

Topological Importance as an Incentive to Cooperate in Mobile Ad Hoc Networks: A Game Theoretic Analysis

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Abstract—Packet forwarding in a multihop wireless network is always prone to jeopardy by the selfish behavior of participating nodes striving to save scarce energy resources. There are many mechanisms which offer appealing incentives to a node in order to instigate cooperation. These incentives are rather unnatural or in some cases the nodes are allured to adapt a tempered proof policy. In this paper, a natural incentive is explored which is applicable to a much general class of networks rather than arrangements that exhibit specific topology configurations. The proposed incentive is associated with the topological strategic position of the source and relay nodes located in the network. It is further argued that an appropriate strength of penalty of betraying can substantially improve the effectiveness of the proposed incentive, even at very low levels of strategic importance.

Keywords—Routing, Game Theory, Mobile Ad hoc Networks, Bayesian Equilibrium, Strategic Importance.

I. INTRODUCTION

Mobile ad hoc networks (MANETs) are visioned as the next generation of communication and sensor networks [1], [2]. The organization of these networks is relatively different from the legacy and prevailing communication networks. There are no stringent infrastructure requirements and these networks are operated and managed without a central governing authority. However, the absence of these elements also opens up a window of new possibilities such as wider coverage, high data rates, peer to peer streaming of data, and sensing physical phenomena from unreachable terrains [3]. In order to turn these ambitions into a reality, there are few challenges that need to be addressed. The fundamental issue to be resolved is the selfishness of autonomous terminals [4]. Every node plays a vital role in the operation and management of the network. The functionality of the network depends upon the individual decisions of the nodes. Since the energy and bandwidth consumption in a network are influenced by the amount of traffic relayed by the nodes, the limited supply of these resources causes a threat which obstructs the altruistic relaying mechanism in the network. Unfortunately, this selfish act is considered as a rational behavior unless any incentive or profit is copped by relaying. There are many methods which make relaying attractive and useful for the nodes, but it requires strong encrypted, temper proof hardware modules to ensure integrity of these mechanisms [5], [8].

In this paper, we formulate a self enforced incentive for relaying in MANETs. A node relays traffic for those peers,

which are of any importance to itself, given that the relaying needs are fulfilled. To quantify this sense of requirement, topological positions can serve as a metric of importance.

Most studies of cooperation rather abstract away the topological influence or consider a fixed configuration [11], [4]. Contrary to both, this paper investigates a general network topological arrangement and exploit this to formulate an incentive. The behavior of the nodes in MANETs resembles social characteristics and game theory turns out to be an instinctive choice to model and analyze these types of systems.

The Section II of the paper presents previous work related to incentive based cooperation schemes. In Section III, the network model is described. The game theoretic model of the network is formulated and analyzed in Section IV. The results are discussed in Section V, and finally, Section VI concludes the paper.

II. RELATED WORK

Enforcing cooperation in ad hoc networks is an attractive topic of research for quite some time [11], [12]. There are several interesting solutions which induces cooperation in MANETs. These cooperation mechanisms can be divided into two broad categories which are based on reputation and credit.

A. Reputation Based Incentives

These mechanisms relate the desire of a node to relay traffic with its reputation. The incentive for a node is to keep relaying an estimated amount of traffic in order to maintain its reputation to an acceptable level [8]. The relay of traffic from a node with high reputation is facilitated by other nodes by virtue of its past behavior, and on the other hand, nodes with poor reputation are isolated from the network participation. In order to achieve the reputation based forwarding methodology, the foremost requirement is the effective implementation of a truthful and cheat proof mechanism. The information pertaining to the reputation is managed locally by each entity, therefore, there is a strong appetite among nodes to cheat the system rather than cooperating honestly [8], [9], [10].

B. Credit Based Incentives

These incentives are influenced by marketing techniques of selling and buying commodities. In most circumstances, a virtual currency (nuglets) is used to offer/use relaying

services [5]. The credit based implementation suffers from the same limitations as reputation based incentives. A cheat proof credit exchange system is an essential requirement for these mechanisms which can only be realized by a cheat proof hardware [5], [6], [7].

