

Performance Analysis of Un-coded Multiband OFDM UWB System

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Abstract: This paper provides analytical derivation for the average pairwise error probability (PEP) of un-coded multiband orthogonal frequency division multiplexing (MB-OFDM) system for Ultra wideband (UWB) communications. To capture the behaviour of UWB channels in terms of cluster arrival rate, ray arrival rate, cluster and ray decay factors, the Saleh and Valenzuela model is employed. It is shown clearly that the system in which the information is not jointly encoded across subcarriers cannot gain performance improvement from the multi path clustering property of UWB channels. Its performance is similar to that of an OFDM system for narrow band communications in a Rayleigh fading channel. The analytical results are compared with computer simulations in order to support the derivation. Some features of UWB channels are also discussed.

Keywords: Un-coded multiband OFDM, Pairwise error probability, Ultra wideband communications, Channel model, joint encoding

I. INTRODUCTION

The demand for transferring data with high rate such as multimedia applications over wireless networks is increasing in recent times. Ultra wideband (UWB) is one of the promising technology acquiring high data rate for short range communication at low transmission power [1],[2]. The traditional design approach is based on sequence of short duration of pulses that occupy the whole allocated spectrum of 7.5 GHz of UWB [3]. Even though the architectural design for this system is simple, it provides little flexibility in spectrum management. In addition, it has drawbacks, such as: need for high complexity Rake receivers to capture multipath energy, high speed analog to digital converters (ADC) and high power consumptions [2]. These concerns motivated a shift in UWB system design approach from traditional single band radio to multiband [4].

In multiband approach, the UWB frequency band is divided into several sub bands. Each sub

band is occupying approximately 500 MHz in compliance with FCC regulations. By interleaving symbols across different sub-bands, UWB system can still maintain the same transmit power as if a large GHz bandwidth was being utilized. Multi band approach allows the information to be processed over a much smaller bandwidth reducing overall design complexity as well as enhancing spectral flexibility and worldwide compliance [5].

MB-OFDM [2],[4] is the promising candidates for PHY layer of short-range high data rate UWB communications. In this design approach to capture the multipath energy efficiently the OFDM has been used to modulate the information in each subband. Moreover, it has all the strength of OFDM technique which has already been proven for wireless communications [6].

This work derives the theoretical average pairwise error probability of un-coded Multiband OFDM system for UWB communication. The theoretical results are compared with simulation results obtained in order to confirm the validity of the computation.

The paper is organized as follows: In section II, the system model is described. In section III the average error rate probability is derived. In section IV analytical and simulation results are presented. Finally conclusions are drawn in section V.

Note that: all the bold faced variables are vectors.

II. SYSTEM MODEL

In this report communication for single antenna multiband OFDM system over Ultra wideband communication is considered. For the Saleh-Valenzuela model [7], the channel impulse response is:

$$h(t) = \sum_{c=1}^C \sum_{k=1}^K \alpha_{ck} \delta(t - T_c - \tau_{ck}) \quad (1)$$

where C is the number of observed clusters, K is the number rays received within the c -th cluster, α_{ck} is the gain of the k -th ray of the c -th cluster, T_c is the time of arrival of the c -th cluster, and τ_{ck} is the delay of the k -th ray in the c -th cluster relative to the cluster arrival time. The cluster and ray arrivals within each cluster are modelled by Poisson processes.

For getting insight to the UWB system and mathematical tractability, the scenario taken in to account is that the amplitude gain $|\alpha_{ck}|$ is modelled as a Rayleigh distribution [8] while the phase $\angle \alpha_{ck}$ is uniformly distributed over $[0, 2\pi]$ and the multipath gain coefficients α_{ck} are zero mean, complex Gaussian random variables with variances as specified in [9]

$$\sigma_{ck} = E[|\alpha_{ck}|^2] = \sigma_{0,0} \exp\left(-\frac{T_c}{\Gamma} - \frac{\tau_{ck}}{\gamma}\right) \quad (2)$$

where $\sigma_{0,0}$ is the mean energy of the first path of the first cluster, Γ is the cluster decay factor, and γ is the ray decay factor. Spatially uncorrelated channel coefficients are assumed and the powers of the multipath components are normalized as:

$$\sum_{c=1}^C \sum_{k=1}^K \sigma_{ck} = 1 \quad (3)$$

The channel frequency response is computed by:

$$H(f) = \sum_{c=1}^C \sum_{k=1}^K \alpha_{ck} e^{-j2\pi f(T_c + \tau_{ck})} \quad (4)$$

