

On Optical ZCZ Codes With a Good Aperiodic Autocorrelation Property

Ainiwan Abudoukeremu*, Shinya Matsufuji*, Takahiro Matsumoto*

* Graduate School of Science and Engineering, Yamaguchi University, Japan

E-mail:{r503we,s-matsu,matugen}@yamaguchi-u.ac.jp

Abstract—A quasi-synchronous optical code division multiple access (OCDMA) system, which uses a set of pairs of binary and bi-phase sequences with a zero correlation zone (Zcz) called an optical ZCZ (O-ZCZ) code as a spreading code, can efficiently mitigate multiple access interferences (MAI). This paper presents O-ZCZ codes of length 2^{n+1} with the minimum zero correlation zone $Z_{cz}=1$, which possess a low aperiodic autocorrelation property. They are derived from complementary sequence pairs of length 2^n that the sum of these aperiodic autocorrelation functions take zero except zero-shift.

Index Terms—sequence design, optical ZCZ code, E-sequence, optical communication

I. INTRODUCTION

In short-range wireless communications, quasi-synchronous code division multiple access (CDMA) systems using a set of sequences with a zero correlation zone (Zcz) called ZCZ code, whose out-of-phase periodic autocorrelation functions take zero for shifts around zero-shift have been discussed, since MAI can be removed [1]-[6]. There are ZCZ codes with a good aperiodic autocorrelation property, which can be used as a synchronization symbol and training ones for channel estimation [7]-[9].

On the other hand, in optical short range wireless communication, similarly OCDMA using optical ZCZ code (O-ZCZ code) is also interesting since MAI is free and system performance is high[10]-[11]. O-ZCZ code which is a set of pairs of binary and bi-phase sequences has large family size rather than optical orthogonal code (OOC) code which is a traditional code for OCDMA [12].

To aim at the further simplification of the hardware and corresponding to a system design flexibility, O-ZCZ code whose any sequence possesses a good aperiodic autocorrelation property is desirable. O-ZCZ code with low aperiodic correlation is derived from a subset of balanced sequences in ZCZ code constructed by using complementary sequences which mean a pair of sequences that the sum of out-of-phase autocorrelation values of the same shifts take zero [13]-[16].

In section 2, ZCZ code and O-ZCZ code are introduced. In section 3, construction method and logic function of ZCZ codes are shown. The balanced condition of O-ZCZ code derived from ZCZ code with low aperiodic auto correlation, which is given by use of complementary sequences including E-sequences whose aperiodic autocorrelation function takes zero at any even shift is concluded. In section 4, aperiodic autocorrelation functions of three types of O-ZCZ codes are evaluated by merit factor. Conclusions are drawn in section 5.

II. RELATION BETWEEN ZCZ CODE AND O-ZCZ CODE

A. ZCZ Code

Let Z be a set of bi-phase sequences of length L defined by

$$Z = \{a^0, a^1, \dots, a^j, \dots, a^{M-1}\}, \\ a^j = \{a_0^j, a_1^j, \dots, a_i^j, \dots, a_{L-1}^j\}, a_i^j \in \{1, -1\}.$$

The periodic correlation function is defined by

$$R_{a^j a^{j'}}(\tau) = \sum_{i=0}^{L-1} a_i^j a_{(i+\tau) \bmod L}^{j'}, \\ = C_{a^j a^{j'}}(\tau) + C_{a^j a^{j'}}(\tau - L),$$

where $C_{a^j a^{j'}}$ is aperiodic correlation function, and be defined as

$$C_{b^{j'} a^j}(\tau) = \begin{cases} \sum_{i=0}^{L-1-\tau} b_i^{j'} a_{(i+\tau)}^j, & 0 \leq \tau \leq L-1, \\ \sum_{i=0}^{L-1+\tau} b_{(i-\tau)}^{j'} a_i^j, & 1-L \leq \tau < 0, \\ 0, & |\tau| \geq L. \end{cases}$$

The set Z is called zero correlation zone code (ZCZ code) denoted by $Z(L, M, Z_{cz})$, if its periodic correlation function satisfies

$$R_{a^j a^{j'}}(\tau) = \begin{cases} L, & \tau = 0, j = j', \\ 0, & \tau = 0, j \neq j', \\ 0, & 1 \leq |\tau| \leq Z_{cz}. \end{cases}$$

It is called autocorrelation function for $j = j'$, crosscorrelation function for $j \neq j'$.