III. NETWORK MODEL

In MANETs, the entities rely on the assistance provided by the rest of the nodes so that the connectivity of a network is sustained (see Figure 1). In this paper, a game theoretic solution is proposed and analyzed for a multihop multipath network. Every node in the network generates its own traffic and relay data of its peers, simultaneously.

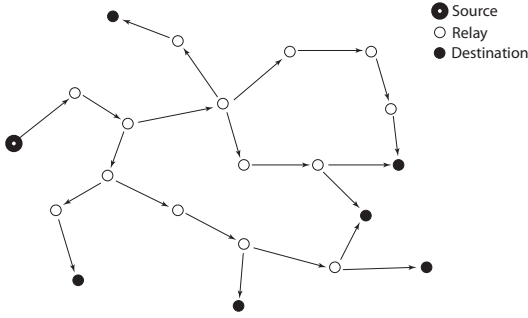


Figure 1. Multihop multipath ad hoc network model

This work uses Multipath Dynamic Source Routing (MP-DSR) protocol for path discovery and maintenance. In MP-DSR, the source node has control over the selection of the transmission path [13]. The source can select a path with maximum power efficiency or Signal to Interference and Noise Ratio (SINR), but these metrics are plausible only if source is confident enough about the packet transmission probability [14]. Hence, a more influential metric of *path reliability* must be used when taking strategic decision of path selection.

Definition 1. *Path reliability* $r_s(d)$ is the product of probabilities of relaying by each entity in a given path from source s to a destination d ,

$$r_s(d) \equiv \prod_{l=1, l \in \text{path}(d)}^d \text{Prob}(\text{node } l \text{ relays}). \quad (1)$$

In considering the reliability for path selection, the selected route may not be optimal in terms of energy consumption or SINR considerations, but the packet loss rate is minimum.

Definition 2. *Intermediate path reliability (IPR)* $r_S^j(d)$ at relay j is the product of probabilities of relaying of each participating node ahead of j in the path from the source node s to the destination d ,

$$r_S^j(d) \equiv \prod_{l=j+1, l \in \text{path}(d)}^d \text{Prob}(\text{node } l \text{ relays}). \quad (2)$$

Where IPR at a destination is defined to be one, i.e., $r_S^d(d) \equiv 1$. The IPRs determine the weight a relay accumulates in a path. The source can be indifferent about the relay decisions of the nodes having very small IPRs in a route. Even if they relay, there is a very minute chance of successful packet delivery. The multipath nature of the network further discriminates between the relaying nodes positioned at different locations in the network.

Definition 3. *Strategic position factor (SPF)* $\beta_s^j(d)$ of a relay j is the ratio of number of paths through j to the destination d , by the total number of possible paths to the destination d for the source s ,

$$\beta_s^j(d) \equiv \frac{\text{Number of paths through } j \text{ to destination } d}{\text{Total paths available to destination } d}. \quad (3)$$

In the light of above definitions, we define the importance of a node for its peers which may cooperate in creating or sustaining a path for a source-destination pair.

Definition 4. *Strategic position importance (SPI)* $\theta_s^j(d)$ is the product of SPF and IPR of a relay j for source s to the destination d ,

$$\theta_s^j(d) \equiv \beta_s^j(d) r_S^j(d). \quad (4)$$

In essence, a particular node can be a relay to multiple destinations for a source.

Definition 5. *Total strategic position importance (TSPI)* of a relay j for source s is the sum of entire SPIs of the relay for all destinations D_S^j of s reached via j ,

$$\Theta_S^j \equiv \sum_{\forall d \in D_S^j} \theta_s^j(d). \quad (5)$$

The TSPI functions as an incentive for nodes to relay. In the proposed network model, every entity transmits within a limited power sphere of energy e , which is assumed to be fixed for each node in the network.

Similar to the energy loss as a consequence of relaying traffic at each node, there is an energy saving gain g per successful multihop packet transmission. For simplicity of the model, we consider the value of g equal for each entity and transmission session. This simplification can be justified if each node has to reach a destination with almost equal number of hops.

