

An Iterative Receiver Design with CFO, SFO, Channel Estimation and Equalization for OFDM Systems over Multipath Fading Channels

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Abstract— An iteration scheme is proposed to robust carrier frequency offset (CFO), sampling frequency offset (SFO) and channel impulse response (CIR) estimation for OFDM systems over synchronization errors and multipath fading channel environments. The development of the estimators involves the following procedures. First, a joint time-frequency cross-correlation scheme is proposed to estimate CFO and SFO, which can resist the multipath fading channel effect. Next, using the estimated CFO and SFO parameters, a least square (LS) algorithm with low dimensional sub-block transformed-preamble is utilized to estimate CIR. Finally, with inter-carrier interference (ICI) cancellation, the proposed estimators can provide more accurate parameters estimation. The main advantage of the proposed estimators is lower computational complexity. Computer simulation results show that the proposed estimators with ICI cancellation can obviously offer better performance and overcome the error floor problem at higher SNR scenario.

Keywords— OFDM, carrier frequency offset, sampling frequency offset, channel impulse response, least square, inter-carrier interference.

I. Introduction

OFDM system is a well-known technique with wide bandwidth efficiency, high transmission capability and robustness against multipath delay. However, OFDM systems are sensitive to carrier frequency offset (CFO) as well as sampling frequency offset (SFO) [1]. These kinds of synchronization errors will break the orthogonality among subcarriers and introduce inter-carrier interference (ICI) into the received signal, which leads to serious performance degradation.

Based on preamble, there are some previous works [2]-[4] on SFO, CFO and channel impulse response (CIR) as well. In their research, a cost function using maximum likelihood (ML) scheme for coarse estimation of CFO and SFO was developed. Then, the recursive least square (RLS) and tracking algorithms are proposed for CFO, SFO and CIR estimation. It truly has high performance quality but needs large computational complexity and technical efforts for parameters estimation.

In this paper, based on preamble sequence, a joint

time-frequency cross-correlation method is proposed to estimate CFO and SFO, which involves the immunity for multipath fading channel effect. It can provide good enough performance and involve lower computational complexity than other schemes. Next, a least square (LS) [5]-[6] with low dimensional sub-block transformed-preamble concept is utilized to estimate the time-domain CIR. Sub-block transformed-preamble not only can reduce the computation of matrix inversion from $N \times N$ to $L \times L$, where N is the total number of subcarriers and L is the number of multipath, but also can reduce noise enhancement. In the previous proposed estimators, ICI will induce error floor problem and conduct the degradation for parameters estimation. In order to enhance the estimation performance, iteration scheme with ICI reconstruction and cancellation [7] is proposed to join the proposed SFO, CFO, and CIR estimators. From simulation results, it is shown that the proposed estimators with ICI cancellation can provide robust performance and overcome the error floor problem at higher SNR scenario.

II. System Model

SFO and CFO are two synchronization errors discussed in this paper, and they are separately caused by the difference with transmitter and receiver sampling frequency and oscillator carrier frequency. Before introduced the proposed algorithm, an OFDM modulated transmit signal should be define first. The l th transmitted OFDM symbol with N subcarriers f_k modulation can be represented as

$$x_l(t) = \frac{1}{N} \sum_{k=0}^{N-1} X_l[k] e^{j2\pi f_k (t-lT_{sym})} \quad (1)$$

where $X_l[k]$ is the transmitted signal in the k th subcarrier and $T_{sym} = (N + G)T_s$ is the duration of the OFDM symbol with the guard interval G samples and T_s being the sampling time at transmitter side. Next, the received signal in the presence of carrier frequency offset Δf can be expressed as

$$y_l(t) = x_l(t)e^{j2\pi\Delta f t} + w_l(t) \quad (2)$$

Then, the received signal is sampled at $t = lT'_{sym} + nT'_s$ over the l th OFDM symbol duration, where $T'_{sym} = (N+G)T'_s$ and $T'_s = T_s(1+\varepsilon_s)$ are the sampling time with offset ε_s , yielding the discrete-time data samples $y_l[n]$:

$$\begin{aligned} y_l[n] &= y_l[lT'_{sym} + nT'_s] \\ &= \frac{1}{N} \sum_{k=0}^{N-1} X_l[k] e^{j2\pi \frac{k}{N} (l(N+G)\varepsilon_s T_s + n(1+\varepsilon_s)T_s)} \\ &\quad \cdot e^{j2\pi (\frac{N+G}{N} \varepsilon_f l + \frac{\varepsilon_f}{N} n)} + w_l[n] \end{aligned} \quad (3)$$

where $w_l[k]$ is AWGN noise and $\varepsilon_f = NT_s\Delta f$ is the normalized frequency offset. After FFT, the received signal with SFO and CFO effects in the frequency domain is given by

$$\begin{aligned} Y_l[k] &= FFT\{y_l[n]\} \\ &= \sum_{n=0}^{N-1} y_l[n] e^{-j2\pi \frac{k}{N} n} \\ &= \sum_{n=0}^{N-1} \frac{1}{N} \sum_{k'=0}^{N-1} X_l[k'] e^{j2\pi \frac{k'}{N} (l(N+G)\varepsilon_f + n(1+\varepsilon_s))} \\ &\quad \cdot e^{j2\pi (\frac{N+G}{N} \varepsilon_f l + \frac{\varepsilon_s}{N} n)} e^{-j2\pi \frac{k}{N} n} + w'_l[k] \\ &= \frac{1}{N} X_l[k] \sum_{n=0}^{N-1} e^{j2\pi \frac{(k\varepsilon_s + \varepsilon_f)}{N} n} e^{j2\pi \frac{N+G}{N} (k\varepsilon_s + \varepsilon_f) l} \\ &\quad + w_l[k] \\ &\quad + \frac{1}{N} \sum_{\substack{n=0 \\ (k' \neq k)}}^{N-1} X_l[k'] e^{j2\pi \frac{k'}{N} n} \\ &\quad \cdot \sum_{n=0}^{N-1} e^{j2\pi \frac{n}{N} (k' - k + k'\varepsilon_s + \varepsilon_f)} e^{j2\pi \frac{N+G}{N} (k'\varepsilon_s + \varepsilon_f) l} \\ &\quad + w'_l[k] \\ &= X_l[k] I_{kk} + \sum_{\substack{k'=0 \\ (k' \neq k)}}^{N-1} X_l[k'] I_{kk'} + w'_l[k] \end{aligned} \quad (4)$$

where

$$\begin{aligned} I_{kk} &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi \frac{(k\varepsilon_s + \varepsilon_f)}{N} n} e^{j2\pi \frac{(N+G)}{N} (k\varepsilon_s + \varepsilon_f) l} \\ &= \frac{1}{N} \cdot \frac{\sin(\pi k \varepsilon_s + \varepsilon_f)}{\sin(\pi k \frac{k\varepsilon_s + \varepsilon_f}{N})} \cdot e^{j\pi k (\varepsilon_s + \varepsilon_f) \frac{(N-1)}{N}} \\ &\quad \cdot e^{j2\pi \frac{(N+G)}{N} (k\varepsilon_s + \varepsilon_f) l} \\ &= A(k) \cdot e^{j\pi k (\varepsilon_s + \varepsilon_f) \frac{(N-1)}{N}} \cdot e^{j2\pi \frac{(N+G)}{N} (k\varepsilon_s + \varepsilon_f) l} \\ I_{kk'} &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi \frac{n}{N} (k'k + k'\varepsilon_s + \varepsilon_f)} \cdot e^{j2\pi \frac{(N+G)}{N} (k'\varepsilon_s + \varepsilon_f) l} \\ &= \frac{1}{N} \cdot \frac{\sin(\pi (k' - k + k'\varepsilon_s + \varepsilon_f))}{\sin(\pi \frac{(k' - k + k'\varepsilon_s + \varepsilon_f)}{N})} \\ &\quad \cdot e^{j2\pi \frac{(N+G)}{N} (k'\varepsilon_s + \varepsilon_f) l} \cdot e^{j\pi (k' - k + k'\varepsilon_s + \varepsilon_f) \frac{(N-1)}{N}} \end{aligned}$$

with $A(k)$ denoting by the magnitude of I_{kk} , i.e.,

$$A(k) = \frac{1}{N} \cdot \frac{\sin(\pi k \varepsilon_s + \varepsilon_f)}{\sin(\pi k \frac{k\varepsilon_s + \varepsilon_f}{N})}$$

