

Blind Equalization and Characteristic Function Based Robust Modulation Recognition

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Abstract—A robust modulation recognition scheme is proposed to identify practical QAM signals, which suffer from various unknown effects including multipath propagation, pulse shaping, time delay, and phase shift. Relying on blind signal processing techniques, the proposed scheme consists of two steps: front-end signal processing followed by modulation classification. At the first step, a fractionally-spaced blind equalizer is applied to accomplish equalization and time synchronization. Phase shift is then estimated and corrected. As required, signal-to-noise ratio is also estimated. At the second step, modulation formats are classified via characteristic function. Simulation results are provided to show the performance of the proposed scheme.

I. INTRODUCTION

Modulation classification (MC) is traditionally used in military applications such as signal monitoring and interception. It has recently received renewed interest and found applications in areas such as adaptive modulation, software-defined radio, and cognitive radio. A comprehensive review of MC is given in [1].

As a modulation classifier typically operates in a non-cooperative environment, the receiver has little *a priori* knowledge about the transmitted signal and also no training is available. Consequently, a practical modulation classifier should be able to perform both blind synchronization and blind channel equalization without the knowledge of modulation format in use. Such a challenging problem has rarely been addressed. Instead, either an AWGN channel without multipath propagation or perfect synchronization is usually assumed in the literature [2]–[6]. To the best of the authors' knowledge, a most relevant work is reported in [7], where blind channel equalization is considered but perfect time synchronization is assumed.

In this work, we consider classifying QAM signals encountered in a practical environment. The received signal is subject to various unknown influences such as multipath propagation, pulse shaping, time delay, and phase shift. A fractionally-spaced blind equalizer (FSBE) is used to handle the multipath channel and the transmit pulse-shaping filter and, at the same time, achieve time synchronization. Then, after phase shift compensation and signal-to-noise ratio (SNR) estimation, a characteristic function (CF) method is adopted to identify the modulation type in use.

II. SYSTEM MODEL

Consider a modulation classifier operating in a practical environment, as illustrated in Fig. 1. The transmitted signal of a frame of N_s symbols may be expressed by

$$s(t) = \sum_{n=0}^{N_s-1} a_n p(t - nT) \quad (1)$$

where T denotes the symbol period, $p(t)$ a pulse-shaping filter, and a_n the n th transmitted symbol drawn from a QAM constellation.

At the receiver, synchronization is not achieved. We assume the received signal is affected by an unknown time delay ε and phase shift φ . In addition, the signal is corrupted by a complex AWGN $\eta(t)$ with zero mean and variance $2\sigma^2$, and an L -path channel with impulse response $\sum_{l=1}^L c_l \delta(t - \tau_l)$, where $\{c_l\}_{l=1}^L$ and $\{\tau_l\}_{l=1}^L$ represent the channel's gains and delays, respectively. For convenience, the pulse-shaping filter and multipath channel are combined to form an equivalent channel described by

$$h(t) = \sum_{l=1}^L c_l p(t - \tau_l). \quad (2)$$

The received signal can now be written as

$$r(t) = \sum_{n=0}^{N_s-1} e^{j\varphi} a_n h(t - nT - \varepsilon) + \eta(t). \quad (3)$$

We assume ε , φ , and $h(t)$ are all fixed within one frame of N_s symbols but they can vary independently from one frame to another. The time delay ε and phase shift φ are uniformly distributed in $[0, T]$ and $[0, 2\pi]$, respectively.

The received signal is oversampled at a rate $\frac{1}{T_s} = \frac{P}{T}$ (P is the oversampling factor), yielding a discrete-time signal model

$$r(kT_s) = \sum_{n=0}^{N_s-1} e^{j\varphi} a_n h(kT_s - nPT_s - \varepsilon) + \eta(kT_s), \quad (4)$$

$$k = 0, 1, 2, \dots, N_s P - 1.$$

A. Fractionally-Spaced Blind Equalizer

A blind equalizer is necessary at the receiver, since both the multipath channel and the pulse-shaping filter are not known.