Definition 6. *Intermediate gain (Ig)* g_s^j is the gain that a source s speculates at any relay j for all its destinations reachable by j , is the product of TSPI and total path gain g ,

$$g_s^j \equiv \sum_{\forall d \in D_S^j} g \theta_s^j(d) = g \Theta_S^j. \quad (6)$$

In MANETs, the nodes can join or leave the network at will, in addition, their movement is not restricted which results in the change in topology as the time progresses.

Definition 7. The measure of the dynamic behavior of the network (δ) is the probability of arriving at same topological/strategic situation in next communication session/period between the same set of nodes.

Highly dynamic networks, such as Vehicular Ad hoc Networks (VANETs), have very small δ , whereas, static networks which include Wireless Sensor Networks (WSNs), have higher values of δ .

This concludes our discussion on the network model used in this work. In the next section, we formally define a game model, which incorporates the TSPIs of the entities in an effort to induce cooperation.

IV. GAME MODEL OF THE NETWORK

Game theory started as a branch of applied mathematics to formulate problems in economics, politics and sociology. In recent years it has been widely used in communication systems, especially for analysis of MANETs. Elements which constitute a game are the players, set of strategies, and their corresponding payoffs [15]. In the proposed network model; players are the nodes, strategy set is $\Omega = \{relay, drop\}$, and the payoff resulting from these set of strategies is the energy savings.

When a node gets a relaying request from another entity, a game starts between these two nodes. We assume that the nodes in a network are synchronized so a *simultaneous decision repeated game* is played between the participating entities. A game can be played in two phases, normal phase or a punishment phase. In the normal phase of the game, the entities make a strategic decision of relaying. On the other hand, when playing in punishment phase, both entities restrain from relaying, because the punishing node acts to punish while the rational response of the entity being punished is also not to relay.

If the game is in the normal phase and any entity drops the packet, then a punishment phase would be triggered for next N periods. After the punishment phase is over, a normal phase resumes. Figure 2 shows a sample of simultaneous game setup between three nodes interacting in pairs. Every strategy $\omega \in \Omega$ has a probability associated with it. Any entity j has a relaying probability z_j as,

$$z_j \equiv \text{Prob}(\omega_j = \text{relay}). \quad (7)$$

In the following sections, without the loss of generality, we consider the game between a pair of i and j entities. Both the nodes are playing the game in the normal phase.

In this game, the nodes play to maximize their individual payoffs, however, if any node drops a packet, the connection is lost from the opponent up to N punishment periods. The future connection consideration involves TSPIs as an incentive.

The transmission gain g , transmission cost e , and dynamic measure δ of the network are *common knowledge* in this game. The payoffs of each entity depends upon the choice of her strategy. In the proposed formulation, we use a little deviation from the standard notation and denote the $|D_x^y|$ by D_x^y . The

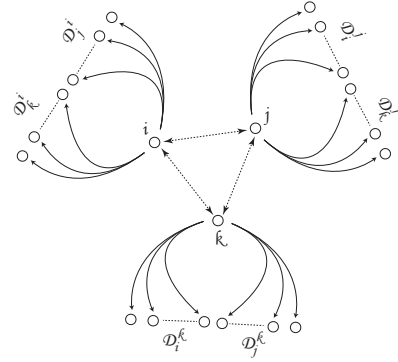


Figure 2. Game between three entities

formulation of the payoffs is broken down into two parts, that is, by deviating (or dropping packets) and by cooperating (or relaying packets).

A. Payoff by Deviation

If node i deviates, it accumulates a certain amount of gain by virtue of not relaying packets of the entity j , not only for the current period but for entire punishment phase plus a probabilistic gain from j 's cooperation. Since there is δ chance of getting the same scenario again in the next period; this makes the speculation as a discounted sum. The total gain of a node which chooses to deviate is given as,

$$G_{devi} = z_j g \Theta_i^j + e D_j^i \left(\frac{1 - \delta^{N+1}}{1 - \delta} \right), \quad (8)$$

where the presentation is made simple by dropping the subscripts of G_{devi} .

