

Quadratic Programming Approach for UWB Pulse Shaping Filter Design with Nonconstant Upper Bounds

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Abstract—In UWB pulse shaping filter design, FIR filter formulated as a semi-definite programming problem has been widely used. Recently, piece-wise continuous bounds has been incorporated into the FIR filter design to avoid power loss due to the constant spectral mask indirectly assumed in the design. In this paper, we formulate the pulse shaping filter design as a semi-infinite quadratic optimization problem using Hermite-Rodriguez basis functions. Computationally effective approach to solving the optimization problem is proposed. Through numerical design examples, we compare the spectral utilization of our approach with those FIR-based pulse shaping filter designs. We demonstrate that our proposed approach is easy to implement and achieves better spectral utilization than a recent result when a suboptimal basic pulse is used.

Keywords—pulse shaping; filter design; semi-infinite programming; ultra-wideband (UWB); Hermite-Rodriguez functions;

I. INTRODUCTION

In UWB systems, unmodulated basic pulse is employed in the transmission and the PSD of the basic pulse determines the PSD of the overall UWB signal. Given the importance of the basic pulse, a lot of research has been done on using finite impulse response (FIR) prefilters to shape the basic pulses [1]–[3]. However, designing the FIR filter coefficients to maximize power while satisfying the spectral mask is not trivial. The traditional FIR design algorithm has been used in [1] but it did not directly maximize the transmit power. In order to directly maximize the power, FIR filter design based on semidefinite programming has been used [2], [3] where the constant FCC spectral constraint was cast into linear matrix inequalities (LMI) framework in their convex optimization formulation [2].

In the most recent work [3], the pulse shaping filter design using FIR filters is improved by incorporating the basic pulse which is commonly generated by analog pulse generators. Incorporating basic pulse in the design leads to piece-wise continuous upper bounds instead of constant spectral mask constraint. Using modified Fourier series expansion to approximate the piece-wise continuous bounds, these bounds were cast in the LMI framework [3]. This approach has achieved high spectral utilization even when a suboptimal basic pulse was used [3].

Maximizing signal power leads to maximizing quadratic function of filter coefficients. To cast this maximizing problem into convex optimization problem, autocorrelation of the filter coefficients was used in [2], [3]. Thus, de-correlation or spectral factorization has to be carried out in order to obtain the designed filter coefficients. This step of the design can be numerically problematic and computationally expensive.

In this paper we investigate the applicability of Hermite-Rodriguez (HR) filter as the pulse shaping filter or transmit filter to shape the basic pulses in the UWB pulse shaping filter design. We directly formulate the pulse shaping filter design problem as a maximization of a quadratic function of the filter coefficients subject to linearizable inequality constraints. Similar to [3], we also incorporate the basic pulse in our formulation. We implement the resulting non-constant bounds and linearize it using the rotation theorem [4]. We show, through both theoretical argument and numerical design examples, that our proposed design is computationally efficient to implement and the design result is either comparable to or better than those reported in [3] especially when the suboptimal basic pulse is incorporated into the design.

The rest of the paper is organized as follows. Section II introduces Hermite-Rodriguez filter using orthogonal HR basis functions. Section III describes the pulse shaping problem with the nonconstant upper bounds. The solution method is then proposed. Design examples will be presented in Section IV. Lastly, Section V concludes the paper.

II. HERMITE-RODRIGUEZ FILTER

In this section, we shall summarize the relevant background material of Hermite Rodriguez functions. For more details, see [5] and the references therein.