Since the cyclic prefix length is assumed to be greater than the length of the channel impulse response, each UWB subband can be divided into a set of N orthogonal narrowband channels in the OFDM system. In the transmitter side, the sequential information can be divided into blocks which will be mapped to $N \times 1$ matrix.

$$\mathbf{X} = [x(0) x(1) \dots x(N-1)]^T \quad (5)$$

where $x(n)$, $n=0, 1, \dots, N-1$ is the complex data to be transmitted over n subcarrier. The data matrix $\mathbf{X}$ is a concatenation of $P = \lceil N/M \rceil$ data matrices as follows:

$$\mathbf{X} = [\mathbf{X}_0^T \mathbf{X}_1^T \dots \mathbf{X}_{(P-1)}^T \mathbf{0}_{(N-PM) \times 1}]^T \quad (6)$$

where $\mathbf{X}_p$ is an $M \times 1$ data matrix defined as

$$\mathbf{X}_p = [x_p(0) x_p(1) \dots x_p(M-1)]^T \quad (7)$$

with

$$x_p(n) \triangleq x(pM + n); p = 0, 1, \dots, P-1 \quad (8)$$

Note that $\mathbf{0}_{m \times n}$ stands for an $m \times n$ all-zero matrix. For different p 's, the data matrices $\mathbf{X}_p$ are designed independently and every data symbol $x_p(n)$ is normalized to have unit energy which imply the data matrix satisfies the following energy constraint for all p .

$$E[\|\mathbf{X}_p\|^2] = M \quad (9)$$

The matrix $\mathbf{X}$ is applied to an N -point IFFT. A cyclic prefix and guard interval is inserted, up-converted to RF, and then the signal will be modulated at each subcarrier for transmission. The signal intercepted will be fed to a correlator or a matched filter after removing the cp and applying FFT block, the output signal at the n th subcarrier of the receiver is specified by

$$y(n) = \sqrt{E_s} x(n) H(n) + z(n) \quad (10)$$

where, $H(n)$ is the channel frequency response at the n th subcarrier, $z(n)$ is the noise sample at the n th subcarrier and is modelled as a complex Gaussian random variable with zero mean and variance N_0 and E_s is the average energy transmitted per symbol. The channel frequency response can be described as

$$H(n) = \sum_{c=0}^C \sum_{k=0}^K \alpha_{ck} e^{-j2\pi n \Delta f (T_c + \tau_{ck})} \quad (11)$$

where $\Delta f = 1/T$ is the frequency separation between two adjacent subcarriers and T is the OFDM symbol period. The channel state information $H(n)$ is considered to be known at the receiver but not at the transmitter.

III. AVERAGE PAIRWISE ERROR PROBABILITY

Let $f(\mathbf{X}_p) = \text{diag}(x_p(0), x_p(1), \dots, x_p(M-1))$ is an $M \times M$ diagonal matrix with the elements of $\mathbf{X}_p$ on its main diagonal. We can re-express (10) in a vector form as:

$$\mathbf{Y}_p = \sqrt{E_s} f(\mathbf{X}_p) \mathbf{H}_p + \mathbf{Z}_p \quad (12)$$

The matrix $\mathbf{H}_p, \mathbf{Y}_p, \mathbf{Z}_p$ in (12) are of the form as $\mathbf{X}_p$, replacing x with H, y and z . The decoding process is performed jointly within each data matrix at the receiver side with a maximum likelihood decoder. The decision rule can be formulated as:

$$\hat{\mathbf{X}}_p = \arg \min_{\mathbf{X}_p} \left\| \mathbf{Y}_p - \sqrt{E_s} f(\mathbf{X}_p) \mathbf{H}_p \right\|^2 \quad (13)$$

Note that the data matrices $\mathbf{X}_p$ are en/decoded independently for different p 's. Pairwise error probability (PEP) can be defined as the probability of decoding erroneously the matrix $\hat{\mathbf{X}}_p$ when $\mathbf{X}_p$ is transmitted [10]. For uncoded multiband OFDM system PEP is similar to the symbol error rate (SER). As stated in [11] the average PEP can be given by:

$$P_e = E \left[Q \left(\sqrt{(\partial/2) \|\Delta_p \mathbf{H}_p\|^2} \right) \right] \quad (14)$$