The loss accumulated at node i is proportional to the amount of plenty imposed as a consequence of betraying. Node i losses a potential relay for the next N periods as a punishment from j . Once again, it is the speculation of next N periods with a discount factor of δ . Hence, the total loss of deviation is given as,

$$L_{devi} = \delta z_j g \Theta_i^j \left(\frac{1 - \delta^N}{1 - \delta} \right). \quad (9)$$

A similar gain and loss is speculated by node j which can be obtained in a symmetric manner.

B. Payoff by Cooperation

If node i decides to cooperate, then it has to speculate a discounted sum of probable gain from j . Since node i can only speculate the gain, there is no effect of future decisions of j on the speculations of i in the present period. This is represented as,

$$G_{coop} = z_j g \Theta_i^j \left(\frac{1 - \delta^{N+1}}{1 - \delta} \right). \quad (10)$$

In order to sustain the cooperation, the loss L_{coop} is the consumed energy for relaying packets of j up to $N+1$ periods, and is given as,

$$L_{coop} = e D_j^i \left(\frac{1 - \delta^{N+1}}{1 - \delta} \right). \quad (11)$$

The inequality, $G_{devi} - L_{devi} < G_{coop} - L_{coop}$ must be true if relaying has to be a rational choice for node i . By substituting expressions of G_{devi} , L_{devi} , G_{coop} and L_{coop} and solving for Θ_i^j we obtain the following inequality,

$$\Theta_i^j > \frac{e}{z_j g} D_j^i \Lambda_\delta(N), \quad (12)$$

where we use $\Lambda_\delta(N)$ to make simplifications of results and isolate the punishment and dynamic measure in a single expression as,

$$\Lambda_\delta(N) \equiv \frac{1 - \delta^{N+1}}{\delta - \delta^{N+1}}. \quad (13)$$

Therefore, it becomes rational and favorable for the node i to relay the packet of node j if TSPI of the entity j satisfies inequality (12).

C. Equilibrium Analysis

In this section, we analyze the properties of the equilibrium of the proposed game. At the equilibrium point, there is no profitable myopic deviation for the players and these players are indifferent among pure strategies [15]. Therefore, at the equilibrium position, while the entity j is playing with an equilibrium probability z_j^* , the equilibrium TSPI Θ_i^{j*} must satisfy (12) with equality, that is,

$$\Theta_i^{j*} = \frac{e}{z_j^* g} D_j^i \Lambda_\delta(N). \quad (14)$$

Hence, the entity i cooperates if j 's TSPI lies above Θ_i^{j*} which is bounded by D_j^i . This links the entity i 's relaying probability with TSPI of the node j as follows,

$$Prob(\Theta_i^{j*} < \Theta_i^j \leq D_j^i) = 1 - Prob(\Theta_i^j \leq \Theta_i^{j*}) = z_i. \quad (15)$$

It is a common knowledge that each node speculates that the TSPIs are sampled from a probability distribution function which is defined by the network topology,

$$P_i(\Theta_i^{j*}) \equiv Prob(\Theta_i^j \leq \Theta_i^{j*}), \quad (16)$$

and as a result, the equilibrium of the game must satisfy (14), and is given as,

$$\Theta_i^{j*} = \frac{e}{g} \frac{D_j^i \Lambda_\delta(N)}{1 - P_j\left(\frac{e}{g} \frac{D_j^i \Lambda_\delta(N)}{1 - P_i(\Theta_i^{j*})}\right)} \quad (17)$$

We consider a case of uniformly distributed TSPIs which is represented as,

$$P_k(\Theta_k^l) \equiv \frac{\Theta_k^l}{D_k^l}. \quad (18)$$

Using a uniform distributions of TSPI and solving the quadratic equation (17) for the equilibrium,

$$\begin{aligned} (\Theta_i^{j*})^\pm &= \frac{g D_i^j D_j^i - e \Lambda_\delta(N) (D_i^{j2} - D_j^{i2})}{2g D_j^i} \pm \\ &\frac{\sqrt{[e \Lambda_\delta(N) (D_i^{j2} - D_j^{i2}) - g D_i^j D_j^i]^2 - 4eg \Lambda_\delta(N) D_j^{i3} D_i^j}}{2g D_j^i}. \end{aligned} \quad (19)$$

The above equilibriums represents the solutions of this game.

