

Introduction of Dual Regularization Parameters to Improve the Performance of MMSE Based Vector Perturbation

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Abstract—This paper proposes a method to improve the performance of minimum-mean square error (MMSE) based vector perturbation (MMSE-VP) by introducing dual regularization parameters of MMSE precoding matrix. First regularization parameter α_1 is set so as to maximize the signal to interference plus noise ratio (SINR) value of each stream to find the optimal perturbation vector. The second regularization parameter α_2 is set as to minimize the total MSE between the transmitted perturbation vector and received perturbation vector to precancel the interference among the streams. The simulated results confirm that the proposed method can improve the performance of MMSE-VP over *i.i.d.* MIMO channel and indoor correlated frequency-selective fading MIMO channel.

I. INTRODUCTION

The technique of multiuser multiple-input multiple-output (MU-MIMO) can be utilized to further improve the downlink transmission rate of each user of the picocell or femtocell system. For the downlink MU-MIMO system, many spatial division multiple access (SDMA) techniques can be selected to improve the MU-MIMO channel capacity [1]. Among them, linear precoding combined with vector perturbation (VP) [2] is an efficient algorithm to achieve MU-MIMO capacity bound using the dirty paper coding (DPC) [3].

VP algorithm includes minimum-mean square error (MMSE) [4] based VP (MMSE-VP) and zero-forcing (ZF) based VP (ZF-VP). Based on ZF-VP, MMSE-VP utilizes an additional parameter α for the MMSE precoding matrix. MMSE-VP is equivalent to ZF-VP if α equals zero. However, MMSE-VP generally outperforms ZF-VP if the α is appropriately selected. Therefore finding an efficient regularization parameter α for MMSE precoding matrix is also a research topic for performance improvement.

The general objective function for finding an optimal α is to maximize the signal to interference plus noise ratio (SINR) of each stream. However this optimal value is still an open issue and only searched with a numerical method [2], [5]. Another objective function for optimizing α is to minimize the total MSE of the transmitted data and received data. To realize such process, many methods have been researched such as VP process with continuous-valued perturbation vector [6] and minimizing the bit-error ratio (BER) of generalized MMSE-VP system as the optimization criterion [7]. Some methods can improve the MMSE-VP performance by shaping

the transmitted symbols with different lattice transformation techniques [8].

Reference [9] showed that finding an optimal α to maximize the SINR value of each stream is equivalent to finding the optimum perturbation vector that minimizes the total mean square error (MSE) of the transmitted data and received data. But the method requires a transformed precoding matrix for searching the optimal perturbation vector. This also means that, using precoding matrix with optimized α based on SINR, the total MSE between the transmitted perturbation vector and received perturbation vector may be not minimized. In such case, the modulo function at receiver sides will deteriorate the BER performance although the SINR value of each stream is maximized. Therefore, it will improve the system performance by combining both objective functions. We will study this topic in this paper.

In this paper, we propose a simple method to improve the performance of MMSE-VP using dual regularization parameters of MMSE precoding matrix. Based on the objective function as SINR maximum, we find the optimal regularization parameter α_1 to maximize the SINR value of each stream by simulation for different modulation. To minimize the total MSE between the transmitted perturbation vector and received perturbation vector, we also find the optimal regularization parameter α_2 . The proposed method utilizes the MMSE precoding matrix with regularization parameter α_1 to realize the optimal perturbation vector search, and utilizes the MMSE precoding matrix with regularization parameter α_2 to pre-cancel the interference among the streams. The simulated results show that the proposed method can improve the performance of MMSE-VP especially for QPSK modulation over *i.i.d.* MIMO channel and indoor correlated frequency-selective fading MIMO channel.

This paper is organized as follows. The MMSE-VP algorithm and QR-decomposition M-algorithm encoder (QRDM-E) [10], which can compromise with the computational complexity (CC) and the diversity order of VP, are simply described in Section II. Then the proposed method is introduced in Section III. The performance of proposed method over flat-fading and frequency-selective fading channels is shown in Section IV. The paper ends with conclusions presented in Section V.

