

Performance Analysis of Multi-Carrier Direct-Sequence (MC DS) CDMA with Fading

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Abstract—The performance of an MC DS CDMA wireless communication system over a Rayleigh Fading Channel is analyzed in the presence of Multiple Access Interference (MAI) and Inter-Carrier Interference (ICI). The Signal to Interference plus Noise Ratio (SINR) is derived in this environment. The analysis is also extended to MC-DS-CDMA system with Rake Receiver. The performance results are evaluated numerically in terms of SINR and BER considering several system parameters like number of users, processing gain, number of sub-carriers and number of rake fingers. The result shows that there are significant deterioration in SINR and BER performance due to fading. Optimum system design parameters are determined from the analytical results for a given performance level.

Keywords - CDMA, SINR, Rayleigh Fading, Rake Receiver

I. INTRODUCTION

A model of MC DS-CDMA with rake receiver operating in a multipath fading environment is presented and an analytical expression of the system will be discussed. During the analysis, the high speed input random data bit stream to the system has been divided into a number of parallel low-speed data streams which is used to modulate a number of orthogonal sub-carriers in correlation with a Pseudo-random Noise (PN) Sequence code for each parallel branch. The modulation considered is QPSK. The output of the transmitter, which is a MC-DS-CDMA signal, is allowed to pass through a Rayleigh fading channel and is received by a Rake receiver. The expression of the output of the Rake receiver has been derived considering the effect of fading. The analytical approach has been carried out for determining the expression of inter carrier interference (ICI) and multiple access interference (MAI) and the expression of the carrier to interference ratio has been found. The expression for the Bit Error Rate (BER) has been derived for a given value of ICI and MAI and given number of sub-carrier and code length.

II. BASIC MODEL OF MC DS-CDMA

Let us consider an MC-DS-CDMA system along with the block diagrams of the transmitter section as shown in Figure 1 and the receiver section as shown in Figure 2. In a specific cell, let for any period of time, there are J-number of users and j-th user is taken as the reference[1,2].

Now, to have a system model, let us make following assumptions: all the users are active at any instant time, all the users' transmitter power levels are equal, all the transmitted

signals are suffering equal amount of fading (i.e. flat fading) at any time and the bit-rate is much larger than chip-rate of the coding PN sequences. Thus, the input data of the j-th user are converted into N_c parallel data streams and each of the parallel data is coded by the channel-respective code of the j-th user. Thus each data bit is split in time domain and then is modulated by the respective sub-carrier. To write the general expression of the sub-carriers, let us consider the following: ω_c = Frequency of the reference channel, $\Delta\omega_c$ = Frequency spacing between two successive channels and ϕ_k = Instantaneous phase angle of the k-th sub-carrier.

