

Robust Modified MMSE Estimator for Comb-Type Channel Estimation in OFDM Systems

Latif Ullah Khan*, M. Inayatullah Babar*, Zeeshan Sabir *

*University of Engineering & Technology, Peshawar, Pakistan

{latifullahkhan, babar , zeeshansabir}@nwfpuet.edu.pk

Abstract— Orthogonal frequency division multiplexing (OFDM) is used for wireless data transfer because of its robustness to narrow band interference, frequency selective fading and spectral efficiency. Channel estimation and equalization in OFDM is necessary to nullify impairments induced by the frequency selective fading environment. Frequency domain comb-type pilot assisted channel estimation has been used in this paper. Pilot subcarriers are used to implement Modified Minimum Mean Square Error (MMSE) estimator algorithm at pilot subcarriers. The performance and complexity comparison is made between the modified MMSE and MMSE estimator for fast fading Rayleigh channel. Linear, Low Pass and spline cubic interpolation techniques have been used with the MMSE estimator. The effect of increase in number of channel taps on the performance of both estimators is studied. Simulation results reveal that the performance of the modified MMSE estimator remains stable with an increase in channel taps, but significant degradation has been observed in the performance of the MMSE estimator with an increase in channel taps.

Keywords— OFDM; MMSE estimator; linear interpolation; low pass interpolation; spline cubic interpolation

I. INTRODUCTION

Bandwidth efficiency and robustness to channel impairments have made orthogonal frequency division multiplexing (OFDM) a preferable candidate for wireless data transfer standards. OFDM is used wide spread in various applications i.e. ADSL, Wi-Fi, WiMAX and power line communications [1]. Broadcasting standards i.e. Digital Video Broadcasting (DVB) and Digital Multimedia Broadcasting (DMB) are using OFDM [2]. In OFDM, the frequency selective channels are transformed to flat fading by dividing the available channel bandwidth into several sub channels. An improvement in the performance of OFDM based communication systems has been seen in the presence of frequency selective fading environments using channel estimation and equalization techniques. In single carrier communication systems, different complex equalization techniques are used for inter symbol interference (ISI) cancellation; however cyclic prefix is used in OFDM for ISI mitigation [3].

Semi-blind, blind and pilot-aided channel estimation techniques are the three categories of channel estimation. The received signal statistics are used for estimating the impulse

response in the blind estimation. Pilot tones are used in pilot-aided channel estimation for estimating effects of channel impulse response variations. Channel estimation capability of blind estimation can be enhanced through the use of Pilots [4].

In [5], comb-type pilot assisted channel estimation over Rayleigh fading channel is used. The interpolation technique proposed in [5] has been compared with second order interpolation technique and time domain interpolation [6]. The proposed interpolation technique has shown to be better performing than second order and time domain interpolations through simulation results. Minimum mean square error (MMSE) estimator outperforms the other estimator (Least Square (LS)) in [7][8]. MMSE estimator uses prior information about the channel statistics. In [9], comb-type and block-type channel estimation techniques are studied and the performance of different one dimensional interpolation techniques i.e. linear, time domain, second order, low pass and spline interpolations are compared for comb-type channel estimation/equalization.

In this paper, 1D frequency domain comb-type channel equalization is used because of its low computational complexity as compared to 2D channel estimation. MMSE channel estimator is modified for effective estimation of the effects of channel impulse response variations. The performance comparison between the modified MMSE channel estimator and conventional MMSE channel estimator is carried-out for channels of different number of taps. Our analysis proves performance of modified MMSE estimator to be fine even for increasing number of taps, however the performance degradation occurs for conventional MMSE estimator with an increase in channel taps.

Rest of the paper is organized as follows: Section II shows overview of the system. Comb-type channel estimation strategy with MMSE estimator, its modified version and interpolation techniques are explained in section III. Simulation results are described in section IV and conclusion of the paper is given in section V.

Notation: I_P stands for $P \times P$ identity matrix. Subscripts ^{H} and ^{T} represents respectively Hermitian transposition and transposition. $E\{\}$ corresponds to the expected value.

