

Complexity Reducing QRM-MLD with Sequential Decision Based on the Estimated Noise Variance

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Abstract—In MIMO systems, maximum likelihood decoding (MLD) shows the best performance of all kinds of detections. However, the complexity of MLD exponentially increases with increasing the number of antenna branches and constellation size. Accordingly, it is impractical to use a full MLD without reducing its computational complexity, because it would be prohibitively large to implement. Recently, the use of QR decomposition with an M-algorithm (QRM-MLD) has been proposed to reduce the system complexity while maintaining the performance of the system. However, QRM-MLD performance depends on the surviving symbol replica candidates. To reduce this problem, in this paper, we propose the complexity reducing QRM-MLD with threshold method using the estimated noise variance. The proposed method shows the superior performance while maintaining the low complexity. From the simulation results, the proposed method achieves 1/10 complexity reduction compared with a full MLD for the threshold index $k_{th} = 3.3$.

I. INTRODUCTION

Recently, the wireless communication technology is required to carry a large amount of digital information with the spread of the mobile phone and wireless LAN. However, the realization of higher data rate transmission per unit frequency is one of the most important issues in wireless communication due to the finiteness of the frequency band. In recent years MIMO (Multiple Input Multiple Output) system has attracted attention since MIMO system achieves higher data rate transmission without increasing frequency bandwidth [1].

MIMO system not only achieves higher data rate transmission but also improve the quality of communication by spatial multiplexing effect. MLD (Maximum Likelihood Decoding) shows the best BER performance [2]. MLD generates replicas of the received signal from candidates of the desired and channel interference signals using pilot symbols. The combination of desired symbols that has the minimum Euclidean distance is selected. However, MLD operation causes exponentially increase of the computation with respect to the number of antenna. Thus, it is difficult to implement MLD algorithm in the mobile devices like Smartphone for the processing delay and power consumption. Therefore, QRM-MLD (QR decomposition with an M-algorithm) has been proposed as a complexity reduction method for MLD and actively studied for the performance enhancement [3-8]. However, the system performance with QRM-MLD depends on the number of surviving symbol replica candidates. When QRM-MLD uses

a small number of candidates, the performance declines, but when there is a large amount of surviving symbol replica candidates, QRM-MLD require a large memory to maintain the branch metrics for the surviving symbol replicas. To reduce the above mentioned QRM-MLD problem, we propose the complexity reducing QRM-MLD using sequential decision with estimated noise variance.

This paper is organized as follows. In Section II, we describe the MIMO-OFDM system and QRM-MLD algorithm. The proposed sequential decision algorithm is described in Section III. In section IV, we show the simulation results. Finally, the conclusion is given in Section V.

II. MIMO-OFDM SYSTEM AND QRM-MLD ALGORITHM

A. Channel model

We assume that the propagation channel consists of L discrete paths with different time delays. The impulse response between the m th transmitter and the n th receiver antenna $\tilde{h}_{m,n}(\tau, t)$ is represented as

$$\tilde{h}_{m,n}(\tau, t) = \sum_{l=0}^{L-1} \tilde{h}_{m,n,l}(t) \delta(\tau - \tau_{m,n,l}), \quad (1)$$

where $\tilde{h}_{m,n,l}, \tau_{m,n,l}$ are the complex channel gain and the time delay of the l th propagation path, respectively. The channel transfer function $h_{m,n}(f, t)$ is the Fourier transform of $\tilde{h}_{m,n}(\tau, t)$ and is given by

$$\begin{aligned} h_{m,n}(f, t) &= \int_0^\infty \tilde{h}_{m,n}(\tau, t) \exp(-j2\pi f\tau) d\tau \\ &= \sum_{l=0}^{L-1} \tilde{h}_{m,n,l}(t) \exp(-j2\pi f\tau_{m,n,l}). \end{aligned} \quad (2)$$