Where $\Delta_p = f(\mathbf{X}_p) - f(\hat{\mathbf{X}}_p)$ denotes the signal difference matrix, $\partial = E_s / N_o$ is the average signal to noise ratio (SNR), and $Q(\cdot)$ is the Gaussian error function which is defined as:

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp(-x^2/2 \sin^2 \theta) d\theta; \quad x \geq 0 \quad (15)$$

We can write (14) as:

$$\begin{aligned} P_e &= E \left[\frac{1}{\pi} \int_0^{\pi/2} \exp \left(-\frac{\partial}{2} \left(\|\Delta_p \mathbf{H}_p\|^2 \right) / 2 \sin^2 \theta \right) d\theta \right] \\ &= E \left[\frac{1}{\pi} \int_0^{\pi/2} \exp \left(-\partial \eta / 4 \sin^2 \theta \right) d\theta \right] \end{aligned} \quad (16)$$

where $\eta = \|\Delta_p \mathbf{H}_p\|^2$. By definition the moment generating function (MGF), $\mathfrak{M}_\eta(S) = E[e^{s\eta}]$

which implies $E[e^{-\partial \eta / 4 \sin^2 \theta}] = \mathfrak{M}_\eta(-\partial/4 \sin^2 \theta)$.

Hence, in similar way the above expression in (16) can be written as:

$$P_e = \frac{1}{\pi} \int_0^{\pi/2} \mathfrak{M}_\eta(-\partial/4 \sin^2 \theta) d\theta \quad (17)$$

According to (11), we can write $\mathbf{H}_p$ as:

$$\mathbf{H}_p = \Psi_p \cdot \mathbf{A} \quad (18)$$

where

$$\begin{aligned} \Psi_p &= \begin{bmatrix} \psi_{p,0}^{T_0+\tau_{0,0}} & \psi_{p,0}^{T_0+\tau_{0,1}} & \cdots & \psi_{p,0}^{T_C+\tau_{C,K}} \\ \psi_{p,1}^{T_0+\tau_{0,0}} & \psi_{p,1}^{T_0+\tau_{0,1}} & \cdots & \psi_{p,1}^{T_C+\tau_{C,K}} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{p,M-1}^{T_0+\tau_{0,0}} & \psi_{p,M-1}^{T_0+\tau_{0,1}} & \cdots & \psi_{p,M-1}^{T_C+\tau_{C,L}} \end{bmatrix}, \\ \mathbf{A} &= [\alpha(0,0) \cdots \alpha(0,K) \cdots \alpha(C,0) \cdots \alpha(C,K)]^T \\ &\text{and} \\ \psi_{p,n} &\triangleq \exp[-j2\pi\Delta f(pM+n)] \end{aligned} \quad (19)$$

Using Eigen value decomposition (EVD), and after some algebraic operations we can approximate the derivation for variable η as follows:

$$\eta \approx \sum_{n=1}^M \lambda_n(\Phi) |\varphi_n|^2 \quad (20)$$

where φ_n are iid complex standard Gaussian random variables with zero mean and unit variance, and $\lambda_n(\Phi)$ are the Eigen-values of the matrix

$$\Phi = \|\Delta_p \Psi_p \sigma^{1/2}\|^2 \quad (21)$$

where $\sigma = \text{diag}(\sigma_{0,0}, \sigma_{0,1}, \dots, \sigma_{C,K})$ is a diagonal matrix obtained from the average powers of multipath components. The Laplace transform of the expression $\{e^{\lambda_n(\Phi)}\} = (1 - s\lambda_n(\Phi))^{-1}$. So by using Laplace transform, the MGF for variable η in (20) can be obtained as:

$$\mathfrak{M}_\eta(s) = E \left[\prod_{n=1}^M (1 - s\lambda_n(\Phi))^{-1} \right] \quad (22)$$