Based on the derivation, SFO and CFO will cause ICI interference, which the serious ICI effect destroys the orthogonality between subcarriers and degrades the receiver efficiency.

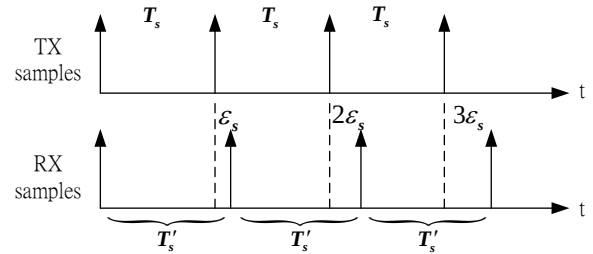


Fig. 1. SFO schematic with $T'_s = T_s(1 + \varepsilon_s)$.

Next, assuming the delay spread of the multipath fading channel is smaller than the guard interval. After removing the guard interval, the post-FFT received signal is given by

$$Y_l[k] = H_l[k] X_l[k] I_{kk} + \tilde{w}_l[k] \quad (5)$$

where $\tilde{w}_l[k]$ represents the combination of ICI part $I_{kk'}[k]$ and noise part of received data in (4), $X_l[k]$ is the transmitted signal or preamble sequence, and $H_l[k]$ is the frequency domain channel matrix.

III. AN ITERATIVE RECEIVER DESIGN WITH CFO, SFO, CHANNEL ESTIMATION AND EQUALIZATION

An iterative receiver is developed whose overall schematic diagram is depicted in Fig. 2. The receiver consists of the joint SFO and CFO estimator, the least square channel estimator with low dimensional sub-block transformed-preamble, channel equalizer, interference reconstruction, and cancellation schemes. Besides, the frame format of the proposed system is shown in Fig. 3, which consists of the two block-type preamble symbols and data symbols.

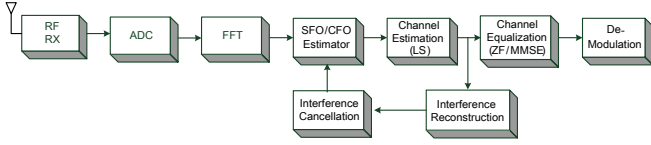


Fig. 2. Block diagram of the proposed receiver.

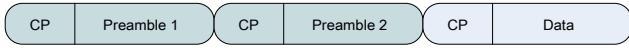


Fig. 3. Frame format of the proposed transceiver.

A. Joint SFO and CFO Estimators

The following development considers the design of the joint SFO and CFO estimation. In order to estimate SFO and CFO parameters and overcome the multipath fading channel effect, the time-frequency cross correlation method with low computational complexity is proposed. Assume that channel is a static channel, i.e., $H_l[k] = H_{l+1}[k]$. For simplifying, the l th and $(l+1)$ th preamble symbols at the k th subcarrier are assigned equivalent, i.e., $X_l[k] = X_{l+1}[k]$. Even through the two preamble symbols are different, the proposed scheme is still available due to the preamble symbols being known at receiver side. Next, according to time-domain cross correlation of two preamble symbols at the same k th subcarrier, the correlated signal can be obtained as

$$\begin{aligned} R[k] &= Y_l^*[k]Y_{l+1}[k] \\ &= |X_l[k]H_l[k]|^2 |A[k]|^2 e^{j2\pi \frac{N+G}{N}(k\epsilon_s + \epsilon_f)} \end{aligned} \quad (6)$$

To eliminate the CFO parameter in the exponential term of $R[k]$, the frequency-domain cross correlation scheme of the time-domain correlated signal $R[k]$ at the k_1 and k_2 subcarriers is proposed. Then, the frequency-domain correlated signal can be expressed as

$$\begin{aligned} Z[k_1] &= R^*[k_1]R[k_2] \\ &= C \cdot e^{j2\pi \frac{N+G}{N}m\epsilon_s} \end{aligned} \quad (7)$$

where, as shown in Fig. 3 with block type preamble symbols, k_1 and k_2 are chosen the upper and lower half subcarriers, respectively, i.e., $k_1 = 1 \sim m$, $k_2 = k_1 + m$, and $m = N/2$.