II. SYSTEM OVERVIEW

The proposed OFDM model is shown in Fig. 1. First of all the input bits are mapped and parallelized.

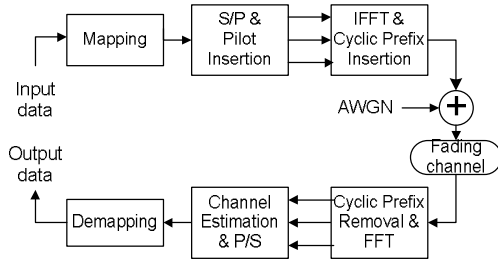


Figure 1. Proposed OFDM model with Comb-type Estimation

The parallelized mapped data symbols X_{Data} and pilot symbols X_{pilots} are given by:

$$X_{Data} = X(I_D) \quad (1)$$

$$X_{pilots} = X(I_P) \quad (2)$$

Where I_D and I_P are the vectors containing the indices of the subcarriers reserved for data and pilots respectively. The output of IFFT is given by:

$$\begin{aligned} x(n) &= IFFT(X) \\ &= \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi kn}{N}} \quad n = 0, 1, \dots, N-1 \end{aligned} \quad (3)$$

Cyclic prefix is appended into the OFDM symbol through inserting the end L_G subcarriers to the start as per the following equation:

$$\tilde{x}(P) = \begin{cases} x(N - L_G + P) & P = 0, 1, 2, \dots, L_G - 1 \\ x(L_G + P) & P = L_G, L_G + 1, \dots, L_G + N - 1 \end{cases} \quad (4)$$

Where L_G is the length of the cyclic prefix and it must be larger than the maximum channel delay spread for the effective cancellation of inter-symbol interference (ISI). Then the OFDM symbol is passed through the fading channel. In the meanwhile additive white Gaussian Noise (AWGN) is added into the signal. At the receiving end, the received signal is given by:

$$y = \tilde{x} \otimes h + w \quad (5)$$

where w is the AWGN vector, h is the vector of channel impulse response and $\otimes$ shows the convolution operation. The data is serialized after the removal of cyclic prefix and FFT. The output of FFT is given by:

$$\begin{aligned} Y(k) &= FFT(y(n)) \\ &= \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j \frac{2\pi kn}{N}} \quad k = 0, 1, \dots, N-1 \end{aligned} \quad (6)$$

(6) can be written in matrix form for the received pilot symbols as:

$$Y_P = X_P H_P + W_P \quad (7)$$

Where

$Y_P = [Y_P(0), Y_P(1), Y_P(2), \dots, Y_P(P-1)]^T$ is the vector containing received pilot symbols.

$X_P = \text{diag}\{[X_P(0), X_P(1), \dots, X_P(P-1)]\}$ is the diagonal matrix having transmitted pilots on its diagonal.

$H_P = [H(0), H(1), H(2), \dots, H(P-1)]^T$ is the vector contains the impulse response of the channel at pilot subcarriers.

$W_P = [W_P(0), W_P(1), W_P(2), \dots, W_P(P-1)]^T$ is the vector with entries as FFT of the AWGN at pilot subcarriers.

To nullify the fading effects of the multipath channel, the frequency response of the channel is necessary to be effectively estimated. We have used MMSE estimator and its modified form for estimation of the impulse response of the channel at pilot subcarriers respectively. Finally, after carrying out the estimation and equalization process, the signal is demapped to yield the output bits.

III. CHANNEL ESTIMATION AND INTERPOLATION TECHNIQUES

One dimensional (1D) Channel estimation in OFDM has two common types i.e. comb-type and block-type based upon arrangement of the pilots. Comb-type estimation process is used for fast fading channels while Block-type estimation is normally used for slow fading channels. Fig. 2 shows the arrangement of pilots for comb-type and block-type channel estimation. In this paper, we have used comb-type estimation due to its effectiveness in the fast fading Rayleigh channel for performance analysis of the OFDM system. Equispaced pilot insertion is adopted because of optimum performance [10]. MMSE algorithm and its modified form are used for estimation of the channel frequency response at pilot subcarrier positions due to its superior performance as compared to least square (LS) estimator [7][8].

