

Probability analysis of isolated node in wireless ad-hoc sensor network with border effect

Nagesh K. N^{*}, Satyanarayana D^{**}, Nagesh Poojary^{*}, Chandrashekar Ramiah^{*} and M.N Giri Prasad^{***}

^{*}Middle East College, Knowledge Oasis, Al Rusyal, Sultanate of Oman

^{**}ECE Department, RGM CET, Nandyal- 528502, A.P, India.

^{***}ECE Department, JNTUCE, Anantapur- 515002, A.P, India.

nagesh@mec.edu.om, nagesh.bk@gmail.com, Chandrashekar@mec.edu.om

Abstract— With the growing importance of wireless ad-hoc networks (particularly, in applications such as wireless local area networks, and ad-hoc sensor networks), it is important to develop an understanding of the isolated node in ad-hoc network with border effect. However it is a challenging task owing to the fact that such analysis must take into account the interactions between wireless physical layer, radio propagation and multiple accesses. Connectivity is one of the important features of wireless sensor network. The nodes are said to be connected in a network if their deterministic distance is less than transmission radius. In this paper, major issues in the probability analysis of isolated node in wireless ad-hoc sensor networks with border effect are presented. Additionally, the associated topics like fading and capacity scaling in different channels are surveyed. The results are obtained using MATLAB communication tool for different values of node density, number of nodes for different channels and comparison of these channels are discussed.

Keywords— Ad-Hoc network, Frame Reception Rate, Deterministic channel, Rayleigh fading, lognormal shadowing.

I. INTRODUCTION

One of the research fields which have gained more attention by the scientific community in the last few years is that of self-organizing large-scale wireless networks, generally referred to as multihop, ad-hoc or packet radio networks. Slightly aside, a growing interest is deferred to wireless sensor networks [1], which, while presenting peculiar features, possess at the same time many of the characteristics of ad-hoc

networks. More concerning issue for such networks, is that of limiting achievable performance, in terms of capacity, connectivity, delay and coverage [2-4]. Most of the literatures still rely on a simplistic model of channel propagation, where the randomness inherently present in radio communications is not considered. Connectivity and coverage in wireless ad-hoc sensor networks have received considerable attention over the last few years. The problem of finding a necessary and sufficient scaling for the transmission range to obtain a fully connected network (in the limiting dense regime) was studied in [6]. Similar results were obtained independently by Penrose in a more abstract framework [7]. Scalings for the case of sparse networks were investigated in [8]. The connectivity achievable in the presence of nodes mobility was first addressed in [9]. The impact of lognormal shadowing on connectivity of ad-hoc networks was first addressed by Bettstetter and Hartmann [10-11] (which extends a previous conference work). In particular, they proposed a numerical method for evaluating the impact of lognormal shadowing with qualitative. In [12] we provided a closed-form expression for such case. The impact of Rayleigh fading on network connectivity was studied by Haennnggi in [15]. All these results can be seen as particular cases of the general case first analyzed in [12].

From the literature it is clear that analysis of node isolation probability has not been explored in the contest of border effect. In this paper influence of border effect is analyzed on node isolation probability for Deterministic channel, Rayleigh fading channel and lognormal shadowing channel.

The remainder of the paper is organized as follows. In Section 2, the preliminary assumptions and model are provided. Analytical evaluation of frame reception rate probability is presented in Section 3. Section 4 describes the numerical and simulation results followed by conclusions in Section 5.

II. SYSTEM MODEL

Consider a wireless multi hop network where the nodes are randomly distributed according to a homogeneous Poisson point process with λ representing the expected number of nodes per unit square ($0 < \lambda < \infty$). Let λ be a two dimensional stationary Poisson point process over $\mathbb{R}^2$. The points of the process represent the location of the nodes. Given a finite subset $A \in \mathbb{R}^2$ of size $v(A)$, the number of nodes in A denoted by $N(A)$ is a Poisson random variable with intensity $\lambda v(A)$. The numbers of nodes in disjoint areas are independent random variables. The network formed by the nodes in A is said to be connected if and only if there is a path between each pair of nodes in A .

The deterministic channel for minimum node degree given by,

$$P_I = e^{-\lambda E[A_o]} = \frac{(\lambda A_o)^n}{n!} e^{-\lambda A_o} \quad (1)$$

Where A_o is expected area given by πr_o^2 , with r_o is concerned radius, n is number of sensor nodes. From equation (1) we have for fading channel minimum node degree given by

$$p(d = n) = \frac{(\lambda \pi u)^n}{n!} e^{-\lambda \pi u} \quad (2)$$

Where u is success probability for given fading medium

From [2] we have while considering border effect

$$E[A_o] = \frac{E[d]}{n} (\pi r_o^2)$$

$$\text{Where } p(d = n) = E(d) \quad p(d = n) = E(d) \quad (3)$$

From Probability given by

$$P_I = e^{-\lambda E[A_o]} \quad (4)$$

III. ANALYSIS OF NODE ISOLATION PROBABILITY

A. Deterministic channel

The entire document should be A channel where the medium between transmitter and receiver is known, as deterministic channel. Let us consider the total area A , which is Area of consideration when nodes are distributed randomly through Poisson distribution. Let A_o be sub area of area A for consideration, then probability that particular node d connected with n nodes is given by

$$P(d = n) = \frac{\left(\frac{A_o}{A} n\right)^n}{n!} e^{-\frac{n A_o}{A}} \quad (3.10)$$

Let $\lambda = (n/A)$ = Node density = Number of nodes per Area

$$P(d = n) = \frac{(\lambda A_o)^n}{n!} e^{-\lambda A_o} \quad (3.11)$$

Where $A_o = \pi r_o^2$, r_o = transmission radius

$$P(d = n) = (\lambda \pi)^n \frac{(r_o^2)^n}{n!} \quad (3.12)$$

Let $P(d = n) = E[d]$ = node degree = A node " d " connected with " n " number of nodes.