Therefore, based on (6) and (7), SFO and CFO parameters can be estimated by

$$\hat{\epsilon}_s = \frac{1}{2\pi \frac{N+G}{N}m} \angle \sum_{k_1=0}^{N/2-1} Z[k_1] \quad (8)$$

$$\hat{\epsilon}_f = \sum_{k=0}^{N-1} \frac{1}{2\pi \frac{N+G}{N}} (\angle R[k] - 2\pi \frac{N+G}{N}k\epsilon_s) \quad (9)$$

B. Channel Estimator

First, concatenating N consecutive symbols $Y_l[k]$ of equation (5) into a vector, the post-FFT received vector signal can be written as

$$\begin{aligned} \mathbf{Y}_l &= \begin{bmatrix} Y_l[0] \\ \vdots \\ Y_l[N-1] \end{bmatrix} \\ &= \begin{bmatrix} I_{0,0} & I_{0,1} & I_{0,2} & \cdots & I_{0,N-1} \\ I_{1,0} & I_{1,1} & I_{1,2} & \cdots & I_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ I_{N-2,0} & I_{N-2,1} & I_{N-2,2} & \cdots & I_{N-2,N-1} \\ I_{N-1,0} & I_{N-1,1} & I_{N-1,2} & \cdots & I_{N-1,N-1} \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} X_l[0] & \cdots & 0 & 0 \\ 0 & X_l[1] & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & X_l[N-1] \end{bmatrix} \begin{bmatrix} H_l[0] \\ \vdots \\ H_l[N-1] \end{bmatrix} + \mathbf{W}_l \\ &= \mathbf{I}_l \mathbf{X}_l \mathbf{H}_l + \mathbf{W}_l \\ &= \mathbf{I}_l \mathbf{X}_l \mathbf{F}_l \mathbf{h} + \mathbf{W}_l \\ &= \tilde{\mathbf{I}}_l \mathbf{h} + \mathbf{W}_l \end{aligned} \quad (10)$$

where $\mathbf{I}_l$ is a ICI interference matrix caused by SFO and CFO, $\mathbf{X}_l$ is the transmitted signal, $\mathbf{H}_l$ is frequency-domain channel effect vector, and $\mathbf{W}_l$ is represented as noise vector. $\mathbf{F}_l$ is a partial FFT matrix with $N \times L$ size, $\mathbf{h}$ is the $L \times 1$ time-domain CIR vector with L multipath channel fading, and $\tilde{\mathbf{I}}_l$ is a sub-block transformed-preamble matrix composed of $\mathbf{I}_l$, $\mathbf{X}_l$ and $\mathbf{F}_l$. It is noteworthy that $\tilde{\mathbf{I}}_l$ is a $N \times L$ matrix.

According to equation (10), a temporal estimation of the CIR can be obtained by least square method as

$$\begin{aligned} \hat{\mathbf{h}} &= \frac{1}{2} (\tilde{\mathbf{I}}_l^H \mathbf{Y}_l + \tilde{\mathbf{I}}_{l+1}^H \mathbf{Y}_{l+1}) \\ &= \frac{1}{2} [(\tilde{\mathbf{I}}_l^H \tilde{\mathbf{I}}_l)^{-1} \tilde{\mathbf{I}}_l^H \mathbf{Y}_l + (\tilde{\mathbf{I}}_{l+1}^H \tilde{\mathbf{I}}_{l+1})^{-1} \tilde{\mathbf{I}}_{l+1}^H \mathbf{Y}_{l+1}] \end{aligned} \quad (11)$$

Because $\tilde{\mathbf{I}}_l$ and $\tilde{\mathbf{I}}_{l+1}$ is a $N \times L$ matrix, it is obvious that the computational complexity in equation (11) with $L \times L$ matrix inversion is lower than the conventional

frequency-domain least square channel estimation with $N \times N$ matrix inversion.