B. MIMO/OFDM system

The coded binary data sequence is modulated, N_p pilot symbols are appended at the beginning of the sequence. The MIMO/OFDM transmit signal for the m th transmit antenna can be expressed in its equivalent baseband representation as

$$\begin{aligned} s_m(t) &= \sum_{i=0}^{N_p+N_d-1} g(t-iT) \cdot \sqrt{\frac{2S}{N_c}} \sum_{k=0}^{N_c-1} d_m(k, i) \\ &\quad \cdot \exp[j2\pi(t-iT)k/T_s], \end{aligned} \quad (3)$$

where N_c is the number of carriers, T_s is the FFT time length, N_d, N_p are the number of data and pilot symbols, S is the average transmitting power, T is the OFDM symbol length, respectively. $d_m(k, i)$ represents the i th symbol of the k th subcarrier. The guard interval T_g is inserted in order to eliminate the inter-symbol interference (ISI) due to multipath fading. Hence, we have

$$T = T_s + T_g. \quad (4)$$

In Eq. (3), $g(t)$ is the transmission pulse given by

$$g(t) = \begin{cases} 1 & \text{for } -T_g \leq t \leq T_s \\ 0 & \text{otherwise} \end{cases}. \quad (5)$$

The received signal for the n th received antenna $r_n t$ is represented by

$$r_n(t) = \sum_{m=0}^{M-1} \int_{-\infty}^{\infty} h_{m,n}(\tau, t) s_m(t - \tau) d\tau + \zeta_n(t), \quad (6)$$

where $\zeta_n(t)$ is additive white Gaussian noise (AWGN) for the n th received antenna. By the FFT operation, the output signal of the n th antenna $\tilde{r}_n(k, i)$ is given by

$$\tilde{r}_n(k, i) = \sqrt{\frac{2S}{N_c}} \sum_{m=1}^M h_{m,n}(k, i) d_m(k, i) + \eta_n(k, i). \quad (7)$$

Eq. (7) can be expressed in the equivalent matrix representation as

$$\mathbf{Y}(k) = \mathbf{H}(k)\mathbf{D}(k) + \mathbf{N}(k), \quad k = 1, \dots, K, \quad (8)$$

where $\mathbf{Y}(k)$ is the $N \times 1$ received signal matrix, $\mathbf{D}(k)$ is the $M \times 1$ transmitted signal matrix, $\mathbf{H}(k)$ is the $N \times M$ channel matrix, $\mathbf{N}(k)$ is the $N \times 1$ noise matrix, respectively.

C. Conventional detection algorithms

1) *MLD*: Assuming that the number of the transmit and receive antennas is N , MLD is represented as

$$u = \arg \min ||\mathbf{Y}(k) - \tilde{\mathbf{H}}(k)\mathbf{D}_q(k)||^2, \quad (9)$$

where $\tilde{\mathbf{H}}(k)$ is the estimated channel matrix, $\mathbf{D}_q(k)$ is the q th symbol candidate for modulation index. MLD algorithm is the optimum method to choose the minimum condition of $||\mathbf{Y}(k) - \tilde{\mathbf{H}}(k)\mathbf{D}_q(k)||^2$. However, MLD requires N^χ times computational complexity, where χ is the modulation index. Then, the number of the computation in MLD increases exponentially with respect to the number of the antenna and modulation index. Hence, it is difficult to implement MLD algorithm in the mobile devices like Smartphone due to the processing delay and power consumption.

2) *QRM-MLD*: QRM-MLD is actively studied as a complexity reducing algorithm for MLD. We can decompose the channel matrix $\tilde{\mathbf{H}}(k)$ in Eq. (9) like following equation with QR decomposition

$$\tilde{\mathbf{H}}(k) = \mathbf{Q}(k)\mathbf{R}(k), \quad (10)$$

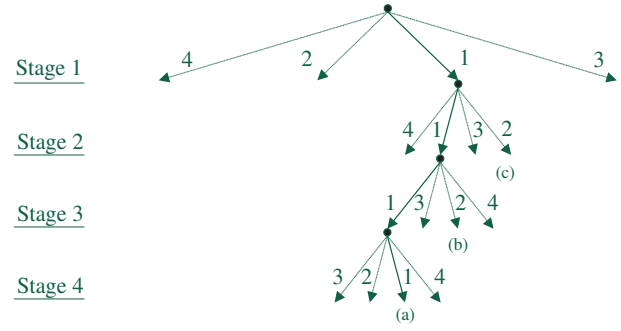


Fig. 1. A tree structure expression for QRM-MLD

where $\mathbf{Q}(k)$ is the $N \times N$ unitary matrix, $\mathbf{R}(k)$ is the $N \times N$ upper triangular matrix. By left multiplying of $\mathbf{Q}^H(k)$ in Eq. (8), $\mathbf{Z}(k) = \mathbf{Q}^H(k)\mathbf{Y}(k)$ can be expressed as

$$\begin{aligned} \mathbf{Z}(k) &= \mathbf{Q}^H(k)\mathbf{Y}(k) \\ &= \mathbf{Q}^H(k)\mathbf{H}(k)\mathbf{D}(k) + \mathbf{Q}^H(k)\mathbf{N}(k) \\ &= \mathbf{R}(k)\mathbf{D}(k) + \tilde{\mathbf{N}}(k). \end{aligned} \quad (11)$$