Note that the periodic correlation function can be expressed by aperiodic correlation function. It is assumed that the both periodic and aperiodic correlation function have similar correlation properties.

B. O-ZCZ Code

Let a^j be such a balanced sequences with same numbers of elements 1 and -1 composing the set Z . Similarly, consider a binary sequence b^j of length L given by

$$b^j = \{b_0^j, b_1^j, \dots, b_i^j, \dots, b_{L-1}^j\}, b_i^j = \frac{1 + a_i^j}{2} \in \{1, 0\},$$

Let A be a set of pairs of binary sequences b^j and bi-phase sequences a^j , written as

$$A = \{(b_L^0, a_L^0), \dots, (b_L^j, a_L^j), \dots, (b_L^{M-1}, a_L^{M-1})\}.$$

The set A is called optical ZCZ (O-ZCZ) code and denoted by $A(L, \hat{M}, Zcz)$, if its periodic correlation function satisfies

$$R_{b^{j'} a^j}(\tau) = \begin{cases} L/2, & \tau = 0, j = j', \\ 0, & \tau = 0, j \neq j', \\ 0, & 1 \leq |\tau| \leq Zcz. \end{cases}$$

The periodic correlation function between b^j and a^j at shift τ is written as

$$\begin{aligned} R_{b^{j'} a^j}(\tau) &= \sum_{i=0}^{L-1} b_i^{j'} a_{(i+\tau) \bmod L}^j \\ &= \sum_{i=0}^{L-1} \left(\frac{1 + a_i^j}{2} \right) a_{(i+\tau) \bmod L}^j \\ &= \frac{1}{2} L a_i^j + \frac{1}{2} R_{a_i^j a_i^j}(\tau), \end{aligned}$$

where

$$L a_i^j = \sum_{i=0}^{L-1} a_i^j.$$

If a_i^j is a balanced ZCZ sequence, then $L a_i^j = 0$.

O-ZCZ code $A(L, \hat{M}, Zcz)$ is restricted by the theoretical bound as

$$\hat{M} < M \leq \frac{L}{Zcz + 1}. \quad (1)$$

where $\hat{M}$ and M denote family size of O-ZCZ code and ZCZ code, respectively.

Note that as O-ZCZ code with low aperiodic autocorrelation is driven from a subset of balanced sequences in ZCZ code with low aperiodic autocorrelation properties.

III. CONSTRUCTION AND LOGIC FUNCTION

In this section, several methods for constructing O-ZCZ code given by ZCZ code with a good aperiodic autocorrelation property are introduced. The balanced condition of O-ZCZ code is theoretically concluded. A given logic function for ZCZ code is explained.

A. General Method

The Sylvester type Hadamard matrix of order 2^n used as a source matrix, is defined by

$$H_n = \begin{pmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{pmatrix}, \quad H_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

with $H_n H_n^T = 2^n I_n$, where I_n denotes a unit matrix of order 2^n .

ZCZ code $Z(2^{n+1}, 2^n, 1)$ is expressed by using a matrix of order $2^n \times 2^{n+1}$ defined by

$$Z = \begin{pmatrix} a^0 \\ a^1 \\ \vdots \\ a^j \\ \vdots \\ a^{2^n-1} \end{pmatrix} = H_n(I_n, D_n). \quad (2)$$

where, D_n is a diagonal matrix of order 2^n whose elements take $(-1, 1, -1, 1, \dots, -1, 1)$.

O-ZCZ code $A(2^{n+1}, 2^n - 2, 1)$ can be driven from a set of ZCZ sequences by removing first and second row, since the remaining rows are balanced in Eq.(2). However, in this construction method, O-ZCZ codes with a good aperiodic autocorrelation property can not be given, although the family size of sequences is large.

Note that aperiodic autocorrelation functions of O-ZCZ codes can be written as

$$C_{b^{j'} a^j}(\tau) = \begin{cases} L/2, & \tau = 0, j' = j, \\ 0, & \tau = 0, j' \neq j, \\ 0, 1, & |\tau| = 1. \end{cases}$$

B. Proposal Method

Since this paper expects O-ZCZ code with good aperiodic correlation, more construction methods will be considered. There is a construction method for ZCZ code which is proposed by T. Hayashida at all [9]. The proposal ZCZ code is constructed by using E-sequence with a good aperiodic autocorrelation property, satisfying $C_{ee}(\tau) = 0$ for even-shift $\tau = 2m(m \neq 0)$.