Notation : We use upper (lower) boldface letters to de-

note matrices (column vectors), sometimes with subscripts to emphasize their sizes. $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ will denote complex conjugate, transpose, conjugate transpose. $\mathbf{E}\{\cdot\}$ and $\text{Tr}\{\cdot\}$ are the expectation operator and matrix trace operator.

II. MMSE BASED VECTOR PERTURBATION (MMSE-VP) AND QRDM-ENCODING

A general model of MU-MIMO system includes a BS with N_t transmitting antennas and K users, each with N_r receiving antennas. We assume that $N_t = KN_r$. The corresponding vector equation is

$$\mathbf{y} = \mathbf{H}\mathbf{u} + \mathbf{n} \quad (1)$$

where $\mathbf{y} = [y_1, \dots, y_{N_t}]^T$, $\mathbf{u} = [u_1, \dots, u_{N_t}]^T$, and $\mathbf{n} = [n_1, \dots, n_{N_t}]^T$. The $N_t \times N_t$ matrix $\mathbf{H}$ has complex elements of MIMO channel. We assume that $E\{\mathbf{n}\mathbf{n}^*\} = \sigma^2 \mathbf{I}_{N_t}$ and the power constraint $E\{|\mathbf{u}|^2\} = 1$. $\mathbf{I}_{N_t}$ is $N_t \times N_t$ identity matrix.

The MMSE precoding matrix [4] is sometimes known as MMSE based channel inversion or MMSE spatial filter. Suppose that BS has perfect CSI $\mathbf{H}$, the MMSE precoding matrix is set as

$$\mathbf{W} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \frac{\alpha}{\sigma^2} \mathbf{I}_{N_t})^{-1} \quad (2)$$

where σ^2 is the signal to noise ratio (SNR) value at receiving antenna of downlink. The regularization parameter α can be used to minimize the total MSE between the transmitted symbols and received symbols.

The BS sets

$$\mathbf{s} = \mathbf{W}\mathbf{x} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \frac{\alpha}{\sigma^2} \mathbf{I}_{N_t})^{-1} \mathbf{x}. \quad (3)$$

After normalized transmit symbol $\mathbf{s}$ as $\mathbf{u} = \sqrt{\frac{1}{\gamma}} \mathbf{s}$ with $\gamma = \|\mathbf{s}\|^2$, the symbol $\mathbf{u}$ will be transmitted over MU-MIMO channel $\mathbf{H}$ to all users.

However, if $\mathbf{H}$ is a high correlated channel matrix, the normalized parameter γ will be largely increased and the data $\mathbf{x}$ will be lost. To mitigate its effect, the transmitted symbol $\mathbf{x}$ can be perturbed with a perturbation vector to reduce the normalized parameter γ . This method has been researched in [2] named as vector perturbation (VP). In the simplest case, let us set $\hat{\mathbf{x}} = \mathbf{x} + \tau\ell$ where τ is a positive real number and decided by the modulation constellation size. ℓ is a N_t -dimensional complex perturbation vector $a + ib$, where a and b are integers. The transmit power γ is computed as

$$\gamma = \|\mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \frac{\alpha}{\sigma^2} \mathbf{I}_{N_t})^{-1} (\mathbf{x} + \tau\ell)\|^2 = \|\mathbf{W}(\mathbf{x} + \tau\ell)\|^2. \quad (4)$$

The core design of VP is to find the optimum choice ℓ which minimizes the transmit power γ , that is

$$\ell = \arg \min_{\ell'} \|\mathbf{W}(\mathbf{x} + \tau\ell')\|^2 = \arg \min_{\ell'} \left\| \frac{\mathbf{H}^H (\mathbf{x} + \tau\ell')}{\mathbf{H}\mathbf{H}^H + \frac{\alpha}{\sigma^2} \mathbf{I}_{N_t}} \right\|^2. \quad (5)$$

If BS uses the identical precoding matrix $\mathbf{W}$ with regularization parameter α to pre-cancel the interference among

streams, the transmitted signal after perturbation vector search as Eq. (5) is

$$\mathbf{u} = \frac{1}{\sqrt{\gamma}} \mathbf{W}(\mathbf{x} + \tau\ell) = \frac{1}{\sqrt{\gamma}} \frac{\mathbf{H}^H (\mathbf{x} + \tau\ell)}{\mathbf{H}\mathbf{H}^H + \frac{\alpha}{\sigma^2} \mathbf{I}_{N_t}}. \quad (6)$$

In such case, the purpose of MMSE-VP is, in essence, the optimization to get smallest normalized transmit power to improve the value of SNR at receiver side.

