

Performance Analysis on Wireless Unicast with Network Coding

Tao Zhang*, Chunlin E*, Bin Wang*

*School of Computer Science, Fudan University
Shanghai, 200433 P.R.China

Email: {09210240051, 09210240008, bwang}@fudan.edu.cn

Abstract—In this paper, network coding is used to improve the wireless unicast performance. We consider a wireless switching network model with erasure channel, where M nodes exchange their packets via relay node. The sender transmits packets to the receiver via relay node independently. The relay node combines packets transmitted to the same receiver from different source nodes to save bandwidth. This paper aims to analyze the performance of wireless unicast. We also analyze the performance of unicast in traditional transmission scheme and compare it with network coding scheme. Our result shows that, in terms of transmission bandwidth, the performance of unicast will be improved highly. The gains depend on the transport protocol, and network environment. In this paper, linear network coding would be used at the relay node for low complexity.

Keywords—Network coding, reliability, wireless networks, unicast, erasure networks

I. INTRODUCTION

The idea of network coding was first proposed by Ahlswede et al. [1], where packets can be jointly combined together and then be transmitted at intermediate node. In [2], mathematical frameworks for multicast using network coding was introduced, and network coding has established its theoretical ability to improve network capacity. With the development of wireless network, network coding used in wireless network has attracted significant attention [3],[4]. Recent research has shown that, network coding can increase throughput over wired and wireless networks. Network coding has been applied to wireless network and is referred to as a means of improving network capacity and network reliability. In [8], they present an efficient distributed randomized approach that asymptotically achieves the capacity of multicast networks with network coding.

In traditional networks, data packets were transmitted from the source node to the destination node by a method known as store-and-forward. In this method, packets received by the intermediate node are stored and a copy is forwarded to the next node. But there is no limitation for packet processing at intermediate nodes except for data replication. Using network coding, the node can combine the packets that are received from incoming channels, and then transmit them through the output channels. Network coding can increase the multicast capacity that has been proved by [1]. But the analysis of the performance of network coding used in network unicast has few results. In [9], a new unicast scheme COPE was proposed where the capacity gain of wireless network with

network coding is achievable. In [3], the capacity gain of network coding over routing for multiple independent unicast transmissions was investigated. This paper focuses on network coding in multiple unicast, where the overall performance of wireless network can be improved compared to the traditional wireless network.

The objective of this paper is to develop a model, and to study the performance gain of wireless unicast using network coding relative to the traditional unicast scheme. Since the wireless communication channels are lossy in real environment, in order to achieve a reliable network communication, automatic repeat request (ARQ) is used for packet retransmission. In this paper, we use a wireless switching network model, where N nodes exchange their packets via a relay node. The previous work on network coding seldom considered the packet-loss rate. In this paper, we assume an erasure model that the channel between each node is unreliable, and the packet has an erasure probability, i.e. packets transmitted on a channel has a possibility to disappear.

Recently, there has been some research on the reliability benefit of network coding in lossy networks [4][6][10]. This paper differs from existing work in this area by using network coding in wireless unicast and considering the correlate between each erasure channel. Previous researches mostly focus on either the wireless broadcast or multicast with time-varying channel conditions, or on the capacity gains from network coding under various different network schedule schemes [8].

This paper proposes a network coding scheme at relay node that do some algebra operation such as linear combination. Instead of storing and forwarding packets, the relay node chooses the packets that would be transmitted to the same receiver, combines them to generate a new packet, and then transmits it to the destination node. We assume that the packets arrive at the relay node randomly. In this proposed transmission scheme, whether a packet is transmitted with or without network coding is determined by the node's buffer queue [5]. When the relay node picks a packet to transmit, if there is other packets transmitted to the same destination, then they would be combined together and be transmitted to the destination.

The rest of the paper is organized as follows. In section II, we introduce the wireless switching network model which serves as a basis for the analysis in following sections, and describe two wireless unicast schemes briefly, whether using

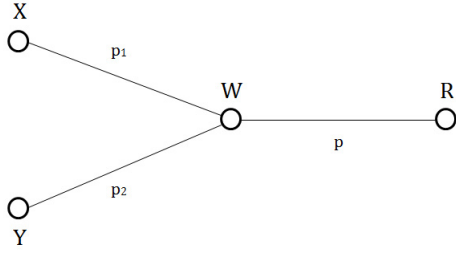


Fig. 1. The wireless switching network model

network coding or not. In section III, we study two unicast schemes, and through the analysis we derive the result for the number of transmissions. In section IV, we summarize the conclusions and discuss the future work.