From Eq. (11) we can see that the variance of $\tilde{\mathbf{N}}(k)$ is equal to $\mathbf{N}(k)$ due to the nature of the unitary transformation. For example, we describe QRM-MLD($S_1, S_2, S_3 = 4, 4, 4$) algorithm, where S_m ($m = 1, 2, 3$) is the surviving symbol replica candidate for the m th stage of M-algorithm, and we show the QRM-MLD(4,4,4) following section. We denote the signal model in Eq. (11) as

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ 0 & R_{22} & R_{23} & R_{24} \\ 0 & 0 & R_{33} & R_{34} \\ 0 & 0 & 0 & R_{44} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{n}_3 \\ \tilde{n}_4 \end{bmatrix}, \quad (12)$$

where $z_j, R_{i,j}, d_j, \tilde{n}_j$ ($i, j = 1, 2, 3, 4$) are $Z, R, D, \tilde{N}$ expressed in the matrix element expression. By considering the fourth row of the Eq. (12), the phase difference between the received signal and the symbol candidates θ_4 is represented as

$$\theta_4 = \arg(z_4 - R_{44}\hat{d}_4), \quad (13)$$

where $\hat{d}_4$ is a symbol candidate and $R_{j,j}$ is a real value. Considering the QPSK, the phase difference between each symbol candidate and the received signal is represented in ascending order as $\hat{d}_{4,1}, \hat{d}_{4,2}, \hat{d}_{4,3}, \hat{d}_{4,4}$ in the first stage. In the QRM-MLD(4,4,4), $\hat{d}_{4,1}$ which makes the phase difference minimum is regarded as the surviving symbol replica candidate to second stage, as shown in Fig. 1. In the second stage, considering Eq. (12), the phase difference between the received signal and the symbol candidates θ_3 is represented as

$$\theta_3 = \arg(z_3 - R_{33}\hat{d}_3 - R_{34}\hat{d}_{4,1}). \quad (14)$$

From Eq. (14), the phase difference is represented in ascending order as $\hat{d}_{3,1}, \hat{d}_{3,2}, \hat{d}_{3,3}, \hat{d}_{3,4}$ in the second stage. Then, $\hat{d}_{3,1}$ is selected as the surviving symbol replica candidate for the third stage. By the same procedure in the third and fourth stages, we can decide the symbol candidate $\hat{d}_{1,1}$ which makes

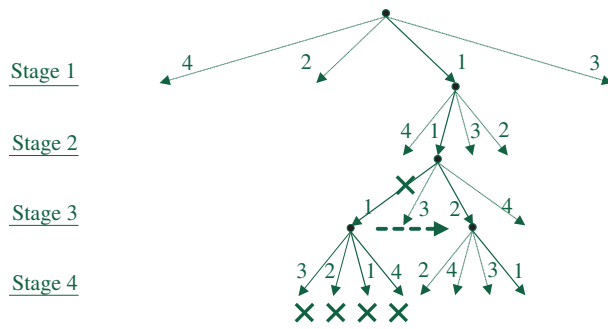


Fig. 2. Update of the tree structure by the sequential decision algorithm

θ_1 minimum in the fourth stage. By QRM-MLD algorithm, $\hat{d}_{1,1}, \hat{d}_{2,1}, \hat{d}_{3,1}, \hat{d}_{4,1}$ are detected. However, the system performance of QRM-MLD depends on the number of surviving symbol replica candidates. When QRM-MLD uses a small number of candidates, the performance declines, but when there is a large amount of surviving symbol replica candidates, QRM-MLD require a large memory to maintain the branch metrics for the surviving symbol replicas. Therefore, for the complexity reducing algorithm without performance degradation, we propose the sequential detection for QRM-MLD algorithm with threshold method using the estimated noise variance and evaluate the system performance following section.