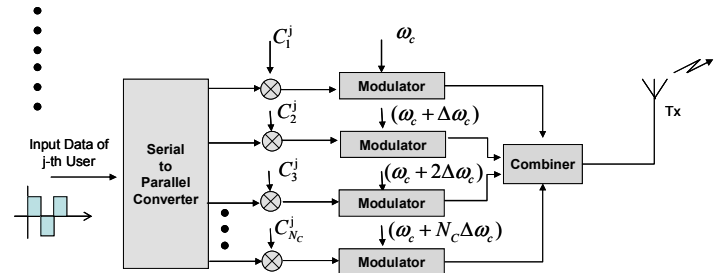


Figure 1. Block Diagram of MC DS-CDMA Transmitter

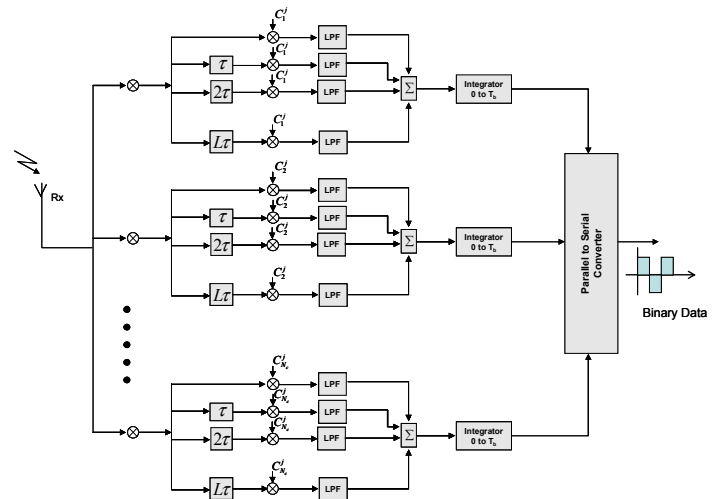


Figure 2. Block Diagram of MC DS-CDMA Receiver

Thus, expression of the transmitted signal of the j-th user is:

$$s_T(t) = \sum_{k=1}^{N_c} \left\{ \sqrt{2P} b_{n,k}^j \left(\sum_{x=1}^N C_{x,k}^j \right) \cos(\omega_c t + k \Delta \omega + \phi_k) \right\}$$

Here, $b_{n,k}^j$ = n-th bit of the j-th user, which is being modulated by the k-th channel and $C_{x,k}^j$ = x-th chip of the k-th section of the j-th user's code.

A. MC-DS CDMA Receiver without Rake

During reception, the receiver of the jth user receives signals transmitted by all the J number of users. Thus, the expression of the received signal is [4]:

$$r(t) = \sum_{m=1}^J \left[\sqrt{2P} \sum_{k=1}^{N_c} \left\{ b_{n,k}^m \left(\sum_{x=1}^{N_c} C_{x,k}^j \right) \cos(\omega_c t + k \Delta \omega_c t + \phi_k) \right\} + \eta(t) \right]$$

After necessary assumptions, the expression of the received signal takes the following form:

$$y(t) = \frac{N \alpha \sqrt{2P}}{2} \sum_{k=1}^{N_c} b_{n,k}^j \cos(\phi_k + \theta) + \sum_{m=1}^{j-1} \left[\frac{\alpha N \mu \sqrt{2P}}{2} \sum_{k=1}^{N_c} b_{n,k}^m \cos(\phi_k + \theta) \right] + \frac{1}{T_b} \int_0^{T_b} \sum_{k=1}^{N_c} \eta(t) \sum_{x=1}^{N_c} C_{x,k}^j \cos(\omega_c t + k \Delta \omega_c t) dt$$

$$\text{or, } y(t) = y_j(t) + y_{MAI}(t) + n(t)$$

Here, $y_j(t)$ = Desired signal of j-th user, $y_{MAI}(t)$ = Multiple Access Interference and $n(t)$ = Noise.

1) The Desired Signal: The desired signal of the j-th user is $y_j(t)$ which is given as follows:

$$y_j(t) = N \alpha \sqrt{P/2} \sum_{k=1}^{N_c} b_{n,k}^j \cos(\phi_k + \theta)$$

Since, $b_n^j = \pm 1$, $\sum_{k=1}^{N_c} b_{n,k}^j = N_c$. Then

$$y_j(t) = N N_c \alpha \sqrt{P/2} \cos(\phi_k + \theta)$$

Thus, for the j-th user, the desired signal power, P_s is given as

$$P_s = \frac{1}{2} \alpha^2 (N N_c \sqrt{P/2})^2 = \frac{1}{4} (\alpha^2 N^2 N_c^2 P) = \frac{1}{4} (\alpha^2 N^2 N_c^2 P)$$

2) Multiple Access Interference:

$$y_{MAI}(t) = \sum_{m=1}^{j-1} \left[\frac{\alpha N \mu \sqrt{2P}}{2} \sum_{k=1}^{N_c} b_{n,k}^m \cos(\phi_k + \theta) \right]$$

So the interference power is given as,

$$P_{MAI} = \sum_{m=0}^{j-1} \left[\frac{1}{2} \alpha^2 \mu^2 N^2 N_c^2 \frac{P}{2} \right] = \sum_{m=0}^{j-1} \left[\frac{1}{4} \alpha^2 \mu^2 N^2 N_c^2 P \right] = \frac{1}{4} \alpha^2 \mu^2 N^2 N_c^2 P (j-1)$$