The symbol $\mathbf{u}$ is transmitted over MU-MIMO channel $\mathbf{H}$ to all users. It can be represented as

$$\mathbf{y} = \frac{1}{\sqrt{\gamma}} \frac{\mathbf{H}\mathbf{H}^H}{\mathbf{H}\mathbf{H}^H + \frac{\alpha}{\sigma^2} \mathbf{I}_{N_t}} (\mathbf{x} + \tau\ell) + \mathbf{n}. \quad (7)$$

To remove the effect of the integer multiple of τ for i th stream, the receiver utilizes the modulo function as

$$f_\tau(y_i) = y_i - \left\lfloor \frac{y_i + \tau/2}{\tau} \right\rfloor \tau \quad (8)$$

where the function $\lfloor \cdot \rfloor$ gives the largest integer less than or equal to its argument. With the modulo function, the receiver can demodulate its own data.

The perturbation vector search of (5) is an N_t -dimensional integer-lattice least-squares problem which can be resolved using the sphere decoder [11]. However, the CC of the sphere decoder is large and limits the application of VP. An efficient sphere decoder algorithm can be realized using the QR-decomposition M-algorithm encoder (QRDM-E) [10] which can compromise with the CC and the diversity order of VP.

QRDM-E limits the integer value a and b of ℓ selected from the symmetric integer set $\mathcal{A}$ as $\mathcal{A} = \{-T, -T+1, \dots, -1, 0, 1, \dots, T-1, T\}$, which reduces the search complexity with a smaller value T . On the other hand, QRDM-E factorizes the matrix $\mathbf{W}$ of (5) into the product of a unitary matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$, thus, the search problem in (5) is simplified to

$$\ell = \arg \min_{\ell' \in \mathcal{A}^{N_t}} \|\mathbf{Q}\mathbf{R}(\mathbf{x} + \tau\ell')\|^2 = \arg \min_{\ell' \in \mathcal{A}^{N_t}} \|\mathbf{R}(\mathbf{x} + \tau\ell')\|^2. \quad (9)$$

For each iterative process, the best M branches that have the least accumulative metrics are retained at each encoding level. We omit the illustration for QRDM-E process and reader can find the clear explanation in [10].

III. THE PROPOSAL MMSE-VP ALGORITHM WITH DUAL REGULARIZATION PARAMETERS

A. Regularization parameter α for optimization of SINR for each stream

As shown in previous section, if BS uses the identical precoding matrix $\mathbf{W}$ with regularization parameter α to pre-cancel the interference among streams, the purpose of VP algorithm is the optimization to get smallest normalized transmit power to improve the value of SNR for each stream. On the other hand, the regularization parameter α of MMSE precoding matrix can be optimized to balance the stream interference and noise power to maximize the SINR value of stream.

To get optimal α to maximize the SINR value of stream, according to Eq. (7), the received signal is written as

$$\begin{aligned} \mathbf{y} &= \frac{1}{\sqrt{\gamma}} \frac{\mathbf{H}\mathbf{H}^H}{\mathbf{H}\mathbf{H}^H + \frac{\alpha}{\sigma^2} \mathbf{I}_{N_t}} (\mathbf{x} + \tau\ell) + \mathbf{n} \\ &= \frac{1}{\sqrt{\gamma}} \left[\mathbf{I}_{N_t} (\mathbf{x} + \tau\ell) + \mathcal{F}(\mathbf{x} + \tau\ell) \right] + \mathbf{n}, \end{aligned} \quad (10)$$

where $\mathcal{F} = \{(\mathbf{I}_{N_t} + \frac{\alpha}{\sigma^2}(\mathbf{H}\mathbf{H}^H)^{-1})^{-1} - \mathbf{I}_{N_t}\}$. The received signal for k th stream is

$$y_k = \frac{1}{\sqrt{\gamma}} [(x_k + \tau l_k) + \langle \mathcal{F}(\mathbf{x} + \tau\ell) \rangle_k] + n_k \quad (11)$$