The *Hermite-Rodriguez filter* of order N is defined as

$$g(t) = \sum_{k=0}^{N-1} x_k \omega_{\lambda,k}(t) = \mathbf{x}^T \mathbf{w}_{\lambda}(t) \quad (1)$$

where

$$\begin{aligned}\mathbf{x} &= [x_0, x_1, \dots, x_{N-1}]^T, \\ \mathbf{w}_\lambda(t) &= [\omega_{\lambda,0}(t), \omega_{\lambda,1}(t), \dots, \omega_{\lambda,N-1}(t)]^T,\end{aligned}$$

x_k is the k^{th} filter coefficient and $\omega_{\lambda,k}(t)$ is the k^{th} order Hermite Rodriguez function given by

$$\omega_{\lambda,k}(t) = \frac{1}{\sqrt{2^k k!}} H_k\left(\frac{t}{\lambda}\right) \frac{1}{\sqrt{\pi \lambda}} e^{-t^2/\lambda^2} \quad k \in [0, \infty). \quad (2)$$

λ is a scaling parameter and $H_k(t)$ is the Hermite polynomials defined recursively as

$$H_k(t) = \begin{cases} 1 & k = 0 \\ 2t & k = 1 \\ 2tH_{k-1}(t) - 2(k-1)H_{k-2}(t) & k \geq 2. \end{cases} \quad (3)$$

The Fourier transform of $g(t)$ is given by

$$G(f) = \int_{-\infty}^{+\infty} g(t) e^{-j2\pi ft} dt = \sum_{k=0}^{N-1} x_k W_{\lambda,k}(f)$$

where $W_{\lambda,k}(f)$ is the Fourier transform of the Hermite-Rodriguez function given by

$$\mathcal{F}\{\omega_{\lambda,k}(t)\} = W_{\lambda,k}(f) = \frac{(-j)^k}{\sqrt{2^k k!}} (2\pi f \lambda)^k e^{-(2\pi f \lambda/2)^2}. \quad (4)$$

Let $\mathbf{W}(f) = [W_{\lambda,0}(f), W_{\lambda,1}(f), \dots, W_{\lambda,N-1}(f)]^T$. Then $G(f)$ can be written as $G(f) = \mathbf{x}^T \mathbf{W}(f)$ and the power spectral density of $g(t)$ can be expressed as

$$\Psi_g(f) = |G(f)|^2 = \mathbf{x}^T \Phi(f) \mathbf{x}$$

where $\Phi(f) = \mathbf{W}(f) \mathbf{W}^T(f)$ is an N by N non-negative definite matrix.

III. PULSE SHAPING PROBLEM

Generally, the basic pulses $p(t)$ used in UWB systems do not only optimally exploit the allowable spectrum but also not comply with some specific constraints such as the FCC spectral mask. An example of a basic pulse $p(t)$ is the Gaussian Monocycle [6]. We will use HR based transmit filters to adapt the basic pulses to a certain spectral constraints. Let $q(t)$ denote the pulses created by the analog pulse generators. Then, following the standard system model [3], [6], $p(t)$ can be expressed as

$$p(t) = q(t) * h(t) = \int_{-\infty}^{+\infty} q(\tau) h(t - \tau) d\tau \quad (5)$$

where

$$h(t) = \sum_{k=0}^{N-1} x_k \omega_{\lambda,k}(t)$$

is the transmit filter represented by (1) with x_k will be the design parameters. The PSD of $p(t)$ is

$$|P(f)|^2 = |H(f)|^2 |Q(f)|^2.$$

Now, let Ω_p denote the sets of passband frequencies and let $S(f)$ denote the spectral masks. Then the optimal pulse shaping problem can be formulated as the following constrained optimization problem

$$\max_{\mathbf{x}} \int_{\Omega_p} |P(f)|^2 df \quad (6)$$

subject to

$$|P(f)|^2 \leq S(f), \quad \forall f \quad (7)$$

i.e., maximizing the power while satisfying the given spectral mask constraint.

The pulse shaping problem (6) and (7) can be converted to the following simplified non-linear optimization problem

$$\max_{\mathbf{x}} \mathbf{x}^T A \mathbf{x} \quad (8)$$

subject to

$$|H(f)| = |\mathbf{x}^T \mathbf{W}(f)| \leq \frac{\sqrt{S(f)}}{|Q(f)|}, \quad \forall f, \quad (9)$$

where

$$A = \int_{\Omega_p} \Phi(f) |Q(f)|^2 df \quad (10)$$

is a positive definite matrix. Note that (8) and (9) define a constrained non-linear optimization problem, where the objective function is quadratic but the constraint in general is non-linear and non-convex.