Further, to deal with the unknown time delay, fractionally-spaced signal processing is employed as it is well known that fractionally-spaced equalizers are insensitive to timing errors.

By dividing the samples $\{r(kT_s)\}$, $\{\eta(kT_s)\}$, and $\{h(kT_s - \varepsilon)\}$ into P subsequences as

$$r_n^{(i)} \triangleq r(nT + iT_s), \quad (5)$$

$$\eta_n^{(i)} \triangleq \eta(nT + iT_s), \quad (6)$$

$$h_n^{(i)} \triangleq h(nT + iT_s - \varepsilon), \quad (7)$$

$$i = 0, 1, \dots, P-1; n = 0, 1, \dots, N_s - 1,$$

the received signal can be equivalently represented in a multichannel form as

$$r_m^{(i)} = \sum_{n=0}^{N_s-1} e^{j\varphi} a_n h_{m-n}^{(i)} + \eta_m^{(i)}, \quad (8)$$

$$i = 0, 1, \dots, P-1; m = 0, 1, \dots, N_s - 1.$$

As illustrated in Fig. 2, the FSBE requires to design P FIR filters with coefficients defined by

$$\mathbf{g}^{(i)} = [g_0^{(i)} g_1^{(i)} \dots g_{N-1}^{(i)}]^T, \quad i = 0, 1, \dots, P-1, \quad (9)$$

where N stands for the order of the FSBE. Using the Godard Constant Modulus Algorithm (CMA) [8], the P filter $\mathbf{g}^{(i)}$, $i = 0, 1, \dots, P-1$ can be obtained by the following stochastic gradient descent algorithm

$$\mathbf{g}^{(i)}(m) = \mathbf{g}^{(i)}(m-1) - \mu y_m (|y_m|^2 - 1) [\mathbf{r}^{(i)}(m)]^*, \quad (10)$$

$$i = 0, 1, \dots, P-1; m = 1, 2, \dots$$

where $\mathbf{g}^{(i)}(m)$ is the filter $\mathbf{g}^{(i)}$ at the m th iteration, μ is the step-size, and

$$\mathbf{r}^{(i)}(m) \triangleq [r_{N-2+m}^{(i)} r_{N-3+m}^{(i)} \dots r_{m-1}^{(i)}]^T, \quad (11)$$

$$y_m = \sum_{i=0}^{P-1} y_m^{(i)} = \sum_{i=0}^{P-1} [\mathbf{g}^{(i)}(m-1)]^T \mathbf{r}^{(i)}(m), \quad (12)$$

where the dispersion constant in CMA is set to 1 in order to let CMA applicable for any unknown QAM constellation.

To measure the performance of the FSBE, a popular figure is the residual inter-symbol interference (ISI). Assuming the combined impulse response (including pulse-shaping filter, multipath channel, and FSBE) is denoted by $\{s_n\}$, ISI is defined by [9]

$$\text{ISI} = \frac{\sum_n |s_n|^2 - \max_n |s_n|^2}{\max_n |s_n|^2}. \quad (13)$$

Clearly, ISI becomes zero if the combined impulse response $\{s_n\}$ reduces to a delta function.

B. Cumulants Based Blind Phase Estimation

The output from the FSBE, $y(n)$, is a sequence of Baud-spaced, time-synchronized, and channel (including pulse-shaping filter) equalized QAM symbols. However, the phase shift φ is not addressed by the FSBE because CMA used is inherently phase-blind. To make things worse, CMA introduces an additional unknown phase shift and power amplification factor. To enable effective modulation classification, we make

use of Cumulants to blindly estimate the total phase shift, denoted by ϕ , and then compensate for it.