3) Noise: The expression for noise power is:

$$n(t) = \frac{1}{T_b} \int_0^{T_b} w(t) \sum_{x=1}^N C_{x,k}^j \cos(\omega_c t + k \Delta \omega_c t) dt$$

Now, the noise power is: $\sigma_n^2 = \frac{N_0}{4T_b}$ where $N_0 = KTR_b$

4) Signal to Interference and Noise Ratio (SINR): Using the above quantities:

$$\text{SINR} = \frac{P_s}{P_{MAI} + \sigma_n^2} = \frac{\frac{1}{4} (\alpha^2 N^2 N_c^2 P)}{\frac{1}{4} \alpha^2 \mu^2 N^2 N_c^2 P (j-1) + \frac{N_0}{4T_b}} = \frac{\alpha^2 N^2 N_c^2 P T_b}{\alpha^2 \mu^2 N^2 N_c^2 P (j-1) T_b + N_0} = \frac{\alpha^2 N^2 N_c^2 E_b}{\alpha^2 \mu^2 N^2 N_c^2 E_b (j-1) + N_0} = \frac{\alpha^2 N^2 N_c^2}{\alpha^2 \mu^2 N^2 N_c^2 (j-1) + \left(\frac{E_b}{N_0} \right)^{-1}}$$

5) BER without Rake receiver: Now, this ξ is a function of α . So, for an instantaneous value of α , the instantaneous Bit Error Rate is

$$P_b(\alpha) = f(\xi) = \frac{1}{2} \operatorname{erfc} \left[\frac{\xi}{\sqrt{2}} \right]$$

$$P_b(\alpha) = 0.5 \times \operatorname{erfc} \times$$

$$\left[\frac{\alpha^2 N^2 N_c^2 (1/\sqrt{2})}{\alpha^2 \mu^2 N^2 N_c^2 (j-1) + \left(\frac{E_b}{N_0} \right)^{-1}} \right]$$

Now, the average bit error rate of the system without rake can be obtained as follows:

$$P_b = \int_0^\infty P_b(\alpha) f(\alpha) d\alpha, \text{ here, } P_b(\alpha) = \text{Instantaneous BER}$$

$f(\alpha)$ = pdf of amplitude distortion coefficient of Rayleigh Fading.

B. Evaluation of Bit Error Rate with Rake Receiver

Let us consider there are rake receivers in each sub-carrier channel which have L no of fingers and they are considered for Maximum Gain Combining [5]. With γ as the instantaneous SINR, the probability of bit error rate at the output is given as follows:

$$P_b(\gamma) = \frac{1}{2^{2L-1}} e^{-\gamma} \sum_{k=0}^{L-1} b_k \gamma^k \quad \text{where, } b_k = \frac{1}{k!} \sum_{n=0}^{L-1-k} \binom{2L-1}{n}$$

So, average bit error probability is: $P_b = \int_0^{\infty} P_b(\gamma) \cdot f(\gamma) d\gamma$

Thus, pdf of γ is given as:

$$f(\gamma) = \frac{d}{d\gamma} F(\gamma) \\ = M \left[\sum_{k=0}^{L-1} \psi_k (1 - e^{-\frac{\gamma}{\Lambda_k}}) \right]^{M-1} \sum_{k=0}^{L-1} \frac{\psi_k}{\Lambda_k} e^{-\frac{\gamma}{\Lambda_k}}$$

Substituting the above values, the expression for Average Bit Error Probability is given as:

$$P_b = \int_0^{\infty} P_b(\gamma) \cdot f(\gamma) d\gamma \\ = \int_0^{\infty} \left(\frac{1}{2^{2L-1}} e^{-\gamma} \sum_{k=0}^{L-1} b_k \gamma^k \right) \cdot \left(M \left[\sum_{k=0}^{L-1} \psi_k (1 - e^{-\frac{\gamma}{\Lambda_k}}) \right]^{M-1} \sum_{k=0}^{L-1} \frac{\psi_k}{\Lambda_k} e^{-\frac{\gamma}{\Lambda_k}} \right) d\gamma$$

III. SIMULATION AND DISCUSSION

The simulation is carried out in MATLAB environment. The simulation of the system without rake receiver is shown at first. Then the system with Rake receiver is simulated. Results have been evaluated numerically and degradation of system performance due to channel impairments has been determined in terms of BER.