III. SEQUENTIAL DETECTION FOR QRM-MLD

If the detected symbol candidates $\hat{\mathbf{D}}_1 = [\hat{d}_{1,1}, \hat{d}_{2,1}, \hat{d}_{3,1}, \hat{d}_{4,1}]$ are equal to the transmit symbols, Euclidean distance $U(k)$ shows only noise component. In this case, $U(k)$ is represented as

$$U(k) = \|\mathbf{Z}(k) - \mathbf{R}(k)\hat{\mathbf{D}}_1(k)\| = \|\tilde{\mathbf{N}}(k)\|. \quad (15)$$

Thus, we can determine the correctness of $\hat{\mathbf{D}}_1$ using the estimated noise variance.

A. Judgment Formula

We need the judgment formula for the decision of the correctness of $\hat{\mathbf{D}}_1$. From Eq.(15), the judgment formula is defined as

$$U(k) < k_{th} * \sigma_v, \quad (16)$$

where σ_v is the estimated noise variance, k_{th} is the threshold parameter.

B. Sequential Decision Algorithm

In the proposed sequential decision algorithm, at first, we decide the correctness of $\hat{\mathbf{D}}_1$ as shown in Fig. 1(a). If correct, $\hat{\mathbf{D}}_1$ is regarded as the correct symbols. If incorrect, $\hat{\mathbf{D}}_1$ is regarded as the incorrect symbol and go to the renew processing of $\hat{\mathbf{D}}_q$. The 2nd ranked symbol $\hat{d}_{1,2}$ at the fourth stage as shown in Fig.1 is updated as $\hat{\mathbf{D}}_2 = [\hat{d}_{1,1}, \hat{d}_{2,1}, \hat{d}_{3,1}, \hat{d}_{4,2}]$.

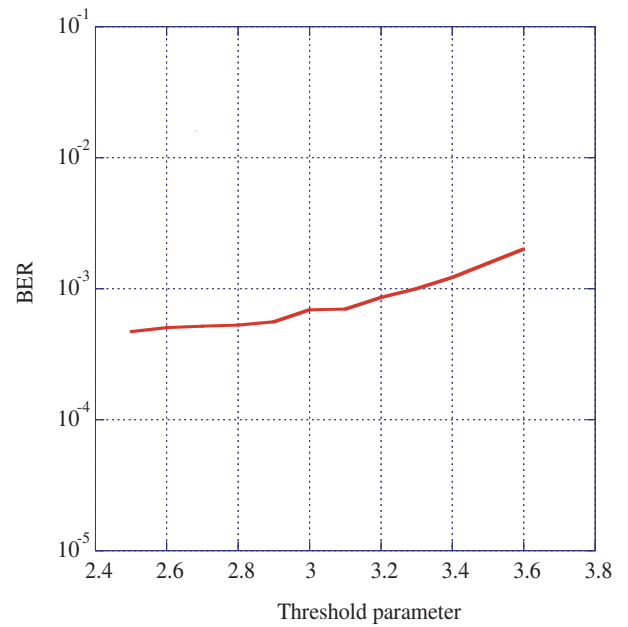


Fig. 3. The BER performance with respect to the threshold parameters

Then, we also check the Euclidean distance $U(k)_2$ of updated symbol candidate $\hat{\mathbf{D}}_2$ which is represented as

$$U(k)_2 = \|\mathbf{Z}(k) - \mathbf{R}(k)\hat{\mathbf{D}}_2(k)\| = \|\tilde{\mathbf{N}}(k)\|. \quad (17)$$

If the fourth ranked symbol at the fourth stage is decided as incorrect, we should update the tree structure as shown in Fig. 2. The second ranked symbol $\hat{d}_{2,2}$ at the third stage is updated as $\hat{\mathbf{D}}_2 = [\hat{d}_{1,1}, \hat{d}_{2,1}, \hat{d}_{3,2}, \hat{d}_{4,1}]$ and we also check the correctness. Sequential decision algorithm continues until appearing the correct judgment. However, if the last, the 4th symbol at the fourth stage, $\hat{\mathbf{D}}_l = [\hat{d}_{1,4}, \hat{d}_{2,4}, \hat{d}_{3,4}, \hat{d}_{4,4}]$ is judged incorrect, we detect the symbol which has the smallest Euclidean distance. In this case, this operation equal to MLD detection.