D. Twin Equilibriums

The twin equilibriums $(\Theta_i^j)^\pm$ denotes two different rational thinkings of the entities while taking the relaying decision. The equilibrium with higher TSPI $(\Theta_i^j)^+$ corresponds to a precautionary rational player, which always tries to play at the a safe side and minimizes its loss in an event of betrayal. On the other hand, the equilibrium with relatively lower TSPI $(\Theta_i^j)^-$ corresponds to a rational player, who is trying to maximize the profit. This player tends to relay at a lower expectation from others to get maximum possible outcome from the current network topology.

E. Thresholds of Equilibrium Existence

As presented in (14), it is evident that no such TSPI exists for any relaying probabilities below $(z_j^*)^{th}$, since the required TSPI is more than the maximum possible TSPI D_i^j (see Figure 3). Similar to the threshold probability, there is a threshold TSPI $(\Theta_i^{j*})^{th}$ at maximum possible relaying probability. Therefore, any feasible equilibrium is higher than or equal to the threshold TSPI.

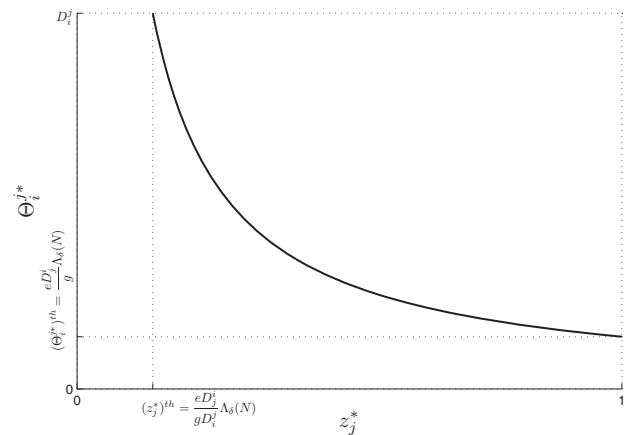


Figure 3. Probability of relaying vs TSPI

V. RESULTS

The derived equilibrium TSPIs of the formulated game in (19) depends upon different parameters of the network. These parameters define a *window* of possible values in which multiple equilibriums exist and remain within plausible limits.

A. Punishment Effectiveness

In the proposed game, the punishment of betraying is implemented as a penalty of N periods of non-cooperation. It is clear from (19) that the effectiveness of punishment depends on the dynamic nature of the network and both of these parameters are encapsulated in $\Lambda_\delta(N)$. An infinite penalty results in $\Lambda_\delta(\infty) = \frac{1}{\delta}$, as shown in Figure 4. Therefore, an *optimal* punishment is to hold the status quo forever for the betraying node, i.e., terminate its connection on betrayal. However, having infinite penalty is not feasible due to dynamic nature of the networks and *dynamic inconsistency* of decisions [16], [17]. The $\Lambda_\delta(N)$ approaches $\frac{1}{\delta}$ very sharply in the beginning but later on convergence to asymptote is very slow, as shown in Figure 4.

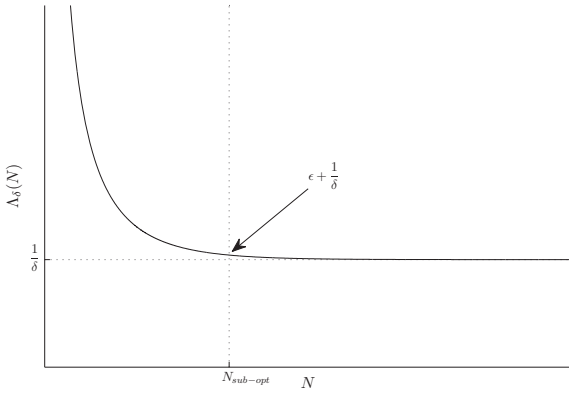


Figure 4. Punishment effect

Therefore, it is required to find a *sub-optimal* punishment that can be advised on a given network situation which extracts the essence of the punishment and at the same time, keeps open the possibility of reconnection after the punishment cycle is over.