II. SYSTEM MODEL

In this section, we introduce a wireless switching network model, where N nodes exchange their packets via a relay node, such as Fig. 1. Transmissions between each node are lossy with losses distributed by independent identical Bernoulli processes with parameter p . In this paper, for easy analysis, we assume that three nodes exchange their packets via the relay node. When the relay node gets chance to transmit packets, it would choose a packet from the top of buffer queue, and transmit it to the destination node. We assume that two senders transmit packets to the receiver independently, i.e. there are two unicast flows with the same receiver.

We shall assume that the erasure probability of each channel is different, i.e. wireless unicasts have different transmission environment[7]. Throughout this paper, we assume that the network has perfect feedback, and don't consider the overhead and complexity of the feedback mechanism. The sender has access to the information about the packet losses, and for simplicity, we assume that all the ACK/NAKs are instantaneous.

Packets transmitted on the channel have a probability to erasure. And the channel erasure possibility is different between each other. A packet loss at channel C follows a Bernoulli trial with parameter p_c . For example, the packet X transmitted on channel XW has a probability p_1 to disappear, and has a probability $(1 - p_1)$ to be received by the relay node W . This assumption can capture the essence of wireless communication.

In order to better analyze the performance of network coding for wireless unicast, we assume that the relay node just forward the packets, and don't in charge of the packets retransmission. That is, if the relay node completes the task of forwarding, it would remove the transmitted packets. If the packets lost on the channel, the receiver would request retransmission of lost packets from the sender. Also, we don't consider the delay on channels, so that when the packet is injected into the channel, the sink node would receive it immediately.

To analyze the performance of wireless unicast, in this paper, we introduce two unicast scheme, the traditional wireless unicast without network coding, and the new unicast scheme with network coding.

A. Unicast Scheme Without Network Coding

In this scheme, the relay node just choose a packet from buffer to send, when it get a chance to transmit a packet at some time. Whenever there is a packet lost, regardless of whether missing or incorrect recipient, the receiver will send a NAK to the sender. Hence, the sender has to retransmit this packet again using the ARQ as before. When the receiver successfully receives the packet, it will send an ACK to the sender. In this scenario, the relay node just stores the packets and forwards them just like the traditional unicast communication. This model is sub-optimal bandwidth utilization, because the sender has to retransmit all lost packets until the receiver receives these packets correctly.

B. Unicast Scheme with Network Coding

In this scheme, network coding will be used in wireless unicast. With network coding, the relay node can combine some packets to be transmitted to the same receiver, and transmit this mixed packet. When the receiver successfully receives packets, it can then decode its packets to recover the original message packets.

The network coding method we use in this paper is as follows. In this paper, two senders transmit packets to the receivers independently, that is two independent unicasts. We suppose that, at the relay node, there are m packets, $X_1, \dots, X_m$, from the sender X , and n packets, $Y_1, \dots, Y_n$, from the sender Y , which are vectors of the same length n over the finite field F_q . If the relay node is not allowed to code, then at any transmitting time, it just transmits a single packet from buffer queue. In this paper, we assume that the buffer queue is sufficiently large.

With network coding, when the relay node get a chance to transmit packets, it choose from buffer queue some packets that are to be transmitted to the same receiver, i.e. $X_{a_1}, \dots, X_{a_i}$, and $Y_{b_1}, \dots, Y_{b_j}$. Specifically, we have

$$X = \sum_{k=1}^i r_k X_{a_k} + \sum_{k=i+1}^{i+j} r_k Y_{b_{k-i}}$$

where coefficients $r_1, \dots, r_{i+j} \in F_q$ and $a_1, \dots, a_i \in \{1, \dots, m\}$, $b_1, \dots, b_j \in \{1, \dots, n\}$. Here $r = (r_1, \dots, r_{i+j})$ is the encoding vector of X . For decoding easily, we assume that the encoding vector is linearly independent. When the relay node complete transmitting the $i + j$ or more linear combinations of $X_{a_1}, \dots, X_{a_i}$ and $Y_{b_1}, \dots, Y_{b_j}$, these packets would be removed from the buffer. When the receiver has received more than $i + j$ packets with linearly-independent encoding vectors, it is able to recover the original message packets.