A. Performance without Rake Receiver

Figure-3 presents the BER vs Eb/No plot without rake receiver for different number of users with fading variance, $\sigma_a=0.2$, bit-rate = 50 kbps, number of subcarrier = 8, length of spreading code per bit, $N = 16$, power = 10 and cross correlation, $\mu = 0.1$. The graph shows that with the increased number of users the BER increases.

Figure 4 presents the BER vs Eb/No graph without rake receiver for 16 users with different fading variances, bit-rate = 50 kbps, number of subcarriers = 8, length of spreading code per bit, $N = 8$, power = 1 and cross correlation, $\mu = 0.1$. The graph shows that with the increased number of channels, there is a considerable improvement in BER.

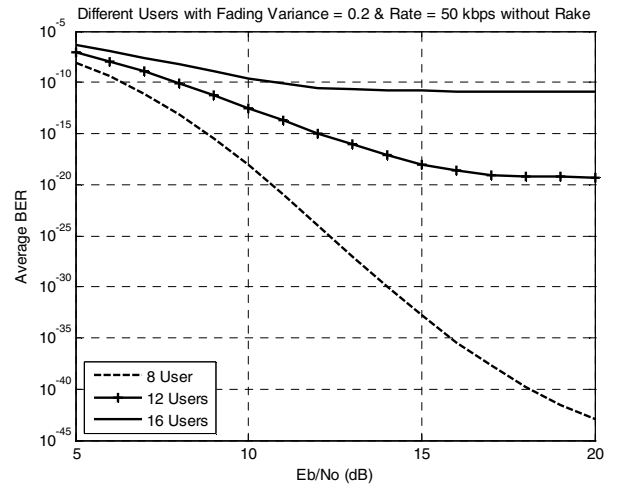


Figure 3. Different Users with Variance = 0.2 and 50 kbps

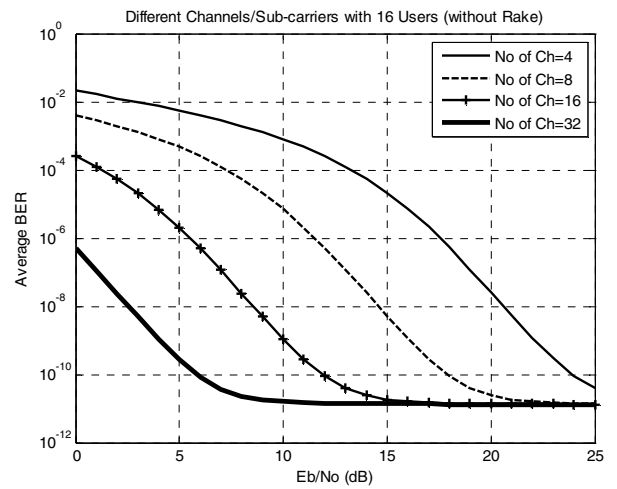


Figure 4. Graph of Different Channels without Rake

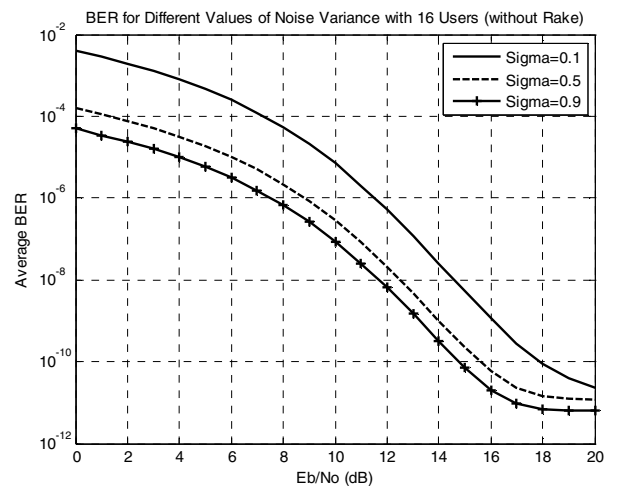


Figure 5. Channels with 16 Users and Different Sigma

Figure 5 presents the BER vs Eb/No plot without rake receiver for different fading variances with 16 users, bit-rate = 50 kbps, number of subcarriers = 8, length of spreading code per bit, $N = 8$, power = 10 and cross correlation, $\mu = 0.1$. The graph shows that with the improvement in fading or improvement in received signal level the BER also improves.