After computation of the channel estimation matrix H_{CE} , the equalization is carried out through following equation:

$$X_e = \frac{Y(k)}{H_{ce}(k)} \quad k \in I_D \quad (8)$$

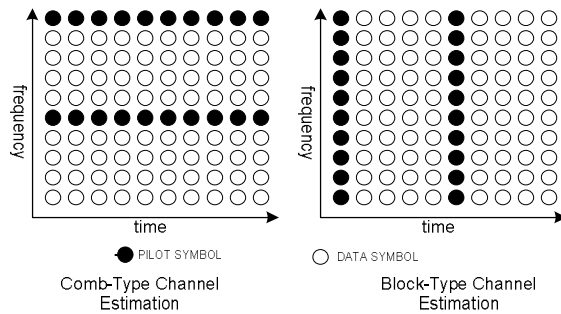


Figure 2. Arrangement of Pilots

A. MMSE Estimator

The frequency domain MMSE estimate of the channel for a Gaussian distributed time domain channel vector h uncorrelated with noise is given by [11]:

$$H_{P,MMSE} = F_N^H R_{HY} R_{YY}^{-1} Y_P \quad (9)$$

Where

F_N is the $P \times P$ matrix $[F_N]_{p,p} = e^{j2\pi(p-1)(p-1)/P}$

$R_{HY} = E\{h_P Y_P\} = R_{HH} F_N X_P^H$ is the received signal vector Y_P cross covariance matrix and the time domain channel vector h_P .

$R_{YY} = E\{Y_P Y_P^H\} = X_P F_N^H R_{HH} F_N X_P^H + \sigma^2 I_P$ is the received signal vector Y_P auto-covariance matrix.

R_{HH} is the auto-covariance matrix for the time domain channel vector h and σ^2 is the variance of noise W_P . R_{HH} and σ^2 are the quantities that are known.

(9) can be written as:

$$H_{P,MMSE} = F_N^H M_{MMSE} F_N X_P^H Y_P \quad (10)$$

Where

$$M_{MMSE} = R_{HH} [(F_N X_P^H X_P F_N^H)^{-1} \sigma^2 + R_{HH}]^{-1} (F_N X_P^H X_P F_N^H)^{-1}$$

The estimated channel frequency response vector H_{CE} for all subcarriers is obtained through the use of interpolation technique. In this paper, we have used linear, low pass and spline cubic interpolation.

B. Modified MMSE Estimator

First of all, define $L \times N$ matrix $[F]_{l,n} = e^{j2\pi(l-1)(n-1)/N}$. Then define $L \times P$ matrix F_K which contains the columns of matrix F corresponding to pilot subcarriers. The time domain channel impulse response estimate using MMSE algorithm is given by:

$$h_{P,MMSE} = M_{MMSE} F_K^H X_P^H Y_P \quad (11)$$

Where

$$M_{MMSE} = R_{HH} [(F_K^H X_P^H X_P F_K)^{-1} \sigma^2 + R_{HH}]^{-1} (F_K^H X_P^H X_P F_K)^{-1}$$

The frequency response of the channel at all positions of the subcarriers is calculated using the following equation:

$$H_{MMSE} = F^H h_{P,MMSE} \quad (12)$$

(12) performs interpolation and conversion from time domain to frequency domain. As (12) performs interpolation inherently therefore there is no need of using interpolation technique and thus get rid of the extra interpolation step as required for conventional MMSE estimator.

C. Interpolation Techniques For Comb-type Channel Estimation

Two consecutive pilot subcarriers are used in linear interpolation for computation of channel frequency response at data subcarriers situated between the pilot subcarriers. Linear interpolation out performs piecewise interpolation [12]. The channel response estimated at a subcarrier K where, $mL \leq K < (m+1)L$, using linear interpolation is given by [9]:

$$\begin{aligned} H(k) &= H(mL + l) = \left(1 - \frac{l}{L}\right) H_P(m) + \frac{l}{L} H_P(m+1) \\ &= H_P(m) + \frac{l}{L} (H_P(m+1) - H_P(m)), 0 \leq l < L \end{aligned} \quad (13)$$

Spline cubic interpolation (*spline* function in MATLAB®) results in continuous and smooth polynomial fitted for the given set of data points. The basic idea governing Spline cubic interpolation is drawing of smooth curves through several points [13]. In low pass interpolation (*interp* function in MATLAB®), after insertion of zeros in the original sequence, the sequence is fed to low pass finite impulse response filter (FIR). The interpolation is based on the minimization of mean square error between the ideal and interpolated points [14].

