\Omega_{sp}}$  and  $\gamma_1$  given in (3), (14) can be rewritten as

$$\gamma_{eq3} = \frac{\gamma_1 \gamma_{rd}}{\gamma_1 \gamma_{sp} + \gamma_{rd} + 1}. \quad (15)$$

Observing  $\gamma_1$  and  $\gamma_{sp}$ , we can see that they have the common term  $|h_{sp}|^2$ , leading to the dependence between them. Due to the small independence between  $\gamma_1$  and  $\gamma_{sp}$ , we can treat them as independent random variables. As a results, the CDF of  $\gamma_{eq3}$  can be approximately as

$$\begin{aligned} F_{\gamma_{eq3}}(\vartheta) &= \int_0^\infty \int_0^\vartheta \overbrace{\Pr \left\{ \gamma_{rd} > \frac{\gamma_1 \gamma_{sp} \vartheta + \vartheta}{\gamma_1 - \vartheta} \right\}}^{=1} \\ &\quad \times f_{\gamma_1}(\gamma_1) f_{\gamma_{sp}}(\gamma_{sp}) d\gamma_1 d\gamma_{sp} \\ &\quad + \int_0^\infty \int_\vartheta^\infty \Pr \left\{ \gamma_{rd} \leq \frac{\gamma_1 \gamma_{sp} \vartheta + \vartheta}{\gamma_1 - \vartheta} \right\} \\ &\quad \times f_{\gamma_1}(\gamma_1) f_{\gamma_{sp}}(\gamma_{sp}) d\gamma_1 d\gamma_{sp}. \end{aligned} \quad (16)$$

After some algebraic manipulations and changing variable, i.e.  $t \rightarrow (\gamma_1 - \vartheta)$ , (16) is rewritten as

$$\begin{aligned} F_{\gamma_{eq3}}(\vartheta) &= 1 - \int_0^\infty \int_0^\infty \Pr \left\{ \gamma_{rd} \geq \frac{\gamma_{sp}(t + \vartheta)\vartheta + \vartheta}{t} \right\} \\ &\quad \times f_{\gamma_1}(t + \vartheta) f_{\gamma_{sp}}(\gamma_{sp}) d\gamma_{sp} dt \\ &= 1 - \int_0^\infty \int_0^\infty F_{\gamma_{rd}}^{(c)} \left( \frac{\gamma_{sp}(t + \vartheta)\vartheta + \vartheta}{t} \right) \\ &\quad \times f_{\gamma_1}(t + \vartheta) f_{\gamma_{sp}}(\gamma_{sp}) d\gamma_{sp} dt. \end{aligned} \quad (17)$$

To solve (17), we first need the CDF of  $\gamma_{rd}$  and the PDF of  $\gamma_1$  and  $\gamma_{sp}$ . The PDF of  $\gamma_1$  can be obtained by taking derivation  $F_{\gamma_{sr}}(\gamma)$  with respect to  $\gamma$ , which is given by [14]

$$f_{\gamma_1}(\gamma) = \frac{\lambda_1}{(1 + \lambda_1 \gamma)^2}. \quad (18)$$

Since  $|h_{rd}|^2$  is an exponential random variable with parameter  $\lambda_{rd}$ , the CDF of  $\gamma_{rd}$  is  $F_{\gamma_{rd}}(\gamma) = 1 - \exp(-\beta_{rd}\gamma)$ , where

$\beta_{rd} = \frac{N_0}{I_p} \Omega_{rp} \lambda_{rd}$ . So the complementary CDF of  $\gamma_{rd}$  is written as  $F_{\gamma_{rd}}^{(c)}(\gamma) = \exp(-\beta_{rd}\gamma)$ . Similarly, the PDF of  $\gamma_{sp}$  is easily expressed as  $f_{\gamma_{sp}}(\gamma) = \exp(-\gamma)$ . Substituting these results into (17) yields

$$\begin{aligned} F_{\gamma_{eq3}}(\vartheta) &= 1 - \int_0^\infty \int_0^\infty \exp \left( -\beta_{rd} \frac{(t + \vartheta)\vartheta + \vartheta}{t} \right) \\ &\quad \times \frac{\lambda_1}{(1 + \lambda_1(t + \vartheta))^2} e^{-\gamma} d\gamma dt. \end{aligned} \quad (19)$$