B. Performance with Rake Receiver

Performance of the system is evaluated using different number of Rake Fingers and compared with a system without Rake.

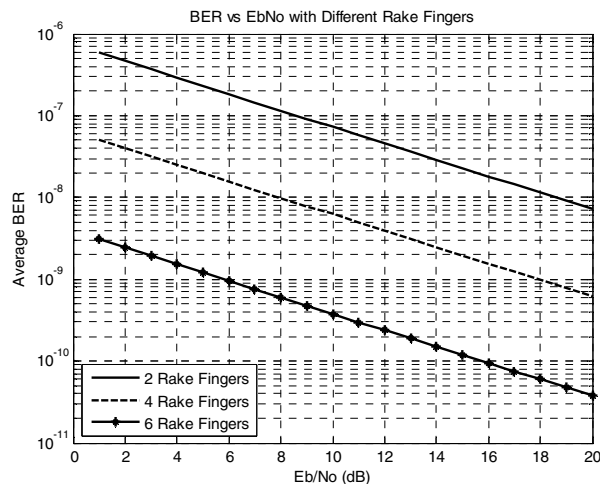


Figure 6. Channels with 16 Users and Different Rake Fingers

Figure 6 presents the BER vs Eb/No plot with rake receiver for different number of Fingers. The simulation is carried out with the fading variance, $\sigma_a=0.2$, bit-rate = 100 kbps, users = 32, number of subcarriers = 32, length of spreading code per bit = 32, power = 10 and cross correlation, $\mu = 0.2$. The results show that with the increased number of Fingers, the BER improves significantly.

Figure 7 presents the BER vs Eb/No plot with and without rake receiver for different number of users. The other parameters are; fading variance, $\sigma_a=0.2$, bit-rate = 50 kbps, number of subcarrier = 8, length of spreading code per bit, $N = 8$, power = 10 and cross correlation, $\mu = 0.1$. The graph shows that there is a significant improvement in terms of BER after taking the effect of Rake receiver.

IV. CONCLUSIONS

A novel theoretical analysis is carried out for an MC-DS CDMA system over a wireless faded system with and without diversity. The diversity is achieved by using Rake receiver. The results are evaluated in terms of BER considering various system parameters to optimize the parameters like number of sub-carriers, the code length and number of rake fingers. The results show that there is deterioration in BER due to fading.

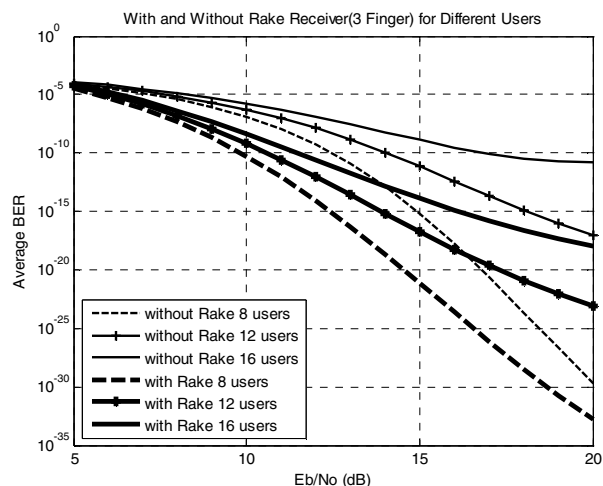


Figure 7. BER vs Eb/No for Different Rake Fingers

The results also show that there is a significant improvement in the system performance in terms of BER with the increased number of subcarriers and Rake receivers with higher number of Fingers. Increasing the number of rake Fingers, more number of users can be accommodated at a given level of BER and for a given number of sub-carriers.

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