Given a sequence of N_s samples $\{y(n)\}$, its second-order moments can be computed as

$$\hat{C}_{21} = \frac{1}{N_s} \sum_{n=1}^{N_s} |y(n)|^2, \quad (14)$$

$$\hat{C}_{20} = \frac{1}{N_s} \sum_{n=1}^{N_s} y(n)^2. \quad (15)$$

Then, its fourth-order Cumulants can be obtained by

$$\hat{C}_{40} = \frac{1}{N_s} \sum_{n=1}^{N_s} y(n)^4 - 3\hat{C}_{20}^2, \quad (16)$$

$$\hat{C}_{42} = \frac{1}{N_s} \sum_{n=1}^{N_s} |y(n)|^4 - |\hat{C}_{20}|^2 - 2\hat{C}_{21}^2. \quad (17)$$

As such, the total phase shift can be estimated by [2]

$$\hat{\phi} = \frac{1}{4} \arg \left(\frac{\hat{C}_{40}}{\hat{C}_{42}} \right). \quad (18)$$

Note that the range of the above phase shift estimator is only $[-\frac{\pi}{4}, \frac{\pi}{4}]$. Fortunately, this is enough for our purpose, since QAM constellations exhibit quadrant symmetry.

C. Subspace Based Blind SNR Estimation

Although there are some MC schemes that do not require the knowledge of SNR, it is expected that better performance can be achieved if SNR can be estimated and properly exploited. In this paper, we consider an SNR-dependent MC scheme. To this end, we need to estimate SNR blindly. In our case, SNR estimation becomes especially challenging in the sense that even the modulation format in use is not known, while conventional SNR estimation schemes [10] all assume the modulation format is known. Here we follow a subspace based approach [11] to tackle this difficult problem.

Referring to (4), we define a length- L_w observation vector

$$\mathbf{r}(n) \triangleq [r(nP) \ r(nP+1) \ \dots \ r(nP+L_w-1)]^T, \quad (19)$$

$$n = 0, 1, 2, \dots$$

By averaging over N_w consecutive observation vectors, the sample auto-correlation matrix is given by

$$\hat{\mathbf{R}} = \frac{1}{N_w} \sum_{n=0}^{N_w-1} \mathbf{r}(n) [\mathbf{r}(n)]^H. \quad (20)$$

Single value decomposition is then performed on $\hat{\mathbf{R}}$, yielding

$$\hat{\mathbf{R}} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H \quad (21)$$

where $\mathbf{U}$ and $\mathbf{V}$ are two unitary matrices and $\mathbf{\Sigma} = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_{L_w}\}$ is a diagonal matrix with diagonal elements arranged in decreasing order.

Applying the Minimum Descriptive Length (MDL) principle, we can estimate the signal space dimension by the the following minimization problem

$$\hat{d} = \arg \min_m N_w (L_w - m) \ln f(m) + \frac{1}{2} m (2L_w - m) \ln N_w, \quad (22)$$

where

$$f(m) = \frac{\frac{1}{L_w - m} \sum_{i=m+1}^{L_w} \lambda_i}{\left(\prod_{i=m+1}^{L_w} \lambda_i \right)^{1/(L_w - m)}}. \quad (23)$$

Given the estimated $\hat{d}$, the variance of the AWGN and the signal power can be respectively calculated as

$$2\sigma^2 = \frac{1}{L_w - \hat{d}} \sum_{i=\hat{d}+1}^{L_w} \lambda_i, \quad (24)$$

$$\hat{E}_s = \frac{1}{L_w} \left\{ \sum_{i=1}^{\hat{d}} \lambda_i - \hat{d} 2\sigma^2 \right\}. \quad (25)$$

The estimated SNR is given by $\hat{\gamma} = \frac{\hat{E}_s}{2\sigma^2}$.

Before proceeding to MC, the received signal is normalized to have an average power $1 + \frac{1}{\hat{\gamma}}$. The aim is to ensure that the estimated QAM constellations have unit average power.