IV. SIMULATION RESULT

In this section, the performance of the proposed algorithm is compared with the conventional MLD algorithm. Table 1 shows the MIMO-OFDM simulation parameters. In this simulation, the number of transmit and receive antennas is

TABLE I
SIMULATION PARAMETERS

Data modulation	QPSK
Frame size	24 symbol ($N_p = 4, N_d = 20$)
Effective symbol duration	8 μs
FFT size	64
Number of carrier	64
Doppler frequency	10Hz
Constraint length (k)	7
Coding rate (R)	1/2
Channel model	15 path
Number of antenna	$Tx = 4, Rx = 4$

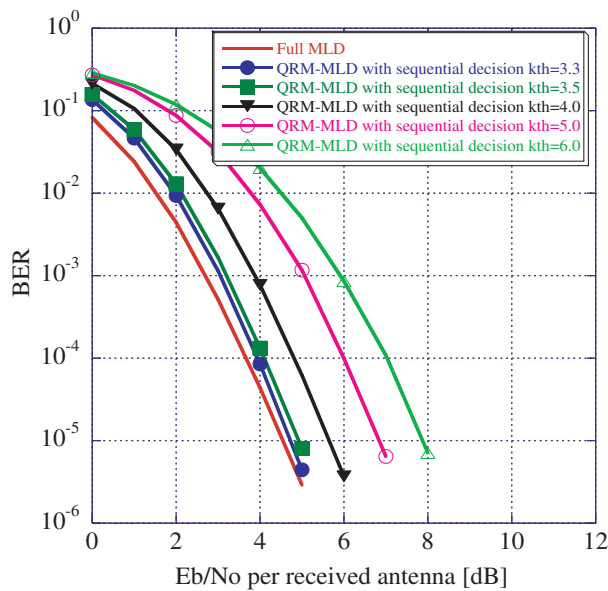


Fig. 4. The BER performance per E_b/N_0 and characteristics comparison with Full MLD

4. And the number of the subcarrier is 64. On the transmitter, the digital data stream is encoded. Here, convolutional code ($R = 1/2, k = 7$) with bit interleaving are used. The coded bits are QPSK modulated and then serial-to-parallel transformed after pilot signals are multiplexed with the modulated bits. The OFDM time signals are generated by an IFFT and transmitted from each transmit antenna branch over the frequency selective and time variant radio channel after cyclic extensions have been inserted. The transmitted signals are subject to broadband channel propagation. In this model, $L = 15$ path Rayleigh fading have exponential shapes. The maximum Doppler frequency is assumed to be 10Hz. The transmitted packet consists of 64 subcarriers and 24 OFDM symbols. On the received antenna, transmitted packets with radio noise are received by each receive antenna. Then, guard interval is removes, and FFT is operated and frequency signals are equalized by the channel state information estimated by the received antenna. Here, QRD-MLD with proposed sequential decision algorithm is used. In this simulation, noise variance set on the transmitter is reused the estimated noise variance.

Fig. 3 shows a BER performance with respect to the threshold parameters. Here, we assumed $E_b/N_0 = 3$ dB. From the simulation results, we can see the BER is increased in high threshold parameters area. This is because the determination accuracy of Eq. (16) is mitigated in high threshold parameters. As a result, incorrect symbols may be detected as correct symbols. In addition, we can see the complexity is reduced in high threshold parameters area.

Fig. 4 shows the BER performance of the conventional full MLD and the proposed method. In the proposed system, the threshold parameters are set to the $k_{th} = 3.3, 3.5, 4.0, 5.0, 6.0$. From the simulation results, we can see the BER performance of the proposed system with the threshold parameter $k_{th} = 3.3$

as same as Full MLD. If k_{th} is high, the proposed system deteriorates the BER performance as shown in Fig. 4. In addition, the required multiplications of the proposed system with the threshold parameters $k_{th} = 3.3, 3.5, 4.0, 5.0, 6.0$ are 2655360, 312520, 105435, 54763, 30790. The BER with $k_{th} = 3.5$ shows 0.4dB penalty compared with Full MLD. However, the required multiplication is smaller than that of the proposed system with $k_{th} = 3.3$.

V. CONCLUSION

In this paper, we proposed the complexity reducing QRM-MLD by sequential decision with noise variance. By reducing the surviving symbol replica candidates with the sequential decision algorithm, we can see the BER performance of the proposed system with the threshold parameter $k_{th} = 3.3$ as same as Full MLD. And, the required multiplication is reduced about 10 times compared with the Full MLD.

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