C. Iterative SFO, CFO, and Channel Estimators

OFDM system is very sensitive to synchronization errors. Thus, the estimated errors in SFO, CFO, and CIR still induce a serious degradation for system performance. In order to enhance the estimation performance, the ICI interference in equation (5) needs to cancel via the estimated SFO, CFO, and CIR parameters in Subsections III.A and III.B. That is, the ICI interference matrix can be constructed by the estimated parameters, i.e.,

$$\hat{\mathbf{Y}}_I = \begin{bmatrix} 0 & \hat{I}_{0,1} & \cdots & I_{0,N-1} \\ \hat{I}_{1,0} & 0 & \vdots & I_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{I}_{N-1,0} & I_{N-1,1} & \cdots & 0 \end{bmatrix} \begin{bmatrix} X_I[0] & 0 & \cdots & 0 \\ 0 & X_I[1] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_I[N-1] \end{bmatrix} \begin{bmatrix} \hat{H}_I[0] \\ \hat{H}_I[1] \\ \vdots \\ \hat{H}_I[N-1] \end{bmatrix} \quad (12)$$

Next, as shown in Fig. 2, the received signal vector can be subtracted by the reconstructed ICI vector in (12), i.e.,

$$\tilde{\mathbf{Y}}_I = \mathbf{Y}_I - \hat{\mathbf{Y}}_I \quad (13)$$

where $\tilde{\mathbf{Y}}_I$ is the received signal with the interference cancellation. Therefore, the k th element of $\tilde{\mathbf{Y}}_I$ can be approached by the desired signal with channel fading and attenuator factor I_{kk} , i.e.,

$$\tilde{Y}_I[k] = H_I[k]X_I[k]I_{kk} + \bar{w}_I[k] \quad (14)$$

where $\bar{w}_I[k]$ is the AWGN noise and the residual ICI interference.

After the ICI interference cancellation in (13), by using $\tilde{Y}_I[k]$ in (14) as the new input of the joint SFO and CFO estimators in (6)-(9), the adverse poor convergence of these parameters can be effectively improved. Moreover, the accurate SFO and CFO can be utilized in (10)-(11), which can enhance the performance of CIR estimation. Note that the proposed SFO, CFO, and CIR estimators in (6)-(14) possess the low computational complexity advantage for OFDM system.

IV. COMPUTER SIMULATIONS

Simulation results are demonstrated to confirm the performance of the proposed receiver over the synchronization errors and multipath fading channel environments. The performance of the proposed SFO, CFO, and CIR estimation method is evaluated in a QPSK OFDM system with $N=128$ subcarriers, normalized SFO

$\varepsilon_s = 0.0001$, and normalized CFO $\varepsilon_f = 0.2$. For the channel model, $L=4$ independent Rayleigh fading paths are generated with the delay chosen from $[0, 3T_s]$, which is smaller than the guard interval $T_G=16T_s$, i.e., $G=16$. A quasi-static fading channel is assumed and the fading gains are i.i.d. complex Gaussian random variables with a zero-mean and unit variance. As a performance index, the simulation results of bit error rate (BER) and mean square error (MSE) versus signal-to-noise ratio (SNR) are conducted to demonstrate the performance of the proposed system. The MSE is defined as

$$\begin{aligned} \text{MSE}_{\varepsilon_s} &= \frac{1}{M} \sum_{i=1}^M \left(\hat{\varepsilon}_s^{(i)} - \varepsilon_s \right)^2 \\ \text{MSE}_{\varepsilon_f} &= \frac{1}{M} \sum_{i=1}^M \left(\hat{\varepsilon}_f^{(i)} - \varepsilon_f \right)^2 \\ \text{MSE}_h &= \frac{1}{ML} \sum_{i=1}^M \left\| \hat{\mathbf{h}}^{(i)} - \mathbf{h} \right\|^2 \end{aligned} \quad (15)$$

where M is the total of Monte-Carlo trials, i.e., $M=3000$ being used to obtain one MSE and BER values, with each trial using a different set of CIR.

