

Enhanced Soft Decoding of Multi-Carrier Systems for Delay Sensitive Applications

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Abstract—Bit interleaving, often used to combat frequency selective fading over multi-carrier channels, can introduce long processing delays, unpractical for a number of real-time applications. When such delays cannot be afforded, the performance gain of soft decoders is more attractive. Unfortunately, even if bit interleaving is not considered, adaptive bit allocation allows only sub-optimal soft decoding. This is a bit-wise processing technique originally proposed for bit interleaved coded modulation. In this paper, an enhanced decoder is proposed. It avoids unnecessary intermediate bit metrics. Instead, it directly and more efficiently computes code metrics, which are what is actually required by the Viterbi decoder.

I. INTRODUCTION

Multi-carrier is a transmission scheme that encompasses single-carrier frequency domain equalization (FDE) [1] and the more popular Orthogonal Frequency Division Multiplexing (OFDM). To remedy to some of its shortcomings, adaptive multi-carrier transmission is often proposed, where bits are assigned depending on the quality of the considered sub-carrier. This is a means to combat frequency (sub-carrier) selective fading since coding alone is not effective to recover burst errors that result from severely faded sub-carriers [2], [3]. Another efficient technique to combat burst errors is bit interleaving which helps spread burst errors [4]. In practice, multi-carrier transmission involves a large number of parallel channels and a huge bit load (up to 2000 bits for ADSL applications). Consequently, bit interleaving (and deinterleaving) may involve a considerable processing time. The subsequent delay may not be compatible with a number of applications (multimedia, for example). Storage (of bit metrics) is also a concern.

One consequence of adaptive bit loading is that jointly optimized coded modulation can no longer be performed. In fact, there is no longer a one-to-one correspondence between codewords and transmitted symbols. In a different context, that of single-carrier systems, the same separation between coding and modulation appears when coded bits are interleaved, the so-called bit interleaved coded modulation (BICM) [4]. As a result, demodulation and decoding have often been performed separately. The decoder, now fed with hard input, performs notoriously poorly compared to the soft input decoder. In multi-carrier systems, separate (de)coding and (de)modulation are still the case even if no bit interleaving is employed, as long as there is no correspondence between the code length and the sub-carriers modulation(s). Hence, hard decision receivers has been the norm in adaptive multi-carrier systems [5].

Near-optimum soft decoding for bit interleaved coded modulation was proposed in [4], [6] and its application to adaptive multi-carrier systems was proposed in [7]. The demodulator computes *soft bit metrics* which are then fed into the (soft input) decoder. In a Viterbi-based solution [4], [6] (often preferred to the complex MAP-based decoder [8]), the soft bit metrics are approximations of the bit likelihoods. These are then converted into soft code metrics then inputted to the Viterbi decoder. The main contribution of this paper is to recognize that if there is no bit interleaving then bit-wise processing is not justified. One may directly compute the soft code metrics and expects this *direct* approach to be more accurate than the *indirect* approach. An empirical evidence is included in this study.

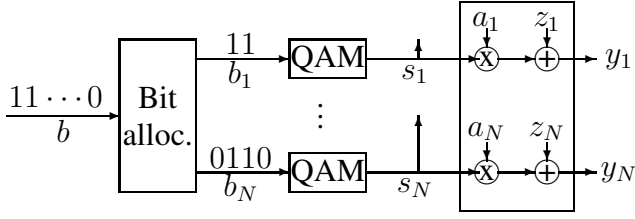


Fig. 1. Adaptive Multi-carrier transmission

After reviewing in Sec. II soft decoding of adaptive multi-carrier systems as proposed in [7], an original soft decoder where the demodulator directly outputs soft code metrics is introduced in Sec. III. The proposed decoder is compared, in Sec. IV, to the BICM-inherited decoder. Finally, a conclusion is given in Sec. V.

II. ADAPTIVE CODED MULTI-CARRIERS

A multi-carrier system can be modeled as a set of parallel inter-symbol interference (ISI)-free single-input single-output (SISO) channels as depicted in Fig. 1. This is, for instance, the case of a (perfectly synchronized) orthogonal frequency division multiplex (OFDM) system [10]. A sufficiently large number of sub-carriers allows the transmitted block to interfere with the preceding block only while the use of a sufficiently long cyclic prefix converts the channel linear convolution into a cyclic convolution, easily diagonalizable using the affordable fast Fourier transform [9]. While an expensive equalizer is required to remove ISI from a SISO channel, a multi-carrier system is inherently interference-free. However, ideally separated sub-channels may also exhibit very unequal responses. In addition to error correcting codes and interleaving [11] (not considered in this work), adaptive transmission is an efficient technique to combat channel frequency selectivity. This consists in assigning to each sub-carrier a different number of bits (or equivalently a different modulation scheme) and a different amplification power, following some optimization criteria (that optimizes power [2], rate [12], BER [13], capacity [14], ...). From the receiver point-of-view, transmission amplification can be assumed as forming part of a larger channel. Hence, it is omitted in Fig. 1.