E-sequences are included in complementary sequences of length $N = 2^n$, that sum of the aperiodic autocorrelation functions is zero everywhere except at zero-shift [16], written as

$$C_{cc} + C_{\hat{c}\hat{c}}(\tau) = \begin{cases} 2N, & \tau = 0, \\ 0, & \tau \neq 0. \end{cases}$$

where c and $\hat{c}$ denote the pair of binary sequences, respectively.

Note that ZCZ code possess a good aperiodic autocorrelation property close to these two sequences, is desired.

The ZCZ codes $Z'(2^{n+1}, 2^n, 1)$ can be expressed by

$$Z' = \begin{pmatrix} a^0 \\ a^1 \\ \vdots \\ a^j \\ \vdots \\ a^{2^n-1} \end{pmatrix} = H_n G_n(I_n, D_n), \quad (3)$$

where G_n is a diagonal matrix of order 2^n whose diagonal elements equal to complementary sequences which including E-sequences expressed by

$$G_n = \begin{pmatrix} g_{00} & \cdots & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \cdots & 0 \\ \vdots & \cdots & g_{xx} & \cdots & 0 \\ 0 & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \cdots & g_{(n-1)(n-1)} \end{pmatrix}. \quad (4)$$

where $g_{xx} = (-1)^{g(\vec{x})}$ is a different expressions which is given in [15]. So that two types of ZCZ codes with good aperiodic correlation properties can be constructed by selecting G_n properly.

If the balanced condition of ZCZ code satisfies

$$\sum_{\vec{x} \in V_n} (-1)^{z'(\vec{x})} = 0. \quad (5)$$

then O-ZCZ codes $A(L, \hat{M}, Z_{cz})$ are exist, and a theoretical conclusion is given without proof.

[Theorem1] In O-ZCZ codes of length $L=2^{n+1}$, $Z_{cz}=1$, when data bits n is even, O-ZCZ codes constructed by using complementary sequences (including E- sequences) are exist, and the total numbers of O-ZCZ sequences are $\hat{M}=2^{n-1}$ in one set, respectively.

At the same time, when n is odd, O-ZCZ code constructed by using complementary (excluding E- sequences) are exist, and the total numbers of O-ZCZ sequences are $\hat{M}=(3/4)2^n$ in one set.

A given logic function for ZCZ code $Z'(2^{n+1}, 2^n, 1)$ based on Eq.(3) and Eq.(4), as

$$z'(\vec{x}) = y_0 y_1 \oplus \dots \oplus y_{n-2} y_{n-1} \oplus \vec{x} \cdot \vec{a} \oplus x_0 x_n \oplus x_n \quad (6)$$

where the $\vec{x} \in V_n$, $y_j \in \{x_0, x_1, \dots, x_{n-1}\}$ with $y_0=x_0$ ($y_{n-1} \neq x_0$), $\vec{a}=(a_0, a_1, \dots, a_{n-1})$ a parameter to give different E-sequences, and $\vec{a} \cdot \vec{x} = a_0 x_0 \oplus \dots \oplus a_{n-1} x_{n-1}$. If $y_0 \neq x_0$ ($y_{n-1} \neq x_0$), the logic function produces complementary sequences excluding E-sequences.

As an example, ZCZ code $Z'(8, 4, 1)$ is generated by the logic function in Eq.(6),

$$z'(\vec{x}) = x_0 x_1$$

which is expressed by

$$Z' = \underbrace{\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}}_{H_2} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}_{G_2} \\ \times \left(\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{I_2}, \underbrace{\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{D_2} \right) \\ = \begin{pmatrix} 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \end{pmatrix}.$$

where G_n used E-sequence as a diagonal matrix. The result of example has shown that O-ZCZ code with $\hat{M}=2$ is exist since it is driven from a subset of balanced sequences in ZCZ code $Z'(8, 4, 1)$ with low aperiodic correlation.

IV. EVALUATION OF O-ZCZ CODES

Three types of O-ZCZ codes are listed in Tabel 1. O-ZCZ1 is a traditional code. O-ZCZ2 a newly given O-ZCZ code

TABLE I
CONSTRUCTION METHODS OF O-ZCZ CODES

Type	G_n
O-ZCZ1	unit matrix
O-ZCZ2	diagonal matrix (E-sequence)
O-ZCZ3	diagonal matrix (Complementary sequence)

using complementary sequences (including E-sequences). O-ZCZ3 given by one of complementary sequences which do not include E-sequences in order to compare with O-ZCZ2.