Next, we consider the linearization of the inequality constraints (9). For any complex number $z = \xi + j\eta$, from the rotation theorem [4] we know that

$$|z| = \max_{0 \leq \theta < 2\pi} \Re\{ze^{j\theta}\}.$$

Using the rotation theorem, the magnitude inequality constraint (9) can be written as

$$\max_{0 \leq \theta < 2\pi} \Re\{\mathbf{x}^T \mathbf{W}(f) e^{j\theta}\} \leq \frac{\sqrt{S(f)}}{|Q(f)|}. \quad (11)$$

Note that the constraint (11) is linear but the number of constraints increases due to the introduction of the new parameter θ . As can be seen that (8) and (11) specify a semi-infinite quadratic programming problem where the number of variables to be optimized is finite but the number of inequality constraints is infinite (depend on θ and f). In numerical optimization, either the parameter θ can be discretized or the approach in [7], [8] can be adopted.

For simplicity, let us discretize the continuous variable θ and consider constraint (11) over a discrete set $\{\theta_k\}_{k=1}^{2p}$ with $\theta_k = \pi(k-1)/p$, where $p \geq 2$ is a sufficiently large integer. Furthermore, let us define

$$M_n^p(f) = \max_{1 \leq k \leq 2p} \Re\{\mathbf{x}^T \mathbf{W}(f) e^{j\theta_k}\}. \quad (12)$$

Table I
SPECTRAL EFFICIENCY, η

$(\tau = 0.087 \text{ ns}) N$	10	20	30
HR (QSIP)	0.4227	0.7850	0.8388
FIR (QSIP)	0.6966	0.8079	0.8172
FIR (SDP) [3]	0.7135	0.8230	-

Table II
SPECTRAL EFFICIENCY, η

$(\tau = 0.15 \text{ ns}) N$	10	20	30
HR (QSIP)	0.3665	0.7672	0.8431
FIR (QSIP)	0.6189	0.8164	0.8485
FIR (SDP) [3]	0.5588	0.6518	-

It can be proved [9], [10] that

$$M_n^p(f) \leq \max_{0 \leq \theta \leq 2\pi} \Re\{\mathbf{x}^T \mathbf{W}(f) e^{j\theta}\} \leq M_n^p(f) \sec\left(\frac{\pi}{2p}\right). \quad (13)$$

Note that $\sec(\pi/(2p)) \rightarrow 1$ (as $p \rightarrow \infty$). This means that for a sufficiently large integer p , M_n^p gives a fairly good estimate of $H(f)$. In fact, for $p = 8$, $\sec(\pi/(2p)) = 1.020$. Hence, instead of considering constraint (11), we shall consider the following strengthened inequality constraint over discrete sets $\{f_l\}_{l=1}^L$ and $\{\theta_k\}_{k=1}^{2p}$

$$\max_{1 \leq k \leq 2p} \Re\{\mathbf{x}^T \mathbf{W}(f_l) e^{j\theta_k}\} \leq \frac{\sqrt{S(f_l)}}{|Q(f_l)| \sec(\frac{\pi}{2p})}. \quad (14)$$

Constraint (14) is equivalent to the following set of linear inequality constraints

$$B_l \mathbf{x} \leq \mathbf{c}_l \quad (15)$$

where, for each l , B_l is a $2p$ by N matrix with

$$\mathbf{b}^T(f_l, \theta_k) = \Re\{\mathbf{W}^T(f_l) e^{j\theta_k}\}$$

as its k th row, and $\mathbf{c}_l$ is a $2p$ -dimensional vector with all its entries

$$c_l = \frac{\sqrt{S(f_l)}}{|Q(f_l)| \sec(\frac{\pi}{2p})}.$$

To summarize, we have the following simplified pulse shaping problem: Given the discrete sets $\{\theta_k\}_{k=1}^{2p}$ and $\{f_l\}_{l=1}^L$, find $\mathbf{x}^*$ which solve the following optimization problem

$$\max_{\mathbf{x}} \mathbf{x}^T A \mathbf{x} \quad (16)$$

subject to

$$B_l \mathbf{x} \leq \mathbf{c}_l \quad (17)$$

for $1 \leq l \leq L$. Note that A is a positive definite matrix. The solution of the optimization problem (16) and (17) is straightforward and can be carried out, for example, using simple MATLAB programs.