Because of the lossy channel, the combined packets may be lost during the transmission, and then lack of adequate

packets, the receiver cannot recover the original message packets. So, the receiver must request the packets again. In this scheme, we assume that the two senders have different erasure probability, and the node has access to this information of erasure. According to this information, the receiver can decide to request packets from the sender with better channel conditions. For example, if $p_1 < p_2$, and $p(i+j) < i$, then the receiver would request packets from X . This retransmitted packets can be combined together again at the relay node. Note that the relay node has to include some information to indicate to the receiver which set of packets it does network coding on.

III. ANALYSIS OF UNICAST SCHEMES

In this section, we are concerned with the transmission bandwidth with or without network coding. In order to facilitate the analysis, we assume that there are two senders X and Y , one receiver R , and one relay node W , i.e., two senders transmit packets to one receiver via relay node independently. We will study the mean transmission number of transmitting one packet to the sender. Then we will characterize the optimal scheduling strategy with and without network coding. For network coding scheme, we propose two retransmission strategies when there are packets lost.

To analyze our proposed scheme, we assume that the erasure possibility over every channel is different. The erasure possibility of channel XW, YW, WR , is p_1, p_2, p , respectively.

A. Unicast Scheme Without Network Coding

Because the sender transmits packets independently, so we just consider one unicast for easy analysis in this scheme. We assume that the sender X wants to transmit a packet to receiver R via relay node W . In the single unicast case, we have the following results:

Proposition 1: The transmission bandwidth of scheme A without network coding is

$$\eta_A = \frac{2 - p_1}{(1 - p)(1 - p_1)} \quad (1)$$

Proof: For scheme A, the sender X transmit a packet to the relay node, and when the relay node W receives this packet, it will be forwarded to the receiver R . If the receiver losses this packet, it will send a NAK to the sender for the retransmission. And the packet will be retransmitted again, until the receiver successfully receives this packet, and send an ACK to the sender.

As described in Section II, loss of the packet at the channel follows a Bernoulli trial with parameter p . So the number of transmission occurring on every channel follows a geometric distribution. We have

$$\begin{aligned} \eta_A &= \frac{1}{1 - p_1} + 1 + \sum_{k=1}^{\infty} \left(\frac{p^k}{1 - p_1} + p^k \right) \\ &= \frac{1}{1 - p_1} + 1 + \left(\frac{1}{1 - p_1} + 1 \right) \lim_{k \rightarrow \infty} \frac{p(1 - p^k)}{1 - p} \end{aligned}$$

$$= \frac{2 - p_1}{(1 - p)(1 - p_1)}$$

From the result, we can see that if the channel is perfect, i.e. $p_1 = p_2 = p = 0$, then the $\eta_A = 2$, so it will use two transmissions to deliver one packet to the receiver. ■

B. Unicast Scheme With Network Coding

Unlike scheme A, scheme B consider two unicasts with the same receiver, and the relay node can do linear network coding operation. In this scheme, we assume that the buffer of relay node is sufficiently large, so the relay node can choose some packets to generate the combined packets. In order to easily recover the original packets, we use the linearly-independently encoding vectors. In order to obtain standardized expression, we assume that the relay node choose equal packets from X and Y , i.e. $i = j$. We now provide an asymptotic result.

Proposition 2: The transmission bandwidth of scheme B with two unicasts where $p_1 \leq p_2$ is

$$\eta_B = \frac{1}{2(1 - p_1)} + \frac{1}{2(1 - p_2)} + 1 + \frac{(2 - p_1)p}{(1 - p_1)(1 - p)} \quad (2)$$

Proof: In this scheme, the packets at the relay node from different unicasts would be combined together using linearly-independent encoding vectors. But the channel is lossy, so some packets would be lost at the channel WR . In this scheme, we assume that the number of requested packets is smaller than the number of packets transmitted by sender X . This is possible, because we can choose more packets from sender X to combine. So, packets from the sender X with better channel condition would be retransmitted again, until the receiver successfully recovers the original message packets.