The aperiodic autocorrelation functions of three types of O-ZCZ codes are evaluated. Two types of criterion are used. One is the merit factor F defined by

$$F = \frac{C_{b^j a^j}^2(0)}{\sum_{\tau=-L+1 \neq 0}^{L-1} C_{b^j a^j}^2(\tau)}, \quad (7)$$

and the other is the maximum absolute aperiodic correlation value C_{max} defined by

$$C_{max} = \max |C_{b^j a^j}|, \quad \text{for } 0 \leq |\tau| \leq L-1.$$

Note that F is desirable to be large as much as possible. On the other hand, C_{max} is desirable to be small.

In order to examine the aperiodic auto-correlation functions of O-ZCZ codes in Table 1, the maximum (max), minimum (min), and average (avg) of merit factors Eq.(7) are discussed. Tables 2 and 3 show the merit factors of O-ZCZ codes of length 32 and 128, respectively. O-ZCZ1 has the lowest merit factors. The maximum and average merit factors of O-ZCZ2 are higher than others. Therefore, O-ZCZ2 has low aperiodic autocorrelation. O-ZCZ3 has the highest minimum merit factors, but the maximum and average merit factors are relatively lower than O-ZCZ2.

TABLE II
MERIT FACTOR OF $A(32, 8, 1)$

	F_{max}	F_{min}	F_{avg}
O-ZCZ1	0.6531	0.1311	0.3921
O-ZCZ2	2.0645	0.8312	1.4479
	2.1333	0.8205	1.4769
O-ZCZ3	1.5610	0.8533	1.2071
	1.3061	0.7191	1.0126
	1.8530	0.7619	1.3076

TABLE III
MERIT FACTOR OF $A(128, 32, 1)$

	F_{max}	F_{min}	F_{avg}
O-ZCZ1	0.2029	0.0317	0.1173
O-ZCZ2	2.1603	0.8791	1.5197
	2.2305	0.8560	1.5433
O-ZCZ3	1.8221	1.2047	1.5134
	1.5851	1.1058	1.3455
	1.7965	1.2047	1.5003

Table 4 shows C_{max} 's, F_{max} 's and family size $\hat{M}$ of all O-ZCZ codes. All the merit factors of O-ZCZ2 codes of length 16 and 64 could not be given, since there exist an immense of O-ZCZ codes for data bits n is odd. O-ZCZ3 can not construct

O-ZCZ code of length $L = 8$, since all the complementary sequences of length 4 are E-sequence.

TABLE IV
APERIODIC AUTOCORRELATION FUNCTIONS OF O-ZCZ CODES

	L	8	16	32	64	128
O-ZCZ1	F_{avg}	0.53	0.55	0.36	0.21	0.12
	C_{max}	3	7	15	31	63
	M	2	6	14	30	62
O-ZCZ2	F_{avg}	1.17	—	1.46	—	1.53
	C_{max}	2	—	6	—	13
	M	2	—	8	—	32
O-ZCZ3	F_{avg}	—	0.84	1.18	0.63	1.45
	C_{max}	—	5	6	20	20
	M	—	6	8	24	32

The Figure 1 shows the aperiodic autocorrelation properties of three types of O-ZCZ codes A(32, 8, 1). It is observed that O-ZCZ2 codes have better aperiodic autocorrelation property than other two O-ZCZ codes, and it could be useful synchronous symbol and training one for channel estimation.

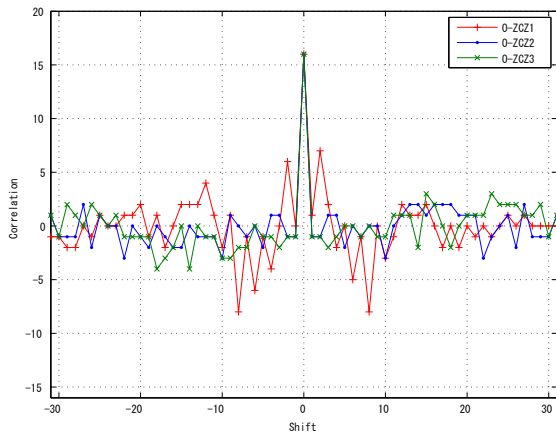


Fig. 1. Aperiodic autocorrelation functions of O-ZCZ codes A(32, 8, 1)

V. CONCLUSIONS

This paper has discussed O-ZCZ codes derived from a subset of balanced sequences in ZCZ code of length 2^{n+1} , $Z_{cz} = 1$ with low aperiodic correlation.