where the notation $\langle \mathbf{C} \rangle_k$ represents the k th row of matrix $\mathbf{C}$. It is clear that $\langle \mathcal{F}(\mathbf{x} + \tau\ell) \rangle_k$ is potentially correlated with x_k and l_k . This correlation can be model as

$$y_k = \frac{1}{\sqrt{\gamma}} [(x_k + \tau l_k) + (\beta_k x_k + \delta_k \tau l_k) + v_k] + n_k \quad (12)$$

where v_k is co-channel interference (CCI) and uncorrelated with x_k and l_k . Here β_k and δ_k represent the correlation coefficients of the term $\langle \mathcal{F}(\mathbf{x} + \tau\ell) \rangle_k$ with x_k and l_k . As shown in Reference [5], the SINR value of k th stream $SINR_k$ is

$$SINR_k = \frac{(1 + \beta_k)^2 \mathbf{E}\{|x_k|^2\}/\gamma}{\mathbf{E}\{|v_k|^2\}/\gamma + \sigma^2}. \quad (13)$$

The regularization parameter α controls the value of γ , the correlation β_k and the variance of v_k . Increasing α generally decreases γ , thus potentially increasing the $SINR_k$, but increases the variance of v_k , thus potentially decreasing the $SINR_k$. Therefore the overall impact on the performance of MMSE-VP is difficult to determine analytically. The optimal value α is generally decided by a numerical method as pointed out in References [2], [5].

B. Regularization parameter α for optimization of total MSE between the transmitted perturbed vector and received perturbed vector

After perturbation vector search, the transmit data $\mathbf{x}$ is perturbed as $\hat{\mathbf{x}} = \mathbf{x} + \tau\ell$. Here $\tau\ell$ can obtain the smallest transmit power γ as shown in Eq. (5). We assume there is a precoding matrix $\mathbf{P}$. Therefore, the received symbol $\hat{\mathbf{x}}$ is represented as

$$\hat{\mathbf{x}}_r = \frac{1}{\sqrt{\gamma_P}} \mathbf{H}\mathbf{P}\hat{\mathbf{x}} + \mathbf{n} \quad (14)$$

where γ_P equals $\mathbf{E}\{|\mathbf{P}\hat{\mathbf{x}}|^2\}$.

Let us design a precoding matrix to minimize the total MSE between the transmitted symbol $\hat{\mathbf{x}}$ and received symbol $\hat{\mathbf{x}}_r$. Such objective function can be represented as

$$\begin{aligned} \{\mathbf{P}_{opt}, \gamma_{opt}\} &= \arg \min_{\mathbf{P}, \gamma} \mathbf{E}\{|\hat{\mathbf{x}}_r - \hat{\mathbf{x}}|^2\} \\ \text{s.t. : } &\mathbf{E}\{|\mathbf{P}\hat{\mathbf{x}}|^2\} = \gamma_P \end{aligned} \quad (15)$$

The solution can be represented as

$$\begin{aligned} \mathbf{P}_{opt} &= \mathbf{H}^H (\mathbf{H}\mathbf{H} + \xi \mathbf{I}_{N_t})^{-1} \\ \gamma_{opt} &= \|\mathbf{P}_{opt} \hat{\mathbf{x}}\|^2 \end{aligned} \quad (16)$$

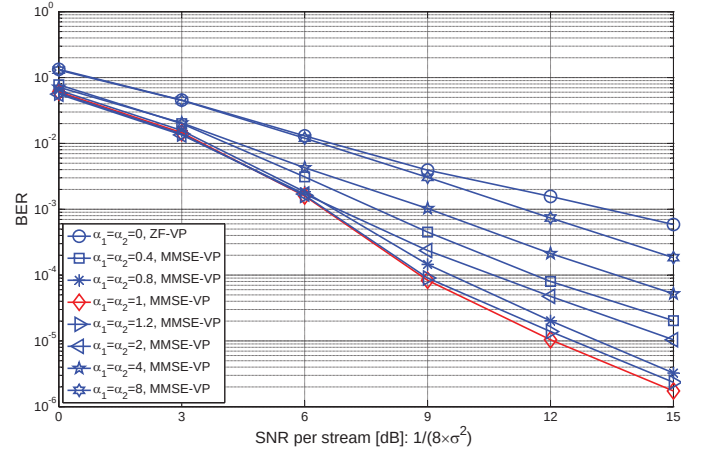


Fig. 1. The simulated BER performance of QPSK modulated MMSE-VP algorithm with different value of α_1 .

where the regularization parameter ξ equals $\text{Tr}\{\frac{1}{\sigma^2} \mathbf{I}_{N_t}\} = \frac{N_t}{\sigma^2}$ [12].