The sub-optimal value of the punishment can be found that results in $\Lambda_\delta(N_{sub-opt}) = \epsilon + \Lambda_\delta(\infty)$, such that, $\epsilon \ll 1$. Figure 5 shows the sub-optimal punishments.

It can be seen in Figure 5 that $N_{sub-opt}$ for static networks is much higher than their dynamic counterparts, since it is essential to use heavy penalties on static networks in which the nodes have a long term relationship.

B. Effect of Network Parameters

There is a bound on the dynamic measure of the network for the existence of a stable equilibrium. This bound can be deduced from the fact that $(z_j^*)^{th}$ must lie away from unity. If we use $N_{sub-opt}$, then $\delta \gg \frac{eD_j^i}{gD_j^i - \epsilon eD_j^i}$. Therefore, δ must

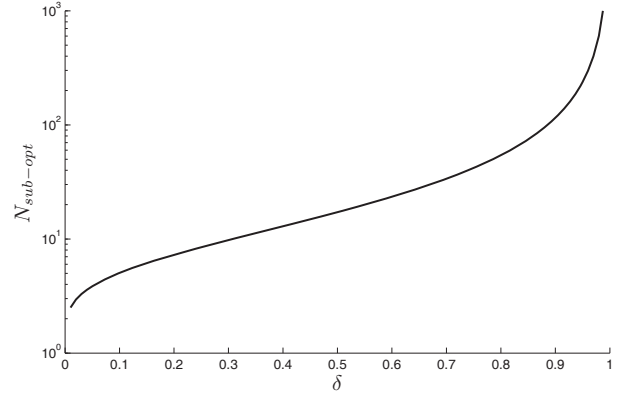


Figure 5. Sub-optimal punishments for $\epsilon = 10^{-5}$

be far more greater than this bound to have a rational feasible equilibrium (see Figure 6).

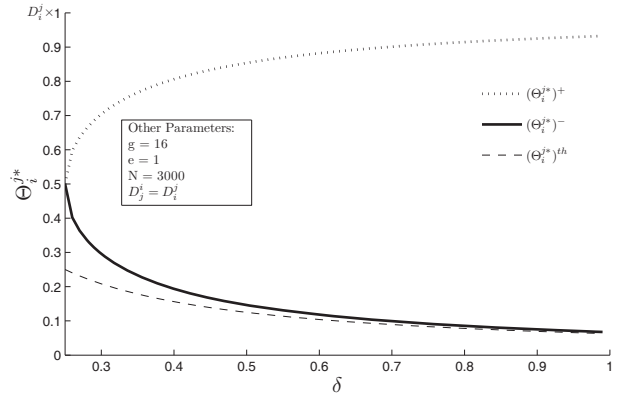


Figure 6. Effect of dynamic measure on the equilibriums

The effect of multihop gain variation is shown in Figure 7. Similar to δ , the gain must also be bounded away from a threshold gain of $\frac{eD_j^i}{D_j^i} \Lambda_\delta(N)$ to have any stable equilibrium. The behavior of both equilibriums is rational as the player is trying to achieve a non-negative payoff by maximizing gain $\left((\Theta_i^{j*})^-\right)$ or minimizing loss $\left((\Theta_i^{j*})^+\right)$.

The transmission cost variation have similar but opposite effect on the equilibriums of the game. The transmission cost must satisfy the inequality $e \ll \frac{gD_j^i}{D_j^i \Lambda_\delta(N)}$ to have a stable equilibrium (see Figure 8).