IV. SIMULATION RESULTS AND ANALYSIS

In this section results have been discussed for the simulations conducted in MATLAB® for the performance evaluation of the proposed modified MMSE and conventional MMSE estimator in OFDM respectively. Parameters specification of the OFDM model used for the simulation purposes is given in table I. We have used the frequency selective Rayleigh fading channel with L taps. The uncorrelated channel taps are complex Gaussian random variables with zero mean.

TABLE 1. SIMULATION PARAMETERS

Parameters	Specifications
Number of subcarriers, N	512
Size of the FFT	512
Length of Cyclic Prefix	¼ i.e. 64
Channel Estimation	Comb-Type
Pilot Ratio	1/8
Digital Modulation Schemes	BPSK, QPSK, 16-QAM, 32-QAM, 64-QAM

We have evaluated performance of the OFDM system using bit error rate (BER) vs Signal to Noise Ratio (SNR) plotted in a graph. The legends Low pass, linear and Spline correspond to low pass interpolation, linear interpolation and spline cubic interpolation respectively.

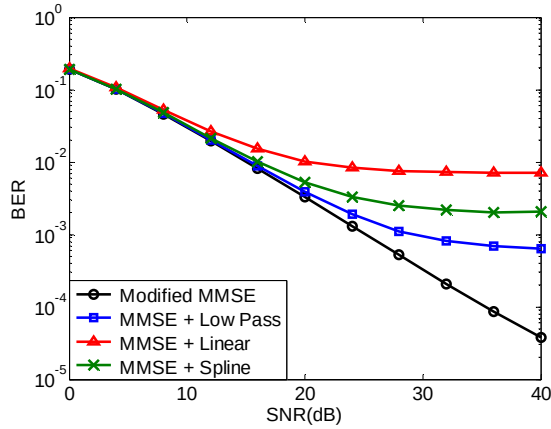


Figure 3. Performance Comparison of modified MMSE & MMSE estimator for channel with 24 taps

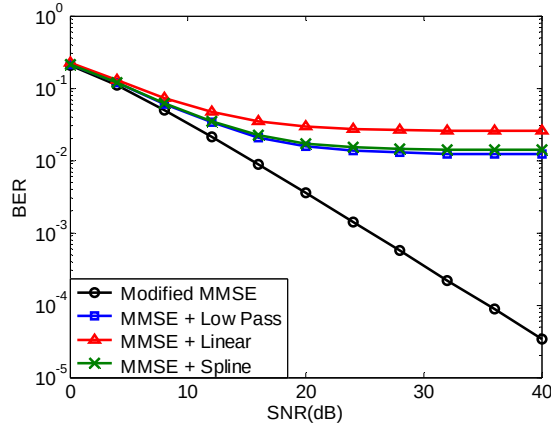


Figure 4. Performance Comparison of modified MMSE & MMSE estimator for channel with 32 taps

We have evaluated the performance of modified MMSE and conventional MMSE estimator for channel with different number of taps, L . As the channel order is different for different environmental conditions and it increases with an increase in adverse effects of the environment. This is the reason for studying the effect of increase in length of channel taps over the performance of the estimators in our analysis. BPSK modulation scheme is used in Figs. 3 to 5. Fig. 3 shows BER performance of modified MMSE and MMSE estimator for BPSK modulated OFDM over Rayleigh fading channel of 24 taps. It is clear that modified MMSE estimator outperforms MMSE estimator with different interpolation techniques. The reason for the performance improvement of modified MMSE over conventional MMSE estimator is the improved interpolation in case of the modified MMSE estimator. The one dimensional interpolation techniques for conventional MMSE Estimator shows their performance from worst to best

