

Matched Filter Bank Implementation on FPGA for a Mutually Orthogonal Set of ZCZ Codes Using Hadamard and ZCZ Codes

Takahiro Matsumoto *, Takahiro Nishikawa *, Hideyuki Torii †, Shinya Matsufuji *

* Graduate School of Science and Engineering, Yamaguchi University,
2-16-1 Tokiwadai Ube, Yamaguchi, 755-8611, Japan

† Department of Information Network and Communication, Kanagawa Institute of Technology,
1030 Shimo-ogino, Atsugi, Kanagawa, 243-0292, Japan

matugen@yamaguchi-u.ac.jp, s028vk@yamaguchi-u.ac.jp, torii@nw.kanagawa-it.ac.jp, s-matsu@yamaguchi-u.ac.jp

Abstract—In this paper, we propose a new structure for the compact matched filter bank of a mutually orthogonal set of zero-correlation zone (ZCZ) codes obtained by Hadamard and ZCZ codes which reduces the number of operation elements such as two-input adders and delay elements. The matched filter banks are implemented on a field programmable gate array (FPGA) with 57,120 logic elements (LEs). A proposed matched filter bank of the sequence of length 128 can be constructed by the circuit scale of about 14 % compared with conventional matched filter bank.

Keywords—Hadamard code, zero-correlation zone (ZCZ) code, mutually orthogonal set, matched filter bank, field programmable gate array (FPGA)

I. INTRODUCTION

The mutually orthogonal set of zero-correlation zone (ZCZ) codes [1], [2], [3] which is obtained by Hadamard and ZCZ codes has zero-correlation zone between codes that belong to the same subset and is orthogonal between codes that belong to the different subset. From this correlation property, the mutually orthogonal set of ZCZ codes can be applied in a user- multi-input multi-output (MIMO) multistage relay transmission system consisting of multiple nodes having an antenna and functions of transmission, reception and relay transmission [4] and digital cellular systems. In this case, it is necessary to lengthen the sequence length to increase the number of users and nodes. Thereby, the matched filter bank which calculates the correlation function between a received signal and all sequences becomes large scale.

In this paper, we propose a new structure for compact matched filter banks of a mutually orthogonal set of ZCZ codes obtained by Hadamard and ZCZ codes which reduces the number of operation elements such as two-input adders and delay elements. These matched filter banks are implemented on a field programmable gate array (FPGA) with 57,120 logic elements (LEs).

II. MUTUALLY ORTHOGONAL SET OF ZCZ CODES USING HADAMARD AND ZCZ CODES

A. Definition of mutually orthogonal set of ZCZ codes

Let a_j^s be a binary sequence of length N whose elements take 1 or -1 , written as

$$a_j^s = (a_{j,0}^s, a_{j,1}^s, \dots, a_{j,i}^s, \dots, a_{j,N-1}^s), \quad (1)$$

$$a_{j,i}^s \in \{1, -1\},$$

where s is the set number, j is the sequence number, i is the order variable and denotes $i \bmod N$. Let A^s be a set of binary sequences a_j^s , written as

$$A^s = \{a_0^s, a_1^s, \dots, a_j^s, \dots, a_{M-1}^s\}, \quad (2)$$

where M is the number of sequences in a subset and is called family size, s is the set number and $0 \leq s \leq S-1$.

A periodic correlation function between sequences at shift i' is defined by

$$\rho_{a_j^s, a_{j'}^{s'}, i'} = \sum_{i=0}^{N-1} a_{j,i}^s a_{j',i-i'}^{s'}. \quad (3)$$

The sequence is called the mutually orthogonal set of zero-correlation zone (ZCZ) code if the periodic correlation functions satisfy

$$\rho_{a_j^s, a_{j'}^{s'}, i'} = \begin{cases} N & ; i' = 0, j = j', s = s', \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq ZCZ, s = s', \\ 0 & ; i' = 0, s \neq s'. \end{cases} \quad (4)$$

B. Construction of mutually orthogonal set of ZCZ codes using Hadamard and ZCZ codes

Let $\mathbf{H}_{N_1}$ be the Sylvester-type Hadamard matrix of size $N_1 \times N_1 = 2^{n_1} \times 2^{n_1}$ with $n_1 \geq 1$, written as

$$\mathbf{H}_{N_1} = [h_0, h_1, \dots, h_j, \dots, h_{N_1-1}]^T, \quad (5)$$

$$h_j = (h_{j,0}, h_{j,1}, \dots, h_{j,i}, \dots, h_{j,N_1-1}), \quad (6)$$

$$h_{j,i} \in \{1, -1\},$$

where the symbol T denotes the matrix transposition, which is defined by

$$\mathbf{H}_{N_1} = \mathbf{H}_{\frac{N_1}{2}} \otimes \mathbf{H}_2, \quad (7)$$