After solving the inner integral with the help of [23, eq. 3.310], (19) can be rewritten as

$$\begin{aligned} F_{\gamma_{eq3}}(\vartheta) &= 1 - \int_0^\infty \lambda_1 \exp \left( -\frac{\beta_{rd}\vartheta}{t} \right) \\ &\quad \times \underbrace{\frac{1}{[(1 + \beta_{rd}\vartheta)t + \beta_{rd}\vartheta^2](\lambda_1 t + 1 + \vartheta\lambda_1)^2}}_{A(t)} dt. \end{aligned} \quad (20)$$

Now, using the partial-fraction expansionthe for  $A(t)$  in (20), we get the desired result, which is expressed as

$$\begin{aligned} F_{\gamma_{eq3}}(\vartheta) &= 1 - \frac{(1 + \vartheta\lambda_1)}{(1 + \vartheta\beta_{rd} + \vartheta\lambda_1)} I_1 \\ &\quad + \frac{\lambda_1 \beta_{rd} \vartheta^2}{(1 + \vartheta\beta_{rd} + \vartheta\lambda_1)^2} (I_2 - I_3), \end{aligned} \quad (21)$$

where  $I_1$ ,  $I_2$ , and  $I_3$  are defined respectively as follows:

$$I_1 = \int_0^\infty \exp \left( -\frac{\vartheta\beta_{rd}}{t} \right) \frac{1}{\left( t + \frac{\vartheta\lambda_1 + 1}{\lambda_1} \right)^2} dt, \quad (22)$$

$$I_2 = \int_0^\infty \exp \left( -\frac{\vartheta\beta_{rd}}{t} \right) \frac{1}{t + \frac{\beta_{rd}\vartheta^2}{1 + \beta_{rd}\vartheta}} dt \quad (23)$$

and

$$I_3 = \int_0^\infty \exp \left( -\frac{\vartheta\beta_{rd}}{t} \right) \frac{1}{t + \frac{\vartheta\lambda_1 + 1}{\lambda_1}} dt. \quad (24)$$

The integral  $I_1$  can be solved in closed-form by first applying the change of variable, i.e.  $t \rightarrow 1/x$ , and then using [23, eq. 3.353.2]. With  $I_2$  and  $I_3$ , their closed-form expression can also obtained by employing the same technique (with the help of [23, eq. 3.352.4]). Plugging the resulting expressions into (21) and after some manipulations, we arrive at the derised result in (25) shown at the top of the next pape. The outage probability expression can be readily obtained by  $P_{out}^3(\gamma_{th}) = F_{\gamma_{eq3}}(\gamma_{th})$ .

#### D. Outage expression for case 4

Substituting (5) and  $P_s = \frac{I_p}{\Omega_{sp}}$  into (1), we have

$$\gamma_{eq4} = \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{sr} + \gamma_{rd} + 1}, \quad (26)$$

where  $\gamma_{sr}$  and  $\gamma_{rd}$  are given in (10) and (15), respectively. Following the same steps as from (7) to (8) and solving the

$$F_{\gamma_{eq3}}(\vartheta) = 1 - \frac{(\vartheta\lambda_1 + 1)}{(1 + \vartheta[\beta_{rd} + \lambda_1])\lambda_1} \left\{ \frac{\lambda_1}{\vartheta\lambda_1 + 1} - \frac{\beta_{rd}\vartheta\lambda_1^2}{(\vartheta\lambda_1 + 1)^2} \exp\left(\frac{\beta_{rd}\vartheta\lambda_1}{\vartheta\lambda_1 + 1}\right) \Gamma\left(0, \frac{\beta_{rd}\vartheta\lambda_1}{\vartheta\lambda_1 + 1}\right) \right\} \\ + \frac{\beta_{rd}\vartheta^2\lambda_1}{(1 + \vartheta[\beta_{rd} + \lambda_1])^2} \left\{ \exp\left(\frac{\beta_{rd}\vartheta\lambda_1}{\vartheta\lambda_1 + 1}\right) \Gamma\left(0, \frac{\beta_{rd}\vartheta\lambda_1}{\vartheta\lambda_1 + 1}\right) - \exp\left(\beta_{rd} + \frac{1}{\vartheta}\right) \Gamma\left(0, \beta_{rd} + \frac{1}{\vartheta}\right) \right\}. \quad (25)$$