The solution also shows that if the previous optimal regularization parameter α for maximizing SINR value obtained by numerical method is identical to the regularization parameter $\xi\sigma^2$, the total MSE between the $\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}_r$ is also minimized. However, as shown in References [2] and [5], the optimal α for maximizing SINR value is generally smaller than N_t which cannot realize both optimization targets.

C. Proposed method with dual regularization parameters

The proposed simple method is realized as following steps.

- (1) We use numerical method to find the efficient regularization parameter α which can achieve better BER performance and named as α_1 for each different modulation.
- (2) For perturbation vector search, the optimum choice ℓ which minimizes the transmit power γ is searched using precoding matrix with regularization parameter α_1 as

$$\ell = \arg \min_{\ell' \in \mathcal{A}_{N_t}} \left\| \frac{\mathbf{H}^H (\mathbf{x} + \tau\ell')}{\mathbf{H}\mathbf{H}^H + \frac{\alpha_1}{\sigma^2} \mathbf{I}_{N_t}} \right\|^2. \quad (17)$$

- (3) The precoding matrix $\mathbf{W}$ is set as $\mathbf{W} = \mathbf{P}_{opt} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \frac{\alpha_2}{\sigma^2} \mathbf{I}_{N_t})^{-1}$ with $\alpha_2 = N_t$ to realize the total MSE optimization. Therefore, after obtains optimum choice $\tau\ell$, BS sets the transmit symbol as

$$\mathbf{u} = \frac{1}{\sqrt{\gamma'}} \mathbf{W} (\mathbf{x} + \tau\ell) = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \frac{\alpha_2}{\sigma^2} \mathbf{I}_{N_t})^{-1} (\mathbf{x} + \tau\ell) \quad (18)$$

where γ' equals $\|(\mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \frac{\alpha_2}{\sigma^2} \mathbf{I}_{N_t})^{-1}) (\mathbf{x} + \tau\ell)\|^2$.

IV. PERFORMANCE EVALUATION OF PROPOSED METHOD OVER THE FLAT-FADING AND CORRELATED FREQUENCY-SELECTIVE FADING MIMO CHANNEL

A. Finding α_1 by numerical method

We utilize numerical method to find efficient value of α_1 . As shown in References [2] and [5], the optimal α_1 is usually

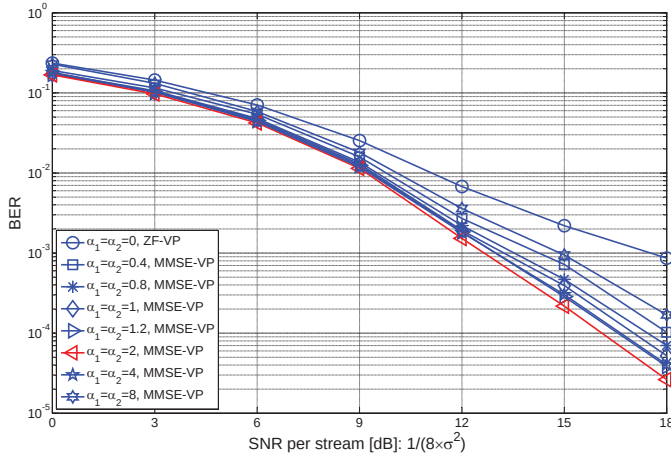


Fig. 2. The simulated BER performance of 16QAM modulated MMSE-VP algorithm with different value of α_1 .