IV. DESIGN EXAMPLES

We design a pulse shaping filter $h(t)$ as a transmit filter *i.e.*, solve (16) and (17) for the filter coefficients. For this particular design, we first fix λ of the HR filter. Since we would like the overall signal to occupy a finite spectrum range *i.e.*, to have essentially zero magnitude after a certain maximum frequency and we know that λ is inversely proportional to the maximum frequency, we set $\lambda = \frac{2}{16\text{GHz}} = 1.25 \times 10^{-10}$. Besides the HR filter, we also design an FIR filter expressed as

$$H(f) = \sum_{k=0}^{N-1} x_k e^{j2\pi k T f}$$

where $T = 0.0333 \text{ ns}$.

The basic pulse $q(t)$ is the Gaussian monocycle whose PSD is given as $|Q(f)|^2 \propto f^2 e^{-\pi(\tau f)^2}$. Similar to [3], the passband is $\Omega_p \in [0, 15] \text{ GHz}$. A in (16) is then computed using (10). The spectral mask used is the FCC indoor limit [11]. B_l and c_l in (17) are computed with $L = 200$ and $p = 8$. The constrained optimization problem (16) and (17) is then solved using Matlab and its Optimization Toolbox.

To compare with the previous work [3], the power utilization efficiency, η is similarly defined as

$$\eta = \frac{\int_{\Omega_p} |P(f)|^2 df}{\int_{\Omega_p} S(f) df}. \quad (18)$$

We also study design examples with $q(t)$ having $\tau = 0.087 \text{ ns}$ and $\tau = 0.15 \text{ ns}$. We plot $S(f)/|Q(f)|^2$ for these two basic pulses in Fig. 1 and Fig. 2 respectively.

- Design with $\tau = 0.087 \text{ ns}$

The PSD of the pulses designed using both HR and FIR filters are plotted in Fig. 3. The spectral efficiency, η for different N s are tabulated in Table I. The η from Table II of [3] is also tabulated for easy comparison. It can be seen that the results reported in [3] is slightly better than our results. However, we note that we use relatively small number of p in the discretization.

- Design with $\tau = 0.15 \text{ ns}$

When we use a suboptimal basic pulse *i.e.*, the pulse with $\tau = 0.15 \text{ ns}$, we achieve much better spectral efficiency. The PSD of the pulses using both HR and FIR filters for $N = 20, 30$ are plotted in Fig. 4. Comparing this with Fig. 9 of [3], it is clear that our designed pulses utilized the allowable spectrum more efficiently. The η for the designed pulses as well as from Table III of [3] are tabulated in Table II.

Our approach is more straightforward than the approach in [3] where the piece-wise continuous constraints were implemented heuristically due to the limitations of the Fourier series. The nonconstant bounds implemented using the approach in [3] were plotted and denoted as $\Gamma(f)$ in Fig. 1. These bounds are optimal for the basic pulse

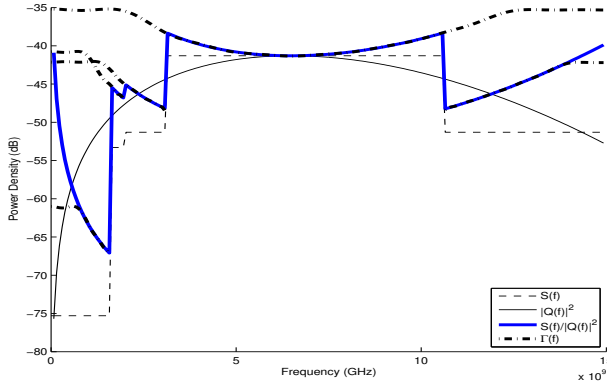


Figure 1. The constraint $S(f)/|Q(f)|^2$ for $\tau = 0.087$ ns.