We assume a k_c/n_c -rate encoder to generate b (b may vary from one block to another) coded bits, which are then dispatched among the N sub-carriers (following some allocation procedure). The b_n bits assigned to the n -th sub-carrier, are mapped into a symbol s_n chosen from the 2^{b_n} -sized QAM constellation S_{b_n} . The transmitted signal is conveyed through the n -th sub-channel (which includes amplification) where it experiences an attenuation a_n and is affected by an additive white Gaussian noise z_n (with variance $\mathbf{E}(|z_n|^2) = N_0$). The n -th sub-channel output is given by

$$y_n = a_n s_n + z_n$$

Straightforwardly extended from BICM, a soft decoder is depicted in Fig. 2. Soft bit metrics were proposed to substitute for the (unavailable) exact bits likelihood [6]. When a soft output decoder is used, its output can be fed back to the demodulator to help refine the (accuracy of the) soft values following the *Turbo* principle [8]. This, however, requires highly complex iterative soft output decoders (MAP, SOVA).

We assume a convolutional encoder and, for simplicity, a Viterbi decoder with hard output. Let $c_n^1, \dots, c_n^{b_n}$ be the bits mapped into the symbol s_n . Let $S_{b_n}^i$ be the set of those signal in S_{b_n} whose labels have a bit "0" at the i -th position. Prior to decoding, coded bits can only be assumed i.i.d. and the coded bit soft metrics, an approximation of $P(y_n | c_n^i = 0)$, are given by $2^{1-b_n} \sum_{s \in S_{b_n}^i} P(y_n | s_n = s)$ [4]. In contrast to BICM, 2^{1-b_n} is not a constant. However, it can still be ignored since the resulting weight increments are path-independent. The soft metric of the coded bit c_n^i is given by

$$\begin{aligned} m_0(c_n^i) &\stackrel{\text{def}}{=} -\log [P(y_n | c_n^i = 0)] \\ &\simeq \frac{1}{N_0} \min_{s \in S_{b_n}^i} |y_n - a_n s|^2 \end{aligned}$$

where the multiplicative constant $1/N_0$ can be dropped. Defined similarly as $m_0(c_n^i)$, $m_1(c_n^i)$ do not verify, in general, $m_0(c_n^i) + m_1(c_n^i) = 1$ [6]. Hence, both (0 and 1) bit metrics need to be computed.

A second step involves the conversion of the bit metrics into code metrics. Soft code metrics can be viewed as a $2^{n_c} \times (b/n_c)$ table where the i -th

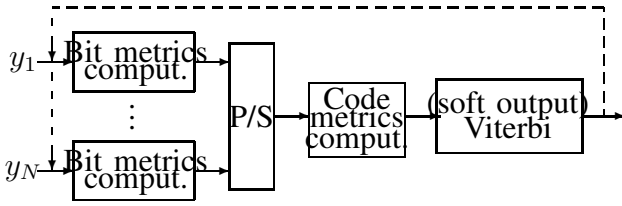


Fig. 2. Soft-decision receiver - Indirect approach

column represents the likelihood of all the possible codewords. Bit metrics $m_0(c_n^i)$ [resp. $m_1(c_n^i)$], $n = 1 \dots N, i = 1 \dots b_n$, are serialized and re-numbered as $m_1, \dots, m_b$ (resp. $m'_1, \dots, m'_b$). They now refer to the soft metrics of the coded bits $c_1, \dots, c_b$. For example, let the following codeword be composed of the bits $\{c_i, \dots, c_{i+n_c-1}\} = \{0 \dots 1\}$. We associated to it the following code metric

$$M(c_i, \dots, c_{i+n_c-1}) \simeq m_i + \dots + m'_{i+n_c-1}$$

The above assumes coded bits to be independent, which is not true, specifically because interleaving is not considered. The so-computed code metrics are then fed to the Viterbi algorithm where they are used to compute the trellis path metrics. We refer to this soft decoder as *indirect* since code metrics, required for decoding, are obtained indirectly (via bit metrics).