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad (8)$$

where the operation $\otimes$ denotes the Kronecker product. h_j is called a Hadamard sequence. The periodic correlation function at $i' = 0$ between h_j and $h_{j'}$ is given by

$$\rho_{h_j, h_{j'}, 0} = \begin{cases} N_1 & ; j = j', \\ 0 & ; j \neq j'. \end{cases} \quad (9)$$

Let $\hat{h}_j$ be a zero-value-inserted sequence of length $N_1 N_2$, which is obtained by inserting $N_2 - 1$ zero value between the adjacent two values of a Sylvester-type Hadamard sequence h_j of length N_1 , written as

$$\begin{aligned} \hat{h}_j &= (\hat{h}_{j,0}, \hat{h}_{j,1}, \dots, \hat{h}_{j,i}, \dots, \hat{h}_{j,N_1 N_2 - 1}) \\ &= (h_{j,0}, \underbrace{0, 0, \dots, 0}_{N_2 - 1}, h_{j,1}, \underbrace{0, 0, \dots, 0}_{N_2 - 1}, h_{j,2}, \dots, \\ &\quad h_{j,N_1 - 2}, \underbrace{0, 0, \dots, 0}_{N_2 - 1}, h_{j,N_1 - 1}, \underbrace{0, 0, \dots, 0}_{N_2 - 1}). \end{aligned} \quad (10)$$

The periodic correlation function at shift i' between $\hat{h}_j$ and $\hat{h}_{j'}$ is given by

$$\rho_{\hat{h}_j, \hat{h}_{j'}, i'} = \begin{cases} N_1 & ; i' = 0, j = j', \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq N_2 - 1. \end{cases} \quad (11)$$

On the other hand, let z_j be the ZCZ code of length N_2 , written as

$$z_j = (z_{j,0}, z_{j,1}, \dots, z_{j,i}, \dots, z_{j,N_2 - 1}), \quad (12)$$

$z_{j,i} \in \{1, -1\}.$

The periodic correlation function at shift i' between z_j and $z_{j'}$ is given by

$$\rho_{z_j, z_{j'}, i'} = \begin{cases} N_2 & ; i' = 0, j = j', \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq Zc. \end{cases} \quad (13)$$

The ZCZ codes are bounded by $M \leq N_2 / (Zc + 1)$, where M is the number of sequences in a sequence family.

For example, a binary ZCZ code z_j of length N_2 with $Zc = 1$ and $M = N_2 / 2$ is generated by a binary sequence α of length N_2 and a Hadamard sequence h_j of length $N_2 / 2$ as

$$z_{j,i} = \alpha_i \cdot h_{j,i \bmod \frac{N_2}{2}}, \quad (14)$$

$$\alpha_i = \begin{cases} h_{0,i \bmod \frac{N_2}{2}} & = 1 \\ & ; 0 \leq i \leq \frac{N_2}{2} - 1, \\ -h_{1,i \bmod \frac{N_2}{2}} & = (-1)^{i+1} \\ & ; \frac{N_2}{2} \leq i \leq N_2 - 1. \end{cases} \quad (15)$$

A binary sequence $a_j^s = (a_{j,0}^s, a_{j,1}^s, \dots, a_{j,i}^s, \dots, a_{j,N-1}^s)$ of length $N = N_1 N_2$ is produced by

$$a_{j,i}^s = \hat{h}_{s,i} * z_{j,i}, \quad (16)$$

where $*$ denotes the convolution as

$$\hat{h}_{s,i} * z_{j,i} = \sum_{k=0}^{N_1 N_2 - 1} \hat{h}_{s,k} z_{j,i-k}, \quad (17)$$

where $z_{j,i} = 0$ with $i < 0$ and $i \geq N_2$. From Eqs. (11), (13) and (16), the periodic correlation function at shift i' between a_j^s and $a_{j'}^{s'}$ is given by

$$\begin{aligned} \rho_{a_j^s, a_{j'}^{s'}, i'} &= \sum_{i=0}^{N-1} a_{j,i}^s a_{j',i-i'}^{s'} \\ &= a_{j,i'}^s * a_{j',-i'}^{s'} \\ &= (\hat{h}_{s,i'} * z_{j,i'}) * (\hat{h}_{s',-i'} * z_{j',-i'}) \\ &= \rho_{\hat{h}_s, \hat{h}_{s'}, i'} * \rho_{z_j, z_{j'}, i'} \\ &= \begin{cases} N_1 N_2 & ; i' = 0, j = j', s = s', \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq Zc, s = s', \\ 0 & ; i' = 0, s \neq s', \end{cases} \end{aligned} \quad (18)$$

with $N = N_1 N_2$. Therefore, the binary sequence a_j^s of length $N = N_1 N_2$ is a mutually orthogonal set of ZCZ codes with $S = N_1$ and $M = N_2 / 2$.