integral with the help of [23, eq. 3.471.9], the CDF of  $\gamma_{eq4}$  can be written as

$$F_{\gamma_{eq4}}(\vartheta) = 1 - 2\sqrt{\beta_{sr}\beta_{rd}\vartheta(1+\vartheta)} \exp(-[\beta_{rd} + \beta_{sr}]\vartheta) \\ \times K_1\left(2\sqrt{\beta_{sr}\beta_{rd}\vartheta(1+\vartheta)}\right), \quad (27)$$

where  $K_1(\cdot)$  denotes the first order modified Bessel function of the second kind. Finally, the outage expression is readily obtained from (27) as  $P_{out}^4(\gamma_{th}) = F_{\gamma_{eq4}}(\gamma_{th})$ .

#### IV. NUMERICAL RESULTS AND DISCUSSION

To investigate the outage behavior of the considered systems, we follow the same network configuration as in [16]. In particular, the SUs and PU-Receiver are arranged into a 2D plane, in which  $s$ ,  $r$  and  $d$  are located at co-ordinates (0,0) and (0,0.5) and (0,1), respectively. The location of  $p$  can be changed to examine the effect of PU on SU-networks. Taking into account the pathloss effect, the average channel power is modeled as  $\Omega_{ab} = 1/D_{ab}^\epsilon$ , where  $D_{ab}$  is the distance between  $a$  and  $b$ , and  $\epsilon$  denotes the path loss exponent, which is set to be 4 in all the numerical results. As benchmarks for comparisons, the plots of outage probability for the considered system with variable gain and direct transmission system are also included and considered.

We first validate the approximation analysis of the case 3 in Fig 2. It is clearly seen that the approximation results are very close to the simulation ones. Only a small gap between the analysis and simulation curves appears at low regime of  $I_p/N_0$ . Besides, the effects of the PU-Receiver positions on SU-networks are also illustrated. It is shown that among three locations of PU-Receiver, the SU-networks achieves the best performance when PU-Receiver is located at (0.66, 0.66). It can be explained by using the fact that with coordinate (0.66, 0.66), the PU-Receiver is farthest away from the SUs, resulting in the transmit power of SUs (the source and the relay) may be maximized. Also in Fig. 2, we prove that the conventional approach considering three quantities  $\frac{I_p}{N_0} \frac{|h_{sr}|^2}{|h_{sp}|^2}$ ,  $\frac{I_p}{N_0} \frac{|h_{sr}|^2}{\Omega_{sp}}$  and  $\frac{I_p}{N_0} \frac{|h_{rd}|^2}{\Omega_{rp}}$  as independent random variables as in [12] cannot be applied for calculating the CDF of  $\gamma_{eq3}$  given in (13).

In Fig. 3, the outage performance of SU-networks under four considered cases are shown and a closer view at low  $I_p/N_0$  regime is further provided in Fig. 4. There is a slight performance loss of the case 2 compared to case 1 as well as case 4 compared to case 3. This shown that the use of the mean channel powers instead of instantaneous CSI of the link  $s \rightarrow p$  to determine the source transmit power does not give much impact on the system performance. It should be noted that the use of mean power  $\Omega_{rp}$  in determining the

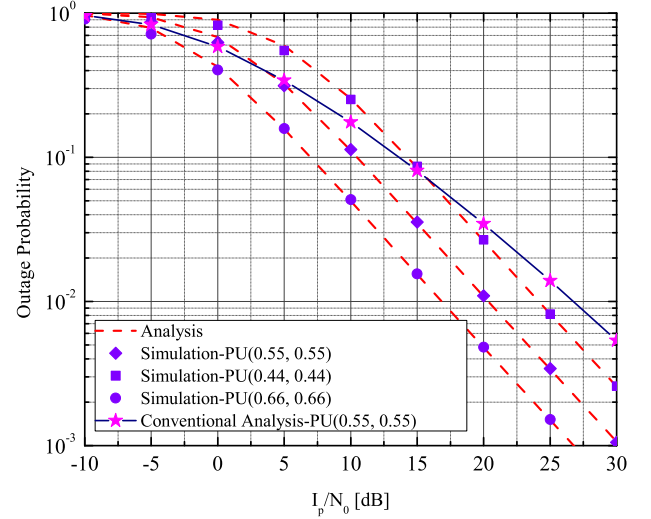


Fig. 2. The validation of approximation analysis in case 3 with  $\gamma_{th} = 1$  dB.