III. A DIRECT APPROACH FOR SOFT DECODING

As an alternative to the indirect approach described above, a new approach that computes code metrics *directly* from the channel measurements is proposed. This is illustrated by Fig. 3. As in Sec. II, coded bits are assumed to be independent. Let $c_i, \dots, c_{i+n_c-1}$ be a codeword for which we want to compute the likelihood

$$M(c_i, \dots, c_{i+n_c-1}) \stackrel{\text{def}}{=} -\log [P(y_1, \dots, y_N | c_i, \dots, c_{i+n_c-1})]$$

Since no bit interleaving is used, the corresponding symbol sequence $s_1, \dots, s_N$ is reduced to a few symbols. The development of the new code metric is inspired by the (BICM) soft bit metrics. The same idea is exploited here, i.e. averaging over symbol sequences that share the same codeword. Complications arise because the (bits of the) considered

codeword may be shared by more than one symbol, depending on the code and the constellation sizes. Furthermore, due to adaptive modulation, symbols conveyed by adjacent sub-carriers may belong to different constellations. To introduce the new metric, we first consider the two simplest scenarios that involves one sub-carrier or two adjacent sub-carriers, then we describe later the formulation of the general case.

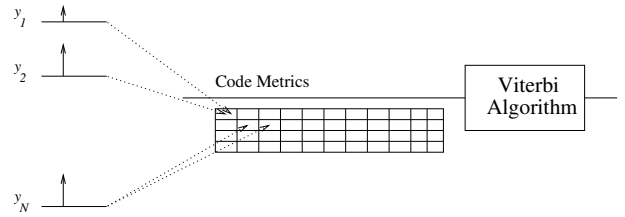


Fig. 3. Soft-decision receiver - Direct approach

If bits $c_i, \dots, c_{i+n_c-1}$ are assumed to be conveyed by a single symbol s_n , then $M(c_i, \dots, c_{i+n_c-1})$ reduces to $-\log [P(y_n | c_i, \dots, c_{i+n_c-1})]$. Let $j, \dots, j + n_c - 1$ be the respective positions of $c_i, \dots, c_{i+n_c-1}$ in the label of s_n . We obtain,

$$\begin{aligned} M(c_i, \dots, c_{i+n_c-1}) &\propto -\log \left[\sum_{s=[xxx c_i, \dots, c_{i+n_c-1} xxx] \in S_{b_n}} P(y_n | s_n = s) \right] \\ &\simeq \max_{s=[xxx c_i, \dots, c_{i+n_c-1} xxx] \in S_{b_n}} |y_n - a_n s|^2 \end{aligned}$$

The notation $s = [xxx c_i, \dots, c_{i+n_c-1} xxx]$ means s is a symbol whose label includes bits $c_i, \dots, c_{i+n_c-1}$ at positions $j, \dots, j + n_c - 1$, respectively.

Under the same assumptions and following the same steps, if $c_i, \dots, c_{i+n_c-1}$ occupy the last u bits of the symbol s_n and the first $n_c - u$ bits of the symbol s_{n+1} , its likelihood is approximated by

$$\begin{aligned} M(c_i, \dots, c_{i+n_c-1}) &\simeq \max_{s_1=[xxx c_i, \dots, c_{i+u-1}] \in S_{b_n}} |y_n - a_n s_1|^2 \\ &\times \max_{s_2=[c_{i+u}, \dots, c_{i+n_c-1} xxx] \in S_{b_{(n+1)}}} |y_{n+1} - a_{n+1} s_2|^2 \end{aligned}$$

In the general case, bits $c_i, \dots, c_{i+n_c-1}$ are shared between a series of successive symbols, say $s_{n_1}, \dots, s_{n_2}$. Only a part of the bits conveyed by s_{n_1} and s_{n_2} belong to the considered codeword. Their

respective labels can be written as $xxx c_i, \dots, c_u$ and $c_v, \dots, c_{n_c-1} xxx$. Similarly, we have