As described in Section II, loss of the packet at the channel follows a Bernoulli trial with parameter p . So the number of transmission occurring on every channel follows a geometric distribution. Then we define that the number of transmissions needed to deliver $2i$ packets to the receiver is the random variable V . We have

$$\begin{aligned} E[Y] &= \frac{i}{1 - p_1} + \frac{i}{1 - p_2} + 2i + \sum_{k=1}^{\infty} 2i \left(\frac{p^k}{1 - p_1} + p^k \right) \\ &= \frac{i}{1 - p_1} + \frac{i}{1 - p_2} + 2i + 2i \left(\frac{2 - p_1}{1 - p_1} \right) \lim_{k \rightarrow \infty} \frac{p(1 - p^k)}{1 - p} \\ &= \frac{i}{1 - p_1} + \frac{i}{1 - p_2} + 2i + 2i \frac{(2 - p_1)p}{(1 - p_1)(1 - p)} \end{aligned}$$

That is we need $E[Y]$ transmissions to complete delivering $2i$ packets to the receiver. So dividing $E[Y]$ by $2i$, we can get

$$\begin{aligned} \eta_B &= E[Y]/2i \\ &= \frac{1}{2(1 - p_1)} + \frac{1}{2(1 - p_2)} + 1 + \frac{(2 - p_1)p}{(1 - p_1)(1 - p)} \end{aligned}$$

From the result, we can see that if the channel is perfect, i.e. $p_1 = p_2 = p = 0$, then the $\eta_B = 2$, equal to scheme A.

But if the erasure probability of channel exists, the number of transmission per one packet is smaller than scheme A. ■

C. Remarks on Unicast with Network Coding

In the previous section, we analyzed the transmission bandwidths of different schemes. From scheme B, using network coding at the relay node, the receiver can get the combined packets that contains the packets from sender with worse channel condition. When the receiver cannot decode the combined packets, it will request the packets again from one sender or all senders dependent on the number of lost packets. For example, like the analysis in scheme B, if $p * 2i$ is smaller than i , then we conclude that retransmission from sender X is enough to decode the entire packets. So sender Y just consume $i/(1 - p_2) + i$ transmissions. And this strategies don't affect the performance of sender X .

The retransmission strategy can be determined by the receiver. If the overall channel condition is worse, then more packets would be lost, and request retransmission, and the retransmission from one sender is not enough to decode the entire packets. In this situation, we find that it is better to retransmit packets as the traditional way.

IV. CONCLUSION

In this paper, we propose a network coding techniques to improve the wireless unicast where the relay node can do network coding to the transmitted packets. Unicast is different from the broadcast. Using network coding method, we analyze the transmission bandwidth of the wireless unicast and the

overall performance of wireless network. From the analysis of scheme A and scheme B, we can see that the unicast with network coding has better transmission bandwidth, and the sender with worse channel conditional will get the same performance as the sender with better channel condition. And the overall performance of wireless unicast can be improved highly. For further research, we can discuss the wireless unicast with other network model. Also we will consider the extensions to the general wireless network.

REFERENCES

- [1] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inform. Theory*, 2000.
- [2] S. Y. R. Li, R. W. Yeung, and N. Cai, "Linear network coding," *IEEE Trans. Inform. Theory*, 2003.
- [3] Z. Li and B. Li. "Network coding: The case for multiple unicast sessions," *Allerton Conference on Communications*, 2004.
- [4] D. Nguyen, T. Nguyen, and B. Bose, "Wireless broadcasting using network coding," in *Proc. NetCod*, san Diego, USA, January 2007.
- [5] C. Yuan and Haibin Kan, "A characterization of solvability for a kind of networks", *Science in China*
- [6] D. S. Lun, M. Médard, and M. Effros, "On coding for reliable communication over packet networks," in *Proc. Allerton*, 2004.
- [7] Haibin Kan and Hong Shen, "A relation between the characteristic generators of a linear code and its dual", *IEEE Transactions on Information Theory*, 2005.
- [8] T. Ho, M. Médard, R. Koetter, D.R. Karger, M. Effros, J. Shi, B. Leong, "A random linear network coding approach to multicast", *IEEE Trans. Inform. Theory*, 2006.
- [9] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Médard, and J. Crowcroft, "Xors in the air: Practical wireless network coding," *ACM SIGCOMM Comput. Commun. Rev.*, 2006.
- [10] H.F. Ma, Haibin Kan, "A New Scheme of Digital Communication Using Chaotic Signals in MIMO Channels", *Science in China*