For $M=1$, that is when the number of jointly encoded subcarriers is one, the Eigen value of a matrix Φ in (21) can be expressed as follows:

$$\lambda(\Phi) = |x - \hat{x}|^2 \cdot \lambda(\Psi_p \sigma \Psi_p^\dagger) = |x - \hat{x}|^2 \quad (23)$$

where the matrix $\Psi_p \sigma \Psi_p^\dagger = \sum_{c=0}^C \sum_{k=0}^K \sigma_{ck} = 1$. It should

be noted that $(\cdot)^\dagger$ represents a conjugate transpose operation. Combining (22) and (17), we can obtain results for computing the average PEP (SER) for multiband OFDM UWB system for any channel

model parameters when there is no coding across subcarriers:

$$P_e = \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{\partial}{4 \sin^2 \theta} |x - \hat{x}|^2 \right)^{-1} d\theta \quad (24)$$

Using substitution and partial fraction integration techniques the above expression can be simplified to:

$$P_e = \frac{1}{4} \left[\int_0^\infty \frac{1}{(-I-4)t^2} dt + \int_0^\infty \frac{1}{1+t^2} dt \right] \quad (25)$$

where $I = \partial |x - \hat{x}|^2$

By solving (25), we can obtain the expression:

$$P_e = (\pi/8) \left(1 - 1/\sqrt{1+4/I} \right) \quad (26)$$

With the assumption of perfect channel estimation of $\mathbf{H}_p$ and additive white noise of zero mean complex Gaussian random variable, the expression $|x - \hat{x}|^2$ can be reduced to $|x|^2$. Hence, for QPSK modulation, (26) can be reduced to:

$$P_e = (\pi/8) \left(1 - 1/\sqrt{1+1/\partial} \right) \quad (27)$$

which is used to trace the theoretical error probability curve for an uncoded MB-OFDM UWB system in Rayleigh fading channel.

IV. ANALYTICAL AND SIMULATION RESULTS

The simulation model is developed based on the following assumptions: perfect channel estimation, no shadowing, Inter symbol interference (ISI) and Inter carrier interference (ICI) are not considered for sake of simplicity, perfect synchronization and no coding.

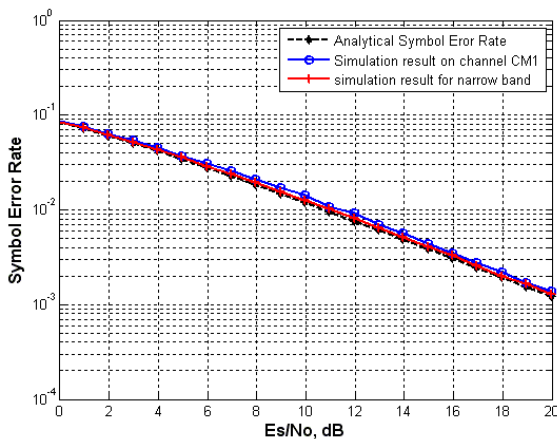


Figure 1. Analytical and simulated SER of Un-coded MB-OFDM over rayleigh fading channels

The simulation for the UWB is partially based on the specifications as in [8] with $N=128$ subcarriers where as for the narrow band it is as per IEEE 802.11a with same number of subcarriers as UWB.

The symbol error rate obtained from simulations results for UWB Channel Model 1(CM1) and for the narrow band system are very close to the analytical symbol error rate which validates the theoretical result.

V. CONCLUSIONS

We can observe from the formulation of expressions (21) and (22) that the Eigen values $\lambda_n(\Phi)$ depend on T_c and τ_{ck} which are based on the Poisson process. For case $M=1$, that is when there is no joint encoding over subcarriers, the expression obtained for the Eigen values does not depend on the variables T_c and τ_{ck} . Hence the analysis resulted in (27) shows that the performance of an uncoded multiband OFDM UWB system doesn't depend on multipath arrival rates or decay factors. Moreover, the performance of the UWB system is same as that of OFDM system for the narrow band communications in a Rayleigh fading environment.

Therefore, for uncoded multiband OFDM UWB system, no performance improvement is obtained from the rich multipath clustering property of UWB channels which means joint encoding of the information across multiple subcarriers is the one that reveals the performance of the system depends on the path arrival rates and decay factors.

It has been introduced and discussed briefly that the performance of the OFDM UWB system can be improved by utilizing channel coding techniques in [12].

Our future work addresses modelling MB-OFDM UWB system for realistic UWB environment and investigates the impact of joint coding across subcarriers in the performance of the coherent detection system.

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