In the first set of simulations, the MSE performance is evaluated as a function of SNR for the proposed joint SFO and CFO estimator with and without interference cancellation. The MSE results shown in Figs. 4 and 5 indicate that the proposed time-frequency cross correlation method can provide good enough performance, i.e., SFO $\text{MSE} = 10^{-7}$ and CFO $\text{MSE} = 6 \times 10^{-4}$ at $\text{SNR}=20\text{dB}$. However, after SNR is larger than 20 dB, the MSE performances of SFO and CFO show the error floor problem due to the ICI effect $\bar{w}_I[k]$ in (5). In order to overcome the problem, with the one time of ICI reconstruction and cancellation by the proposed joint SFO and CFO estimator, a more accurate SFO and CFO estimation results can then be acquired. After doing two times of iteration parameters estimation results can then be obviously obtained to guarantee no error floor at higher SNR and extremely better than without ICI cancellation or one iteration ICI cancellation.

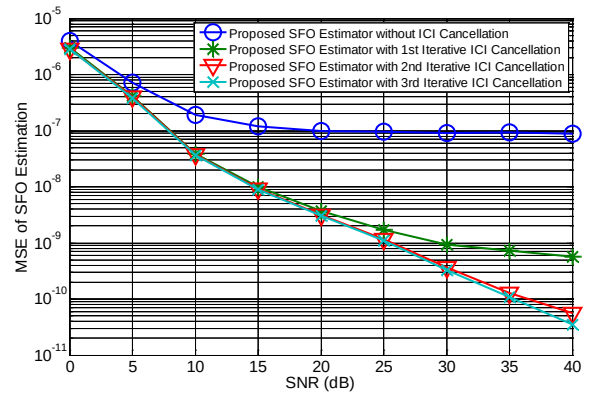


Fig. 4. MSE performance of proposed SFO estimation with SFO=0.0001 and CFO=0.2.

In the second set of simulations, the MSE performances

of the proposed CIR estimator with and without interference cancellation are shown in Fig. 6. As observed, the proposed least square CIR estimator with low computational complexity can provide CIR MSE performance about 2×10^{-3} at SNR=20dB. However, the error floor phenomenon still exists at higher SNR. As expected, the proposed CIR estimator with interference cancellation achieves better performance than those without it, especially after two times of iteration.

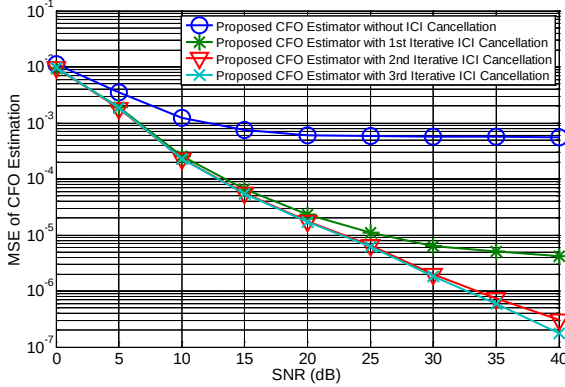


Fig. 5. MSE performance of proposed CFO estimation with SFO=0.0001 and CFO=0.2.

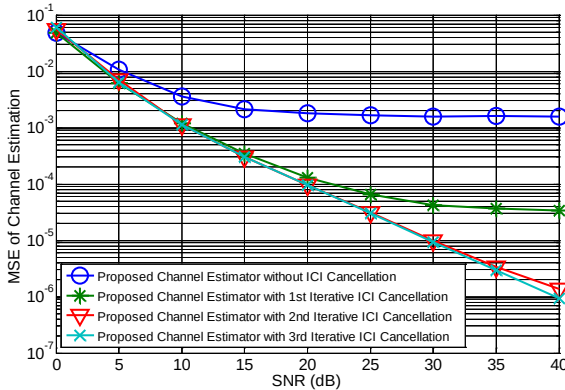


Fig. 6. MSE performance of least square CIR estimation.