III. MATCHED FILTER BANK FOR A MUTUALLY ORTHOGONAL SET OF ZCZ CODES

A. Conventional matched filter bank for a mutually orthogonal set of ZCZ codes

The matched filter bank is a set of matched filters to calculate the correlation function between a received signal and all sequences.

The transfer function of matched filter bank for a mutually orthogonal set of ZCZ codes a_j^s is written as

$$\begin{aligned} A_j^s(Z) &= a_{j,0}^s Z^{-(N-1)} + a_{j,1}^s Z^{-(N-2)} + \dots \\ &\quad + a_{j,N-2}^s Z^{-1} + a_{j,N-1}^s \\ &= \sum_{i=0}^{N-1} a_{j,i}^s Z^{-(N-1-i)}. \end{aligned} \quad (19)$$

Therefore, a filter bank is constructed by arranging $(S \cdot M)$ direct-type matched filters in parallel as shown in Fig. 1. The direct-type matched filter of a sequence of length N is shown in Fig. 2, and is constructed from $N-1$ adders and $N-1$ delay elements. Therefore, the numbers N_{add} of two-input adders and N_{del} of delay elements in the conventional filter bank are

$$\begin{aligned} N_{add} = N_{del} &= (N-1) \cdot M \cdot S \\ &\leq \frac{N(N-1)}{Zc+1}. \end{aligned} \quad (20)$$

It is not easy to realize the filter bank because the adders and delay elements increase as $O(N^2)$.

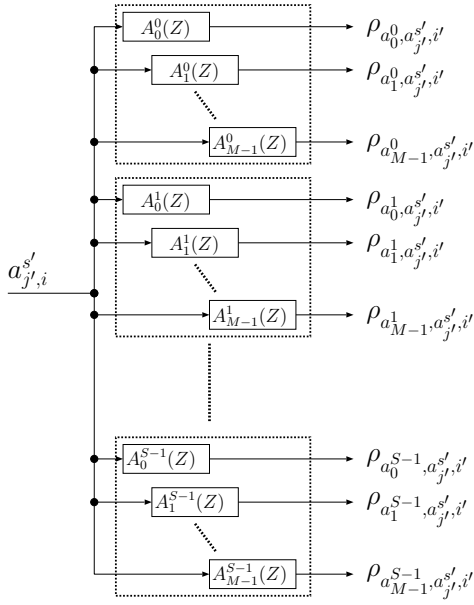


Fig. 1: A conventional matched filter bank of a mutually orthogonal set of ZCZ codes a_j^s with family size M and set number S .

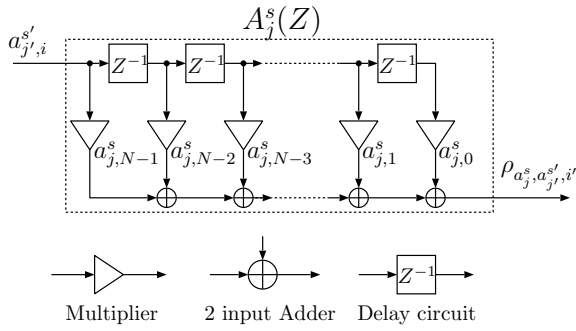


Fig. 2: A direct-type matched filter of a sequence a_j^s of length N .

B. Proposed matched filter bank of a mutually orthogonal set of ZCZ codes

From Eq. (16), the periodic correlation function $\rho_{a_{j,i}^{s'}, a_{j,i}^{s'}}$ shown in Eq. (18) is written as

$$\rho_{a_{j,i}^{s'}, a_{j,i}^{s'}} = (\hat{h}_{s,i'} * z_{j,i'}) * a_{j,i}^{s'}. \quad (21)$$