D. CF Based Modulation Classification

After front-end signal processing, the received signal is equalized and synchronized. The modulation type is ready to be identified. There are a number of MC methods in the literature. Among them, Cumulants and maximum-likelihood methods are widely used. However, the cumulants approach does not perform well in classifying high-order QAM constellations and the maximum-likelihood method is computationally demanding. In this paper, we adopt a CF based method proposed in [12]. It is worth pointing out that there are two main differences between our approach and the one in [12]. First, we use the amplitude of CF to do MC, whereas in [12] the phase of CF is considered. Second, in our approach we need to blindly estimate SNR whereas in [12] SNR is not exploited.

We start with an ideal case. Assuming a received signal can be described by

$$x_n = a_n + \eta_n, \quad n = 0, 1, \dots \quad (26)$$

where a_n is drawn from a QAM constellation with unit average power and η_n is a complex Gaussian random variable with zero mean and variance $1/\sqrt{\gamma}$ (γ denotes the SNR). The CF of x_n is defined as

$$\begin{aligned} \Phi_x(\omega_1, \omega_2) &\triangleq \mathbb{E} \left\{ e^{j(\omega_1 \text{Re}[x_n] + \omega_2 \text{Im}[x_n])} \right\} \\ &= \mathbb{E} \left\{ e^{j(\omega_1 \text{Re}[a_n] + \omega_2 \text{Im}[a_n])} \right\} e^{-\frac{1}{4\gamma}(\omega_1^2 + \omega_2^2)} \\ &= |\mathbb{E} \{ \cos(\omega_1 \text{Re}[a_n]) \cos(\omega_2 \text{Im}[a_n]) \}| e^{-\frac{1}{4\gamma}(\omega_1^2 + \omega_2^2)} \end{aligned} \quad (27)$$

where $\mathbb{E}[\cdot]$ denotes expectation, and $\text{Re}[\cdot]$ and $\text{Im}[\cdot]$ represent the real and imaginary parts of a complex number, respectively. In the above derivation, the quadrant symmetry of QAM constellations has been applied.

We propose to use the amplitude of CF, $|\Phi_x(\omega_1, \omega_2)|$, to perform MC. In other words, $|\Phi_x(\omega_1, \omega_2)|$ is served as a two-dimensional feature function, which is determined by SNR and some QAM constellation employed. It is clear that

$|\Phi_x(\omega_1, \omega_2)|$, particularly its high frequency part, is significantly attenuated by the Gaussian noise.

Next, the output samples $\{z_n\}$ from the front-end signal processing block is used to obtain an estimated feature function, i.e.,

$$\begin{aligned} |\hat{\Phi}_z(\omega_1, \omega_2)| &= \left| \mathbb{E} \left\{ e^{j(\omega_1 \text{Re}[z_n] + \omega_2 \text{Im}[z_n])} \right\} \right| e^{-\frac{1}{4\hat{\gamma}}(\omega_1^2 + \omega_2^2)} \\ &= |\mathbb{E} \{ \cos(\omega_1 \text{Re}[z_n]) \cos(\omega_2 \text{Im}[z_n]) \}| e^{-\frac{1}{4\hat{\gamma}}(\omega_1^2 + \omega_2^2)} \end{aligned} \quad (28)$$

where in implementation, the frequencies ω_1 and ω_2 are truncated up to ω_{max} and discretized with sampling interval $\Delta\omega$.

Given the theoretical $|\Phi_x(\omega_1, \omega_2)|$ and the estimated $|\hat{\Phi}_z(\omega_1, \omega_2)|$, a classification decision can be made, based on certain criterion. We notice that in statistics, Bhattacharyya distance is widely used to measure the similarity/difference of probability distributions [13]. Therefore, Bhattacharyya distance is adopted to characterize the difference between $|\Phi_x(\omega_1, \omega_2)|$ and $|\hat{\Phi}_z(\omega_1, \omega_2)|$ as follows

$$D_B = -\ln \left(\sum_{\omega_1=0}^{\omega_{max}} \sum_{\omega_2=0}^{\omega_{max}} \sqrt{|\hat{\Phi}_z(\omega_1, \omega_2)| \cdot |\Phi_x(\omega_1, \omega_2)|} \right). \quad (29)$$

Note that, to use Bhattacharyya distance, $|\Phi_x(\omega_1, \omega_2)|$ and $|\hat{\Phi}_z(\omega_1, \omega_2)|$ should be normalized to mimic probability distributions. As expected, if the theoretical and estimated feature functions are equal, their difference is zero. A final MC decision is made by choosing the QAM constellation with minimum Bhattacharyya distance.