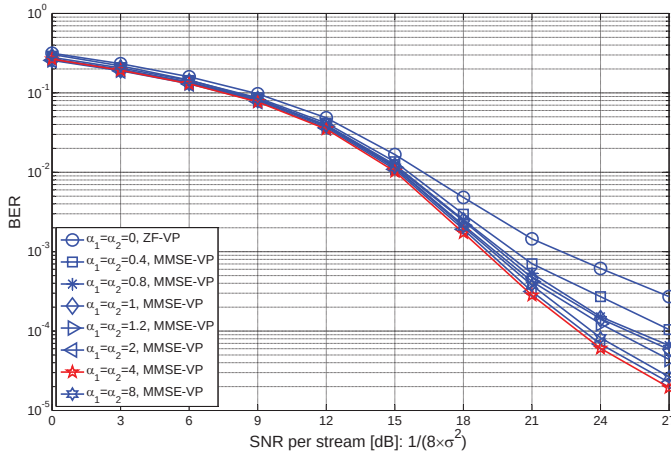


Fig. 3. The simulated BER performance of 64QAM modulated MMSE-VP algorithm with different value of α_1 .

smaller than N_t . We simulate the BER performance of VP algorithm with different value of α over *i.i.d.* MIMO channel. We assume $N_t = 8$ and $(T, M) = (3, 7)$ for QRDM-E algorithm. The simulated BER results are shown in Fig. 1 for QPSK modulation, Fig. 2 for 16QAM modulation and Fig. 3 for 64QAM modulation. According to the simulated results, the optimal values of α_1 for MMSE-VP algorithm over (8×8) *i.i.d.* channel are approximated as 1, 2 and 4, respectively.

B. Performance evaluation of proposed method over flat-fading MIMO Channel

We use Fig. 4 and Fig. 5 to show the simulated results of proposed method over *i.i.d.* MIMO channel with different modulation. We also assume $N_t = 8$ and $(T, M) = (3, 7)$ for QRDM-E algorithm. To show the impact of α_2 , we choose different values to compare the BER performance. Here the original method in Figures refers to that MMSE-VP algorithm transmits the data with the optimal performance as shown in Figs. 1-3. The simulated results show that the proposed method can improve the performance of MMSE-VP especially

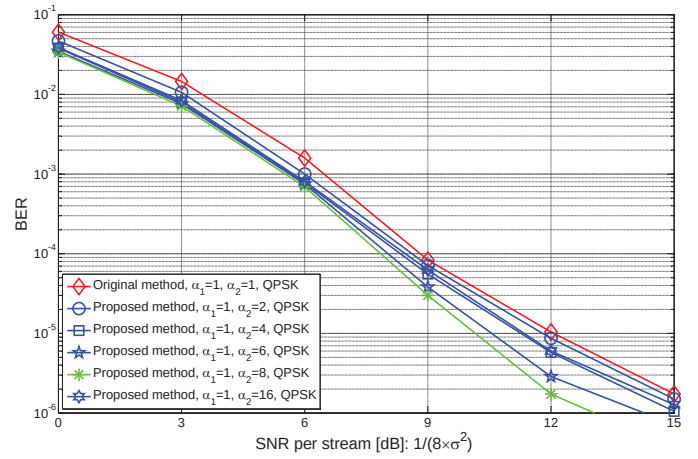


Fig. 4. The simulated BER performance of QPSK modulated MMSE-VP algorithm with different value of α_1 and α_2 .

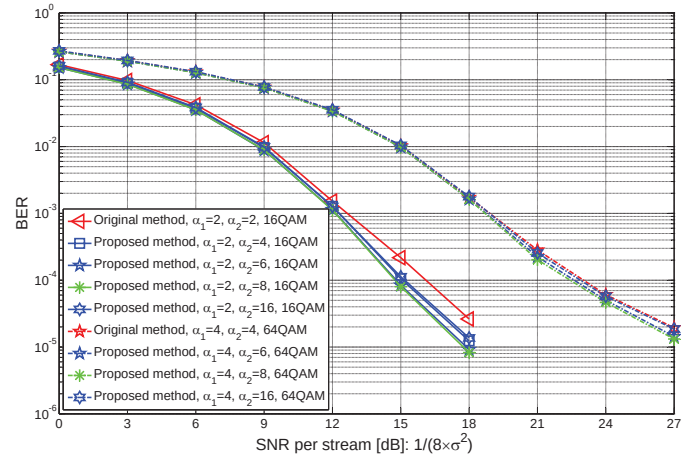


Fig. 5. The simulated BER performance of 16QAM and 64QAM modulated MMSE-VP algorithm with different value of α_1 and α_2 .

for QPSK modulation. The reason maybe due to that, for modulation with large number of constellation points, the effect of modulo function has less impact on the interior constellation points than that of boundary constellation points even if the MSE of between the transmitted perturbed vector and received perturbed vector is slightly increased. On the other hand, all simulated results show the best performance when $\alpha_2 = 8$.