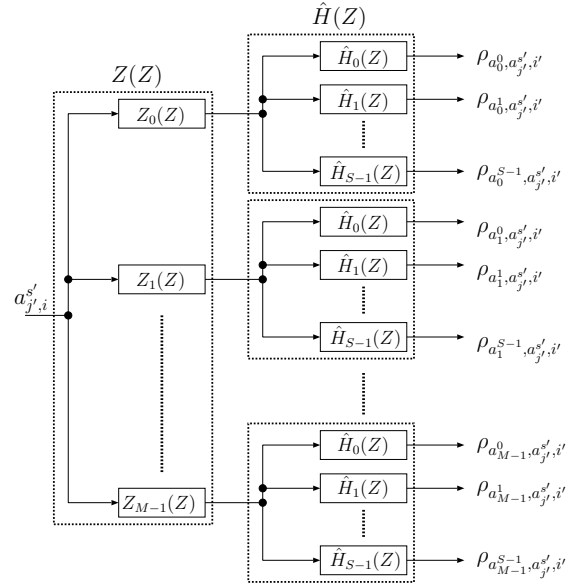


Fig. 3: A proposed matched filter bank of a mutually orthogonal set of ZCZ codes a_j^s with family size M and set number S .

Therefore, the transfer function of matched filter bank of a mutually orthogonal set of ZCZ codes a_j^s is written as

$$\begin{aligned} A_j^s(Z) &= \hat{H}_s(Z) \cdot Z_j(Z) \\ &= (h_{s,0}Z^{-\{N_2(N_1-1)\}} + h_{s,1}Z^{-\{N_2(N_1-2)\}} + \dots \\ &\quad + h_{s,N_1-1})(z_{j,0}Z^{-(N_2-1)} + z_{j,1}Z^{-(N_2-2)} \\ &\quad + \dots + z_{j,N_2-1}) \\ &= \left[\sum_{i=0}^{N_1-1} h_{s,i}Z^{-\{N_2(N_1-1-i)\}} \right] \\ &\quad \cdot \left\{ \sum_{i=0}^{N_2-1} z_{j,i}Z^{-(N_2-1-i)} \right\}. \end{aligned} \quad (22)$$

Figure 3 shows a proposed matched filter bank of a mutually orthogonal set of ZCZ codes a_j^s with family size M and set number S . In addition, matched filter banks of Hadamard and ZCZ codes can reduce a circuit size applying the fast algorithm to an M-sequence-type Hadamard matrix [5]. Figures 4 and 5 show a compact matched filter bank of Hadamard and ZCZ codes, respectively [6]. Generally, the numbers N_{add} of two-input adders and N_{del} of delay elements in the filter bank of a mutually orthogonal set of ZCZ codes with $Zcz = 1$ are the following:

$$\begin{aligned} N_{add} &= \frac{N_2}{2} \log_2 N_2 + \frac{N}{2} \log_2 \frac{N}{N_2}, \\ N_{del} &= N_2 - 1 + \frac{N_2}{2}(N - N_2). \end{aligned}$$

In short, the numbers N_{add} of two-input adders and N_{del} of delay elements have been decreased from $O(N^2)$ to $O(N \log_2 N)$ and $O(N)$, respectively.

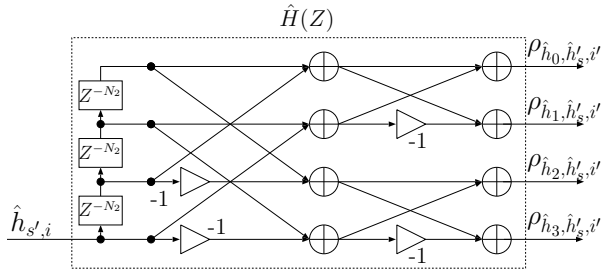


Fig. 4: A matched filter bank of a Hadamard codes $\hat{h}_s$ with length 4 and family size 4.

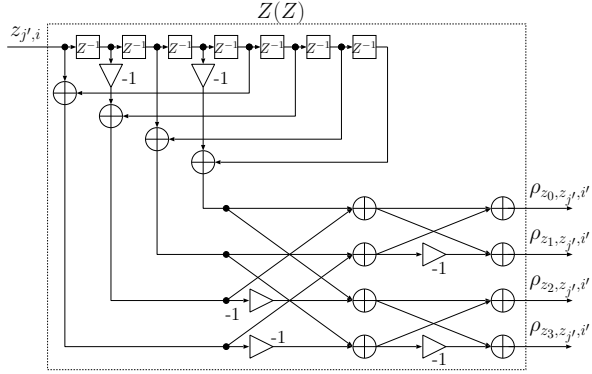


Fig. 5: A matched filter bank of a ZCZ codes z_j with length 8, family size 4 and $Zcz = 1$.