III. SIMULATION RESULTS AND DISCUSSION

Simulations are carried out to study the performance of the proposed MC scheme. At the transmitter side, pulse-shaping is realized by a square-root raised cosine filter with a roll-off factor 0.25, truncated in $[-5T, 5T]$. The multipath channel is described by $[-0.005 - 0.004j, 0.009 + 0.030j, -0.024 - 0.104j, 0.854 + 0.520j, -0.218 + 0.273j, 0.049 - 0.074j, -0.016 + 0.020j]$, which is taken from [14]. Although in our simulations the channel is fixed, the proposed scheme is applicable to multipath fading channels, provided that fading is slow enough so that the channel impulse response is approximately fixed during a frame of N_s symbols.

At the receiver, we assume the frame length is $N_s = 20000$, the oversampling factor is $P = 4$, and the step-size for CMA is $\mu = 5 \cdot 10^{-4}$. We set the equalizer order to $N = 10$, since it is shown in [15] that the FSBE is insensitive to the difference between the channel order L and the equalizer order N . Moreover, the channel order can be blindly estimated as in [16]. For the SNR estimation, we choose $L_w = 80$ and $N_w = 10000$.

We first examine the performance of the front-end signal processing part in Figs. 3-5, where square 16QAM is employed and $2\sigma^2 = 0.01$. Fig. 3 shows the residual ISI as a function of the number of iterations. We can see that the FSBE converges after around 10000 iterations. In Fig. 4 a snapshot of

TABLE I

CONFUSION MATRIX FOR ONE DIMENSIONAL CF WITH $2\sigma^2 = 0.01$ AND 100 TRIALS.

Input	Output				
	16QAM	64QAM	256QAM	32QAM	128QAM
16QAM	100	0	0	0	0
64QAM	0	51	49	0	0
256QAM	0	0	100	0	0
32QAM	0	0	0	100	0
128QAM	0	0	36	0	64

TABLE II

CONFUSION MATRIX FOR ONE DIMENSIONAL CF WITH $2\sigma^2 = 0.001$ AND 100 TRIALS.

Input	Output				
	16QAM	64QAM	256QAM	32QAM	128QAM
16QAM	100	0	0	0	0
64QAM	0	100	0	0	0
256QAM	0	0	100	0	0
32QAM	0	0	0	100	0
128QAM	0	0	0	0	100

TABLE III

CONFUSION MATRIX FOR ONE DIMENSIONAL CF WITH $2\sigma^2 = 0.1$ AND 100 TRIALS.

Input	Output				
	16QAM	64QAM	256QAM	32QAM	128QAM
16QAM	82	18	0	0	0
64QAM	0	61	7	21	11
256QAM	0	30	16	40	14
32QAM	0	0	0	84	16
128QAM	0	0	0	51	49

TABLE IV

CONFUSION MATRIX FOR TWO DIMENSIONAL CF WITH $2\sigma^2 = 0.01$ AND 100 TRIALS.

Input	Output				
	16QAM	64QAM	256QAM	32QAM	128QAM
16QAM	100	0	0	0	0
64QAM	0	77	23	0	0
256QAM	0	0	100	0	0
32QAM	0	0	0	100	0
128QAM	0	0	0	0	100

the combined impulse response s_n after convergence is illustrated. Clearly, apart from a complex number that is inherent to blind channel estimation, the combined impulse s_n seems like a delta function, indicating that the FSBE satisfactorily compensates for the multipath channel and pulse-shaping filter. In Fig. 5 we show that after equalization and phase correction, the square 16QAM constellation is recovered.