C. Performance evaluation of proposed method over correlated frequency-selective fading MIMO Channel

The simulation specifications are given in Table 1. We choose one indoor correlated channel model to compare the proposed method and original method. The channel model is Winner II A1 LOS [13]. The average delay spread is 40 [ns], average AoA/AoD are 44/45 [Deg.], and the average K-factor is about 5-6[dB]. We assume each subcarrier has perfect CSI.

The simulated results are shown in Fig. 6. The values of α_1 and α_2 are set based on previous simulation over *i.i.d.* MIMO channel. The simulated results also confirm that using

TABLE I
SIMULATION SPECIFICATIONS

Sampling rate	30.72 Msamples/s
FFT size	2048
Length of cyclic prefix	4.7 μ s
Number of subcarriers	1200
Number of antennas	8 (BS) 2 (UE)
Number of users (N)	4
Direction of UEs	-67.5, -22.5, 22.5 and 67.5 (Deg.)
Modulation scheme	QPSK, 16QAM, 64QAM
Channel coding	Turbo code
Coding rate	3/4
Frame size	3597+3 parity bits (QPSK) 3597+3 parity bits (16QAM) 5397+3 parity bits (64QAM)
Interleaver	Random interleaver
Decoding algorithm	soft-output Viterbi algorithm
Number of decoding iterations	6
Array configuration	Equally spaced linear array
Antenna spacing	1 λ DL carrier frequency (BS) 0.5 λ DL carrier frequency (UE)
Carrier frequency	3.36 GHz (DL) 3.26 GHz (UL for analog feedback)
Spatial filtering	MMSE
Estimation of average received SNR	Perfect
Perturbated vector search	QRDM-E [10], $(T, M) = (3, 7)$

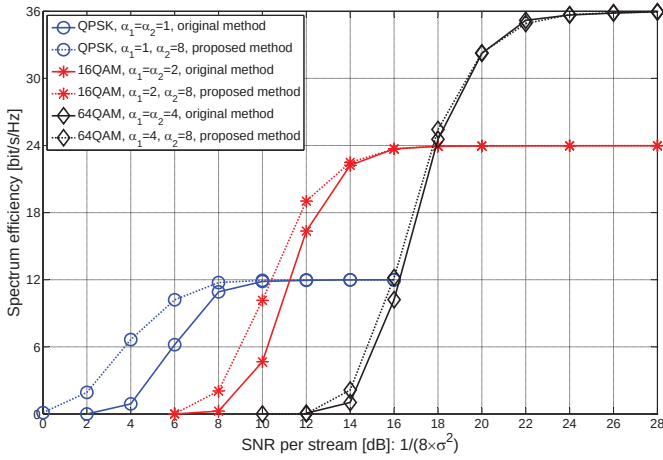


Fig. 6. The spectrum efficiency performance comparison of MMSE-VP algorithm with and without two-stage optimization method.

different values of α_1 and α_2 based on two-stage optimization increases the spectrum efficiency of MMSE-VP especially for QPSK modulation.

V. CONCLUSIONS

We have proposed a simple method to improve the performance of MMSE-VP using dual regularization parameters of MMSE precoding matrix. The proposed method utilizes the MMSE precoding matrix with regularization parameter α_1 which can maximize the SINR value of each stream to find the optimal perturbation vector. For MMSE precoding matrix to pre-cancel the interference among the streams, MMSE-

VP uses the MMSE precoding matrix with regularization parameter α_2 which can minimize the total MSE between the transmitted perturbation vector and received perturbation vector. The simulated results confirm that the proposed method can improve the performance of MMSE-VP especially for QPSK modulation over *i.i.d.* MIMO channel and indoor correlated frequency-selective fading MIMO channel.

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