amplifying gain in four investigated cases is equivalent to the use of that to specify the transmit power at the relay. So, by comparing the outage probability curves of the case 3 and 4 to the case of using variable gain, it can be concluded that the using statistical information of the interference links (from the secondary source and relay to PU-Receiver) for determining the transmit power of SUs as well as the amplifying gain at the relay does not impact substantially to the system. Through the comparisons between the cases 1 and 2 with the cases 3 and 4, we conclude that the instantaneous CSI of the first relaying link (from the secondary source to relay) plays a vital role in specifying the amplifying gain. As shown in these figures, using mean power of the first relaying link to calculate the fixed gain given in (4) causes a significant performance loss. It is also shown that at high value of  $I_p/N_0$  (high maximum tolerable interference at PU), the system performance as using fixed gain in (4) is even inferior to the direct transmission scenario while at lower  $I_p/N_0$  value range, both the cases are almost identical. So the fixed gain calculated by averaging the individual random components of the variable gain in (4) is not applicable for underlay cognitive dual-hop relaying system.

#### V. CONCLUSION

We have studied the outage performance of cognitive underlay dual-hop relaying systems. The effects of using average channel power instead of instantaneous CSI on the network performance are also investigated. Based on the numerical

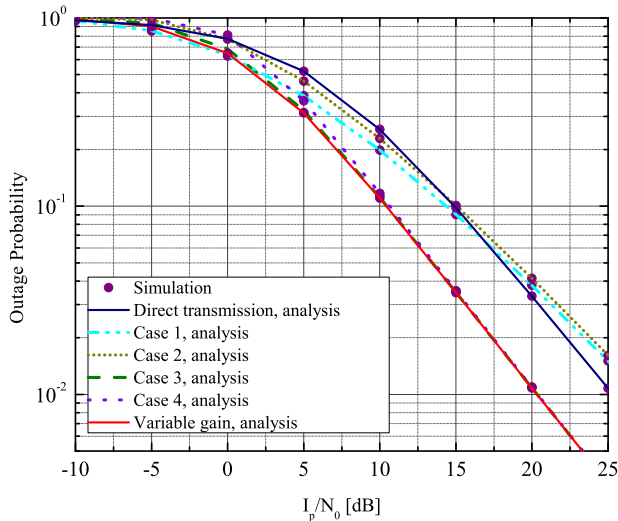


Fig. 3. Comparison of outage performance among considered cases with PU-Receiver at (0.55, 0.55) and  $\gamma_{th} = 1$  dB.

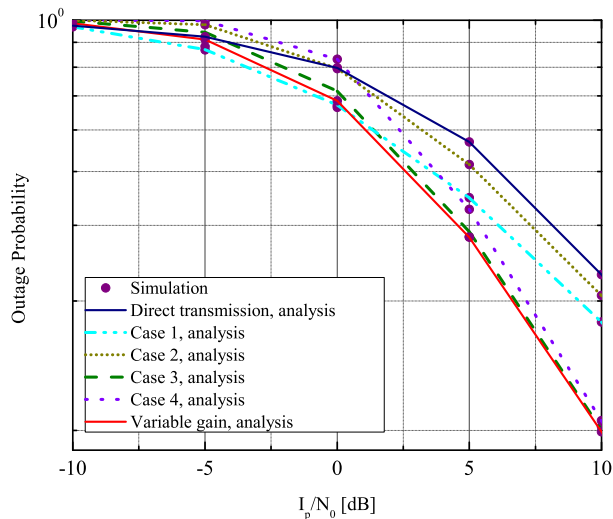


Fig. 4. A closer view of Fig. 3 in low value range of  $I_p/N_0$ .

results, there are three observations worth noting: a) The use of instantaneous CSI for the data link, i.e.  $s \rightarrow r$ , in determining the amplifying gain will give about 5 dB gain over the fixed gain case, b) With interference links, the use of average CSI does not affect much on the secondary network performance and c) The simulation results are in excellent agreement with the analysis results confirming the correctness of the analysis approach proposed in the paper.

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