Finally, the load ratio $\frac{D_j^j}{D_j^i}$ also has a pronounced affect on the equilibriums. Since the game is symmetric in sense of equilibriums, therefore, this ratio is doubly bounded $\frac{g}{e\Lambda_\delta(N)} \ll \frac{D_j^j}{D_j^i} \ll \frac{e\Lambda_\delta(N)}{g}$ to induce a stable equilibrium in the game for both players (see Figure 9).

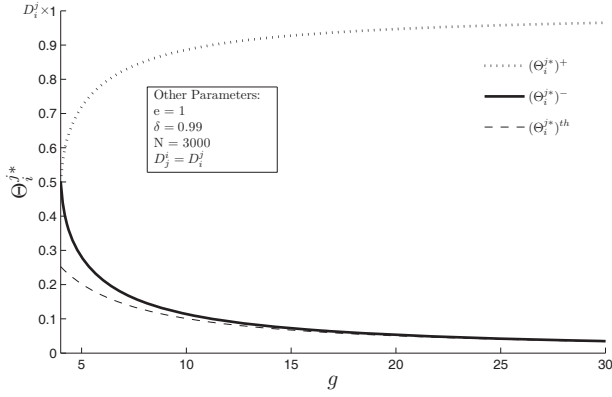


Figure 7. Effect of relaying gain on the equilibriums

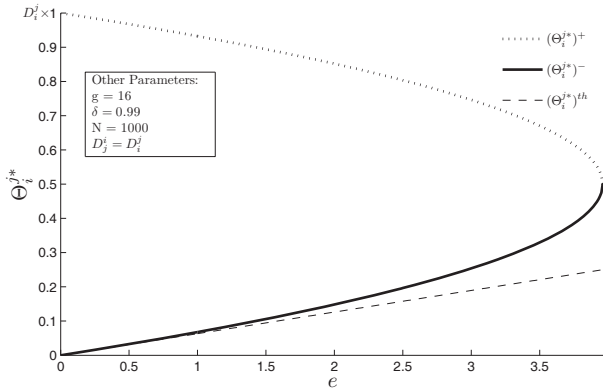


Figure 8. Effect of transmission cost on the equilibriums

C. Calculation of Incentives

The calculation of TSPIs of the entities can be easily performed by using (5), and is given as,

$$\Theta_i^j = \sum_{\forall d \in D_i^j} \beta_i^j(d) \prod_{l=j+1, l \in \text{path}(d)}^d \left(1 - \frac{\Theta_l^{i*}}{D_l^i}\right). \quad (20)$$

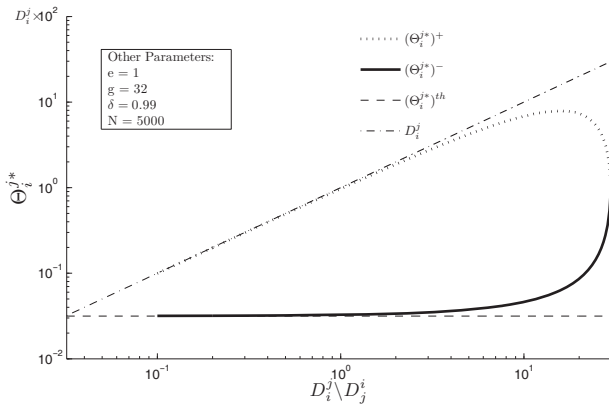


Figure 9. Effect of load ratio on the equilibriums

The information related to SPFs is available from the routing table maintained by each node. The equilibrium TSPIs are a function of common knowledge parameters and the destinations of each player which can be extracted from the path discovery traffic. As a result, there is no requirement of temper proof hardware to realize cooperation in the network.

VI. CONCLUSIONS

In this paper, we define and analyze a game for ad hoc networks, which introduces an incentive associated with topological strategic position, to relay traffic. The presented game is very simple to realize and there is no need of any specialized hardware modules to mitigate cheating, retain integrity and induce cooperation. The information available from the routing protocol is sufficient for the nodes to evaluate the importance of the peer nodes. Any ad hoc network with nodes meeting threshold importance for each other can exhibit cooperative behavior. As a result, the cooperating behavior becomes a rational choice in that network.

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