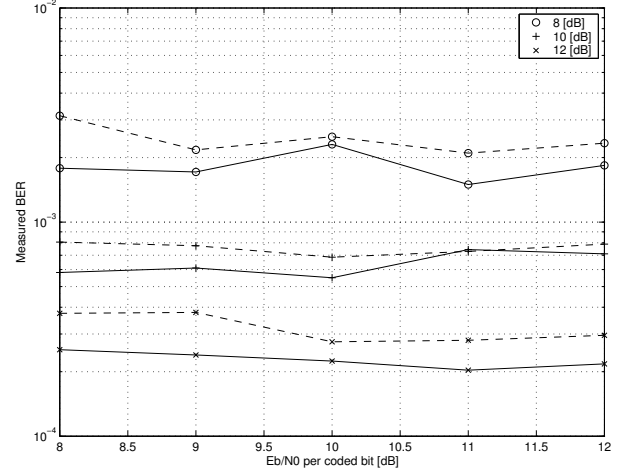
$$\begin{aligned} & M(c_i, \dots, c_{i+n_c-1}) \\ & \simeq \max_{s_1=[xxx c_1, \dots, c_u] \in S_{b_{n_1}}} |y_{n_1} - a_{n_1} s_1|^2 \\ & \times \max_{s_2 \in S_{b_{(n_1+1)}}} |y_{(n_1+1)} - a_{(n_1+1)} s_2|^2 \dots \\ & \times \max_{s_{(n_2-n_1)} \in S_{b_{(n_2-1)}}} |y_{(n_2-1)} - a_{(n_2-1)} s_{(n_2-n_1)}|^2 \\ & \times \max_{s_{(n_2-n_1+1)}=[c_v, \dots, c_U xxx] \in S_{b_{n_2}}} |y_{n_2} - a_{n_2} s_{(n_2-n_1+1)}|^2 \end{aligned}$$

The so-computed soft code metrics can now be summed up by the Viterbi algorithm to compute path metrics in the code trellis, and perform maximum likelihood sequence estimation.

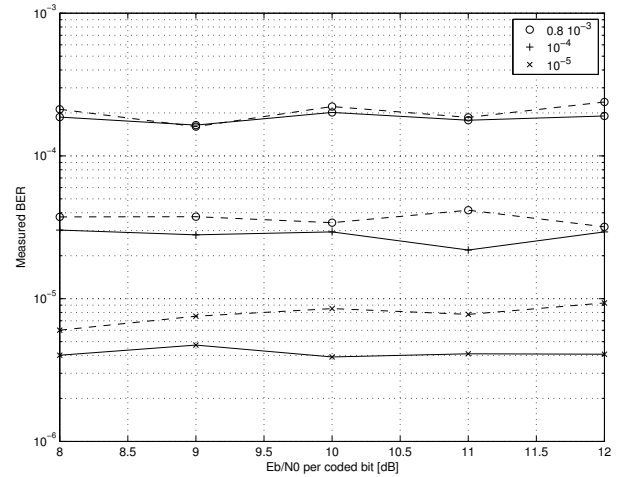
IV. SIMULATIONS

The transmitter of Fig. 1 is simulated with a $1/2$ -rate convolutional encoder (generator matrix 57 in octal representation) generating fixed-sized blocks of $b = 768$ bits. Bits from one block are dispatched among a set of $N = 256$ sub-carriers. After bit allocation, BPSK, QPSK, cross 8-QAM, 16-QAM, cross 32-QAM and/or 64-QAM (or no modulation at all if no bits are assigned to the sub-carrier) are conducted. Constellations are normalized to have an energy per coded bit equal to 1. The sub-carriers (single-tap) responses, $a_1, \dots, a_N$, are generated as (normalized) independent random Rayleigh variables. This corresponds to a non light-of-sight propagation, a model suitable for reception in dense urban area and indoor environments [10]. Monte Carlo runs are repeated until 100 erroneous information bits are counted.

Two strategies are considered for bit (and power) allocation. A uniform allocation is performed by equally assigning the $b = 768$ bits to the $N = 256$ sub-carriers, i.e. 3 bits per sub-carrier. An adaptive allocation technique is also used [15], that is specifically designed for coded multi-carrier system using soft-decision receivers. Simulation results of the multi-carrier system using uniform and adaptive allocation are presented in Fig. 4-(a) and 4-(b), respectively. Simulation results confirm that the direct soft decoding approach proposed in this paper provide some improvement compared to the indirect approach.



(a) Uniform allocation. The legend shows the transmission SNR per coded bit.



(b) Adaptive allocation. The legend shows the target BER.

Fig. 4. Soft decoding using the indirect and direct approaches are presented by dashed and solid lines, respectively.

V. CONCLUSION

Large sized multi-carrier systems may not tolerate the considerable delay resulting from bit interleaving. Hence, there is a much stronger interest for soft decoding. We propose to directly compute code metrics rather than via bit metrics if one should apply techniques extended from the (single-carrier) bit interleaved coded modulation. Simulations show, as expected, a noticeable decoding gain.

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