O-ZCZ code derived from the Sylvester type Hadamard matrix has $2^n - 2$ sequence pairs with a poor autocorrelation property. On the other hand, O-ZCZ code derived from an Hadamard matrix derived from complementary sequences pairs has 2^{n-1} sequence pairs with almost good aperiodic autocorrelation for even n and $(3/4)2^n$ ones for odd n , respectively. There exist O-ZCZ codes of 2^{n-1} sequences with good aperiodic autocorrelation for even n which are derived from an Hadamard matrix derived from E-sequences. It can be assumed that the proposal O-ZCZ codes with a good aperiodic correlation property can efficiently mitigate the MAI in quasi-synchronous OCDMA communication systems.

REFERENCES

- [1] P. Fan, and M. Darnell, "Sequence Design for Communication Applications," Research Studies Press LTD, 1997.
- [2] K. Takatsukasa, S. Matsufuji and N. Kuroyanagi, "Ternary ZCZ sequence sets for cellular CDMA systems," IEICE Trans. on Fundamentals of Electron., Communications and Computer Sciences, Vol. E85-A, No. 9, pp. 2135-2140, 2002.
- [3] N. Suehiro, "A signal Design Without Co-channel Interference for Approximately Synchronized CDMA Systems," IEEE Journal On Selected Areas in Communications, Vol. 12, No. 5, pp. 837-841, 1994.
- [4] T. Hayashida, S. Matsufuji, T. Matsumoto and P.Z. Fan, "ZCZ Codes With a Good Aperiodic Autocorrelation Property," Proc. CICC-ITOE2011, Vol.2, pp.362-365, 2011.
- [5] K. Takatsukasa, S. Matsufuji, Y. Tanada, "Formalization of Binary Sequence Sets with Zero Correlation Zone," IEICE Trans. Fundamentals of Electronics, Communications and Computer Sciences, vol. E87-A, no.4, pp.887-891, 2004.
- [6] P.Z. Fan, N. Suehiro, N. Kuroyanagi and X.M. Deng, "A class of binary sequences with zero correlation zone," IEEE Electron. Lett., Vol. 35, No. 10, pp.777-779, 1999.
- [7] S. Matsufuji, T. Matsumoto, T. Hayashida, T. Hayashi, N. Kuroyanagi and P.Z. Fan, "On a ZCZ Code Including a Sequence Used for a Synchronization Symbol," IEICE Trans. Fundamentals of Electronics, Communications and Computer Sciences, Vol. E93-A, No.11, 2010.
- [8] X. Cai, Q.C. Chen, P.Z. Fan and S. Matsufuji, "Enhanced frequency offset estimation based on pilot signal design over flat-fading channel," Proc. of IET CCWMC2009, No. 1149, 2009.
- [9] T. Hayashida, S. Matsufuji, T. Matsumoto, P.Z. Fan, "Construction of ZCZ Codes With Low Aperiodic Autocorrelation," Journal of Signal Processing, Vol.16, No.6, pp.-, Nov. 2012.
- [10] T. Matsumoto, S. Tsukiashi, S. Matsufuji and Y.Tanada, "A Design of a Digital Matched Filter Bank for an Optical ZCZ Code Using a Sylvester Type Hadamard Matrix," Proceedings of The Second International Workshop on Sequence Design and Its Applications in Communications, pp.184-188, 2005.
- [11] T. Matsumoto, S. Matsufuji, "Code Generator Implementation on FPGA for an Optical ZCZ Code Using a Sylvester Type Hadamard Matrix," Proc. of IWSDA07, pp.228-232, 2007.
- [12] Chung F.R.K, "Optical Orthogonal Codes:Design, Analysis, and Applications," IEEE Trans. Inform. Theory,VOL.35, No.3, pp.595-604, 1989.
- [13] X.H. Tang, P.Z. Fan, and S. Matsufuji, "Lower bounds on the maximum correlation of sequence set with low or zero correlation zone," IEEE Electron. Lett., Vol. 36, No. 6, pp. 551-551, 2000.
- [14] T. Hayashi, "Binary sequences with orthogonal subsequences and zero correlation zone," IEICE Trans. on Fundamentals of Electron., Communications and Computer Sciences, Vol. E85-A, No. 6, pp.1420-1425, 2002.
- [15] S. Matsufuji, T. Takamoto, K. Fanakoshi, "Properties of Even-Shift Orthogonal Sequences," Proc. of IWSDA07, pp.181-184, 2007.
- [16] M.J.E. Golay, "Complementary series", IRE Trans. Inform. Theory, vol.IT-7, pp.82-87, 1961