IV. MATCHED FILTER BANK IMPLEMENTATION ON FPGA

Matched filter banks of the sequence of length $N = 32, 64, 128$, the family size $M = 4$, set size $S = 8, 16, 32$ and $Zcz = 1$ have been implemented on a field programmable gate array (FPGA) corresponding to 57,120 logic elements (LEs) which is the basic building blocks of a FPGA, containing a 4-input look up table (LUT), a register, and additional logic. This FPGA has 782 pins which the user can freely use. The word-length of the input and output is 10 bits, respectively. Table I shows the resultant specification of matched filter banks. Figure 6 shows the number of logic elements (LEs) of matched filter banks of the sequences of length 32, 64 and 128, and the family size 4, and the set size 4, 8 and 16, respectively. As

TABLE I: Specifications of matched filter banks of mutually orthogonal sets of ZCZ codes.

Spreading sequence	mutually orthogonal set of ZCZ codes
Sequence length N	32, 64, 128
Zero correlation zone Zcz	1
Family size M	4
Set size S	4, 8, 16
Input output word-length	10bit
Samples per chip	1
FPGA	Altera, Stratix, EP1S60F1020C5
Max. logic elements(LEs)	57,120
Max. pins	782

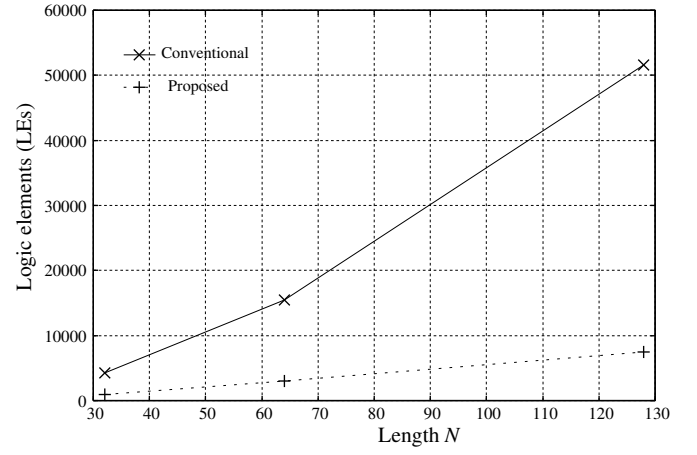


Fig. 6: Logic elements (LEs) of matched filter banks of mutually orthogonal sets of ZCZ codes of $Zcz = 1$.

a result of implement on FPGA, the proposed matched filter banks can reduce logic elements (LEs) than the conventional matched filter banks.

V. CONCLUSIONS

In this paper, we have proposed compact matched filter banks of a mutually orthogonal set of ZCZ codes obtained by Hadamard and ZCZ codes with $Zcz = 1$, which are constructed by applying a fast algorithm to an M-sequence-type Hadamard matrix, in which the numbers of two-input adders and delay elements are decreased from $O(N^2)$ to $O(N \log_2 N)$ and $O(N)$, respectively. The conventional and proposed matched filter banks are implemented on an FPGA, and the proposed matched filter banks have fewer logic elements than conventional matched filter banks.

ACKNOWLEDGEMENTS

This work is supported by the Ministry of Education, Culture, Sports, Science and Technology (MEXT), Grant-in-Aid for Young Scientists (B) (24760301), and VLSI Design and Education Center (VDEC), the University of Tokyo in collaboration with Mentor Graphics, Inc..

REFERENCES

- [1] A. Rathninkumar and A. K. Chaturvedi, "Mutually orthogonal sets of ZCZ sequences," *Electron. Lett.*, vol. 40, no. 18, pp. 1133–1134, Sept. 2004.
- [2] F. X. Zeng, "New perfect polyphase sequences and mutually orthogonal ZCZ polyphase sequence sets," *IEICE Trans. Fundamentals*, vol. E92-A, no. 7, pp. 1731–1736, July 2009.
- [3] Y. Li, C. Xu, and K. Liu, "Construction of mutually orthogonal zero correlation zone polyphase sequence sets," *IEICE Trans. Fundamentals*, vol. E94-A, no. 4, pp. 1159–1164, April 2011.
- [4] R. Kunimitsu, T. Matsumoto, S. Matsufuji and N. Kuroyanagi, "Proposal on User-MIMO Multistage Relay Transmission System having Route Separation Function," *IEICE Tech. Report*, RCS2011-243, pp. 63–67, Dec. 2011 (in Japanese).
- [5] N. Suehiro, "Fast algorithms for M-sequence type and E-sequence type Hadamard matrices," *Proc. of ISSSTA'92*, pp. 55–58, 1992.
- [6] T. Matsumoto and S. Matsufuji, "Compact matched filter banks of optical ZCZ codes using fast algorithm for M-sequence type Hadamard matrix," *J. Signal Processing*, vol. 14, no.6, pp. 427–432, Nov. 2010.