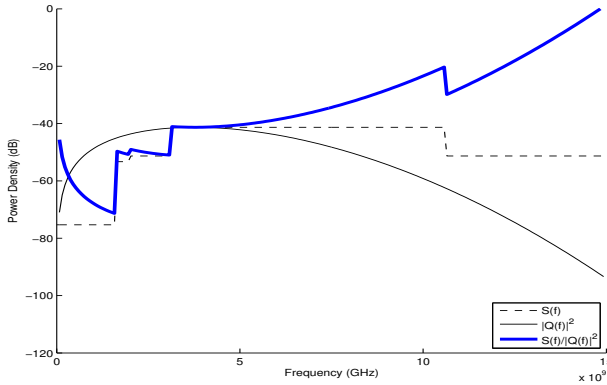


Figure 2. The constraint $S(f)/|Q(f)|^2$ for $\tau = 0.15$ ns.

with $\tau = 0.087$ ns but not optimal for $\tau = 0.15$ ns. In our proposed approach, we can easily implement different bounds for the different basic pulses. This is the reason we achieved much better results for the less optimal pulse.

To get a sense of the size of the parameters involved in solving (16) and (17), let's compare it with the total elements of semidefinite matrices in [3]. Approximately, there are $2pL$ parameters in our approach and $(i+1)N^2 + i(N-1)^2$ variables in [3] where i is the number of bounds. Thus, we have 3200 parameters with $p = 8$ and $L = 200$, and there are 4205 variables in [3] with $N = 20$ and $i = 5$.

V. CONCLUSION

In this paper we have applied orthogonal Hermite Rodriguez basis functions to design a pulse shaping filter for UWB communication systems. We directly formulated the design problem to maximize the transmit power in terms of filter coefficients, thus avoiding the spectral factorization step. We incorporated the basic pulse into the design and using the rotation theorem, the resulting nonlinear piecewise continuous bounds have been converted into linear constraints with the addition of a new parameter. Design examples using both FIR and HR filter have demonstrated that the proposed design approach is more straightforward

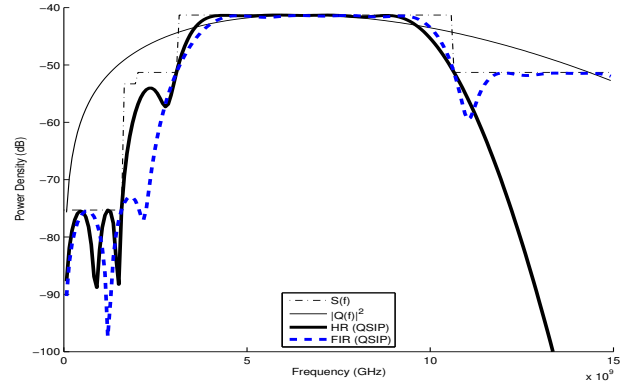


Figure 3. Plot of PSD for $\tau = 0.087$ ns ($N = 20$).

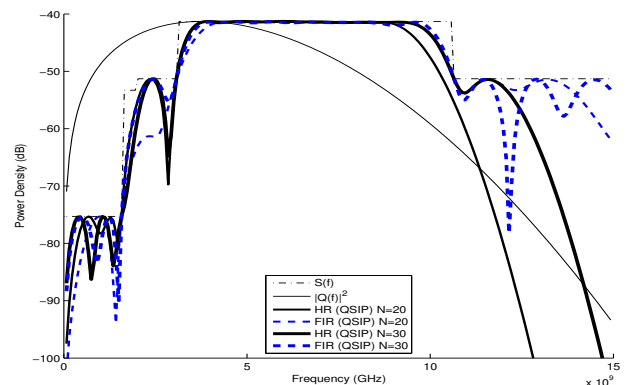


Figure 4. Plot of PSD for $\tau = 0.15$ ns.

and achieve comparable spectral utilization with the recent work in [3]. When a suboptimal basic pulse is used, we achieved better spectral utilization.

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