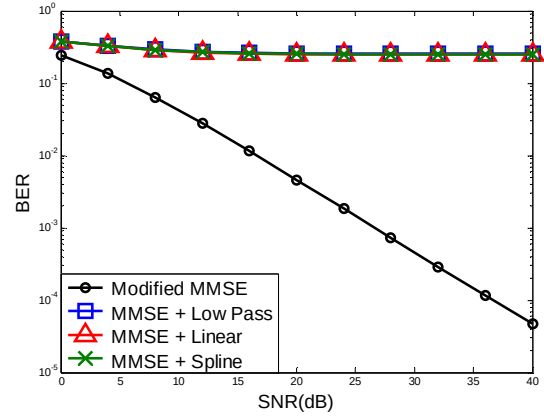


Figure 5. Performance Comparison of modified MMSE & MMSE estimator for channel with 64 taps

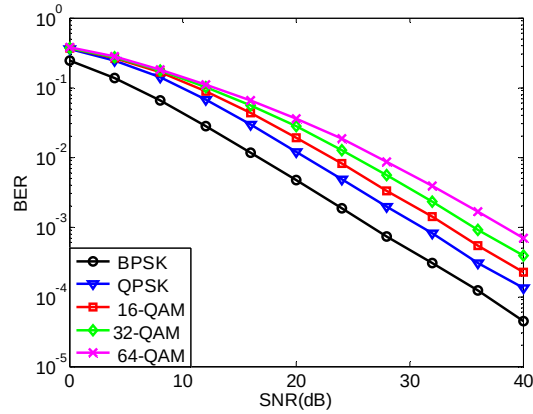


Figure 6. Performance Comparison of Uncoded OFDM for modified MMSE estimator for channel with 64 taps

is in the following order i.e. linear, spline and low pass. In [9][13], simulation results show that the performance of different one dimensional interpolation techniques for comb-type channel estimation is in the following order from worst to best. i.e. linear, second order, time domain, spline and low pass. Therefore it is concluded that the performance of modified MMSE is better than the conventional MMSE estimator with low pass, linear, second order, time domain and spline interpolation technique.

By comparing Fig. 3 to 5 it becomes obvious that the modified MMSE estimator shows a stable performance for an increase in length of channel taps; however the performance of conventional MMSE estimator degrades significantly with an increase in channel taps. Fig. 5 shows the performance of both estimators for channel with 64 taps. It is clear from the Fig. 5 that the performance of modified MMSE estimator remains stable while the performance of conventional MMSE

degrades such that it becomes useless. The BER performance of the conventional MMSE estimator with different interpolation techniques for large number of channel taps degrades to an extent such that it seems to overlap with each other. Fig. 6 illustrates the performance of uncoded OFDM with BPSK, QPSK, 16-QAM, 32-QAM and 64-QAM modulation schemes for modified MMSE estimator over channel with 64 taps. The reason of the performance degradation for higher modulation schemes is the close placement of constellation points.

Table II shows the complexity comparison of the conventional MMSE and proposed modified MMSE estimator. The size of the matrix M_{MMSE} is equal and increases proportionally with an increase in channel taps for both estimators. Hence the complexity of both estimators is same.

TABLE 2. COMPLEXITY COMPARISON OF THE ESTIMATORS

Channel Taps, L	Estimator	Size of M_{MMSE}
24	Modified MMSE	24X24
	MMSE	24X24
32	Modified MMSE	32X32
	MMSE	32X32
64	Modified MMSE	64X64
	MMSE	64X64

V. CONCLUSIONS

In this paper, we have modified the MMSE estimator for comb-type estimation in OFDM. Linear, Low-pass and Spline interpolation techniques are used with the MMSE estimator. The performance of modified MMSE estimator is compared with conventional MMSE estimator for channels of different number of taps. It is observed that the performance of the modified MMSE estimator remains stable with the increase in length of channel taps, however conventional MMSE estimator shows a significant performance degradation with an increase in channel taps.

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