In the third set of simulations, the BER performances of the proposed system with ZF equalizers are shown in Fig. 7. For performance comparison, the results obtained with the ideal MMSE and ZF equalizers are included. The ideal MMSE and ZF equalizers are implemented by the true SFO, CFO, and CIR parameters to obtain the detected signal. The results given in Fig. 7 show that the proposed techniques without doing ICI cancellation has error floor effect as SNR larger than 25dB because of non accurate parameters estimation results. If the proposed techniques involve ICI cancellation, then the BER curve will be close to ideal equalizers. Noted that, with ICI cancellation by the proposed system, an accurate SFO, CFO, CIR estimate can then be obtained to guarantee a better BER performance than without ICI cancellation.

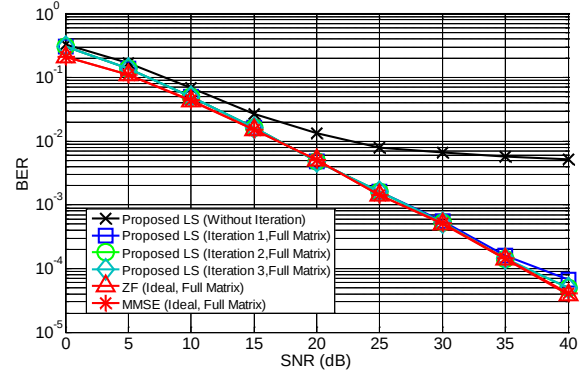


Fig. 7. BER performance of proposed receiver design.

V. CONCLUSIONS

An iterative receiver design with CFO, SFO, and CIR estimation is proposed for OFDM systems over synchronization errors and multipath fading channel environments. First, the novel joint time-frequency cross-correlation method with low complexity advantage can provide good enough performance for SFO and CFO estimation. Next, the time-domain least square CIR estimator with low computational complexity can perform accurate performance. Finally, with the ICI reconstruction and cancellation, the proposed SFO, CFO, and CIR estimators can offer the obvious performance improvement at higher SNR scenario. In the future, the proposed schemes with SFO, CFO and CIR estimation will be extended to use for cell-edge user communication under OFDMA downlink systems.

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REFERENCES

- [1] T. D. Chiueh and P. Y. Tsai, *OFDM baseband receiver design for wireless communication*, pp. 86-90, John Wiley, Dec. 4, 2007.
- [2] N. L. Hung, L. N. Tho, and C. K. Chi, "Joint channel estimation and synchronization with inter-carrier interference reduction for OFDM," *IEEE ICC*, pp. 2841-2846, June 2007.
- [3] N. L. Hung, L. N. Tho, and C. K. Chi, "Vector RLS-Based joint estimation of channel response and frequency offsets for MIMO-OFDM," *IEEE Globecom*, pp. 2801-2805, Nov. 2007.
- [4] N. L. Hung, L. N. Tho, and C. K. Chi, "Joint channel estimation and synchronization for MIMO-OFDM in the presence of carrier and sampling frequency offsets," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 6, pp. 3075-3081, July 2009.
- [5] Y. S. Cho, J. W. Kim, W. Y. Yang, and C. G. Kang, *MIMO-OFDM wireless communications with Matlab*, pp.153-180, Nov. 2010.
- [6] J. J. van de Beek, O. Edfors, M. Sandell, and S. K. Wilson, "On channel estimation in OFDM systems", *IEEE VTC*, vol.2, pp. 815-819, July 1995.
- [7] J. H. Deng, S. M. Liao, Y. W. Kuo, J. K. Hwang, and Y. L. Chiu, "An iterative receiver with intercarrier interference cancellation for OFDM communication systems with carrier frequency and timing offsets over multipath fading channels," *IEEE ISCE*, pp. 346-350, May 2009.