Next, confusion matrices are used to investigate the performance of the CF based MC scheme. Five types of QAM constellations are taken as an example, including three square QAM formats (16QAM, 64QAM, and 256QAM) and two cross QAMs (32QAM and 128QAM). We assume each frame of transmitted symbols consists of $N_s = 20000$ QAM symbols. The first 10000 symbols are used for blind equalization and synchronization. The remaining 10000 symbols are used to calculate CF's and then perform MC. Since the two dimensional CF discussed in Section II.D is computationally intensive, our simulation results are mainly based on one-dimensional CF, for which only ω_1 and the real part of the received signal need to be considered. We choose $\omega_{max} = 15\pi$ and $\Delta\omega = \frac{\pi}{10}$ when evaluating related CF's.

Simulation results are shown in Tables I-III with AWGN variance $2\sigma^2 = 0.01, 0.001$, and 0.1 , respectively. We see from Table I that while 16QAM, 256QAM and 128QAM are correctly recognized, the probability of correct classification (Pcc) for 64QAM and 128QAM is relatively low. They both are likely to be misclassified as 256QAM. In Table II, it is clear that all QAM constellations are perfectly identified, suggesting that the proposed scheme works especially well if SNR is high. On the other hand, when SNR is relatively low, we can see in Table III that the Pcc performance for all QAM formats deteriorates. The reason is that as SNR decreases, more difference between QAM constellations gets lost due to the increasing attenuation effect of AWGN. Finally, two-dimensional CF is calculated in Table IV. Comparing with Table I, we see that better performance can be obtained with two-dimensional CF. This is, however, at the cost of much high computational complexity.

IV. CONCLUSION

Based on on advanced signal processing techniques, a robust modulation classification scheme has been proposed to identify practical QAM signals, which suffer from various unknown effects including multipath propagation, pulse-shape filtering, time delay, and phase shift. The proposed scheme relies heavily on a fractionally-spaced blind equalizer, which is able to accomplish blind equalization and blind time synchronization. After phase shift correction and SNR estimation, MC is performed via the CF method. The proposed scheme is supported by simulation results.

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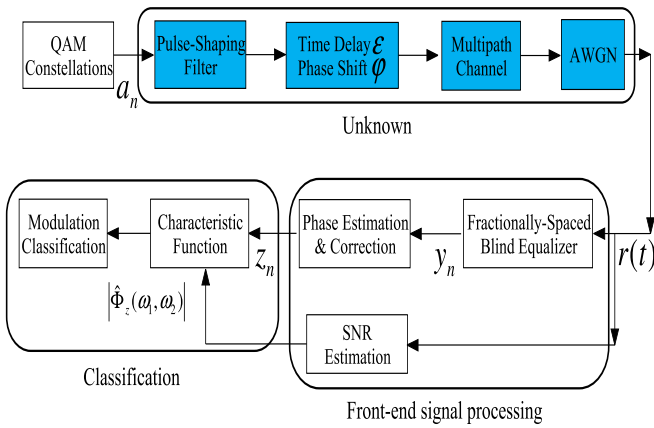


Fig. 1. Block diagram of the modulation classification system.

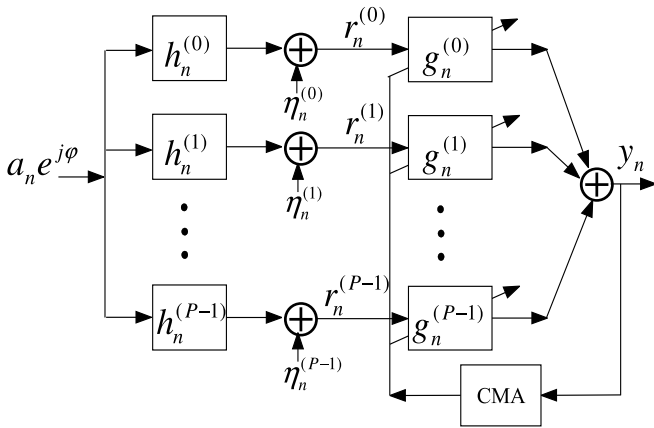


Fig. 2. Fractionally-spaced blind equalizer with CMA.

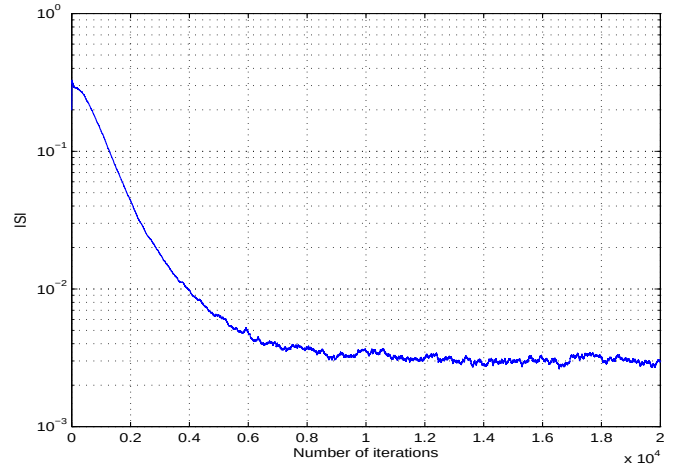


Fig. 3. Learning curve of the fractionally-spaced blind equalizer with order $N = 10$ and step-size $\mu = 5 \cdot 10^{-4}$ ($2\sigma^2 = 0.01$ and square 16QAM is transmitted).

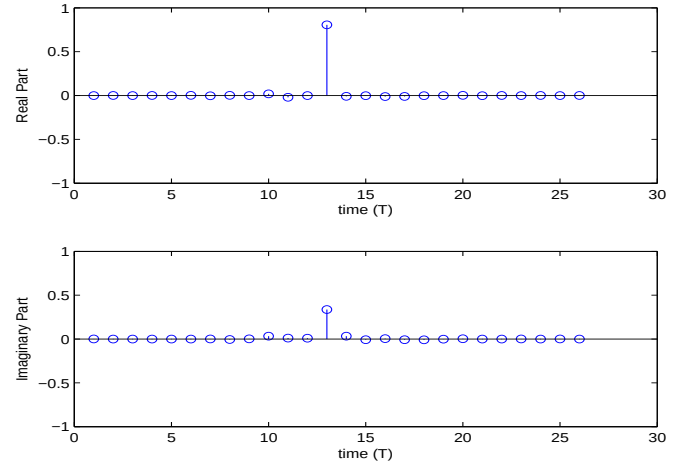


Fig. 4. Real and imaginary parts of the combined (pulse-shaping filter, multipath channel, and fractionally-spaced blind equalizer) impulse response ($2\sigma^2 = 0.01$ and square 16QAM is transmitted).

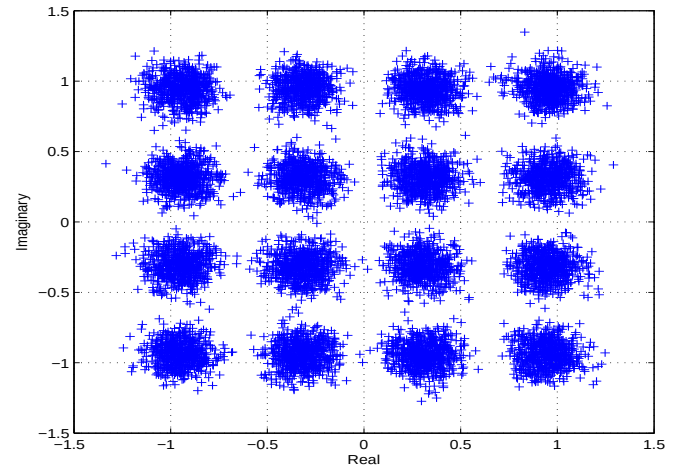


Fig. 5. Constellation of received signal samples after phase estimation and correction ($2\sigma^2 = 0.01$ and square 16QAM is transmitted).