We have from [2], with consideration of border effect,

$$E[A_o] = E[d] \left(\frac{\pi r_o^2}{n!} \right) \quad (3.13)$$

$$E[A_o] = \frac{\pi r_o^2}{n} \left\{ \frac{(\lambda \pi r_o^2)^n}{n!} e^{-\lambda \pi r_o^2} \right\} \quad (3.14)$$

From [3], node isolation probability is given by $P_I = \exp(-\lambda \pi E[r^2])$]

(3.15)

$\pi E[r^2]$ = Average area without border effect.

The node isolation probability with border effect is

$$P_I = \exp \left[-\lambda \left\{ \pi \frac{r_o^2}{n} \left(\frac{(\lambda \pi)^n (r_o^2)^n}{n!} e^{-\lambda} \right) \right\} \right] \quad (3.16)$$

For $n = 15, \lambda = 2, P_I = 0.9994$

For $n = 20, \lambda = 3, P_I = 0.9995$

B. Rayleigh fading channel

If the signal arrived at receiver consists of several multipath and there is no line of sight between transmitter and receiver then such type of channel is called Rayleigh fading channel.

The probability density equation of SNR for Rayleigh fading is given by $f_{\bar{\gamma}}(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}$ (3.20)

$\bar{\gamma}$ = Average SNR

γ = Instantaneous SNR

Node isolation probability [3] is given by

$$P_I = \exp(-\lambda \pi E[r^2]) \quad (3.21)$$

$\pi E[r^2]$ = Average area without border effect.

A node is able to make connection with any other node is given by success probability. Since it is fading channel, node degree can be defined as

$$P(s) = \text{Success Probabi} = P(\lambda \geq \psi) = 1/\lambda \int_{\psi}^{\infty} e^{-\frac{\gamma}{\psi}} d\gamma \quad (3.22)$$

$$Ps(\bar{\gamma}) = (1/\bar{\gamma}) \left[\frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{-\frac{1}{\bar{\gamma}}} \right] = (1/\bar{\gamma}) [(-\bar{\gamma})(e^{-\infty} - e^{-\frac{\psi}{\bar{\gamma}}})] \quad (3.23)$$

$$Ps(\bar{\gamma}) = (-1)[0 - e^{-\frac{\psi}{\bar{\gamma}}}] = e^{-\frac{\psi}{\bar{\gamma}}} \quad (3.24)$$

$$\text{Let } Ps(\gamma) = \mu \text{ and from [4] node degree is given by } [d] = \frac{(\lambda\pi\mu)^n}{n!} e^{-\lambda\pi\mu} \quad (3.25)$$

$$\text{From [2], we have with border effect, } E[A_o] = \frac{E[d]}{n} (\pi r_o^2) \quad (3.26)$$

Node isolation probability is given by

$$P_I = \exp \left((-\frac{\lambda\pi r_o^2}{n}) E[d] \right) \quad (3.27)$$

$$\text{For } n = 15, \lambda = 2, P_I = 0.9780$$

$$\text{For } n = 20, \lambda = 3, P_I = 0.9709$$

C. Log-normal shadowing channel

Log-normal shadowing channel describes the random shadowing effects which occur over a large number of measurement locations and it has the same transmitter-receiver separation, but different levels of clutter on the propagation path [6].

The Probability density function of log-normal shadowing is given by

$$f(\lambda) = \frac{\varepsilon}{\sqrt{2\pi}} \frac{1}{\sigma\lambda} \exp(-(10\log_{10} \lambda - \mu)^2 / 2\sigma^2), \text{ for } \lambda > 0 \quad (3.30)$$

$$Ps(\gamma) = P(\lambda \geq \psi) = \text{Success probability}$$

$$Ps(\gamma) = \int_{\psi}^{\infty} \frac{\varepsilon}{\sqrt{2\pi}} \frac{1}{\sigma\lambda} \exp(-(10\log_{10} \lambda - \mu)^2 / 2\sigma^2) d\lambda \quad (3.31)$$

$$Ps(\gamma) = P(\lambda \geq \psi) = \text{Success probability}$$

$$Ps(\gamma) = \int_{\psi}^{\infty} \frac{\varepsilon}{\sqrt{2\pi}} \frac{1}{\sigma\lambda} \exp(-(10\log_{10} \lambda - \mu)^2 / 2\sigma^2) d\lambda \quad (3.32)$$

Case (1): When $\mu = 0$

$$Ps(\gamma) = \frac{\varepsilon}{\sqrt{2\pi}} \int_{\psi}^{\infty} \frac{1}{\sigma} \exp(-(10\log_{10} \gamma)^2 / 2\sigma^2) d\gamma \quad (3.33)$$

$$\text{Let } \frac{10\log_{10} \gamma}{\sigma} = t$$

$$\ln \left(\frac{\lambda}{\sigma} \right) = t$$

$$\ln \lambda = \sigma t$$

Differentiate (a) with respect to t

$$(1/\lambda) d\gamma = \sigma dt$$

$$d\gamma = \sigma \lambda dt$$

$$Ps(\gamma) = \frac{\varepsilon}{\sqrt{2\pi}} \frac{1}{\sigma} \int_{(\ln \psi / \sigma)}^{\infty} \frac{1}{\gamma} \exp \left(-\frac{t^2}{2} \right) \lambda \sigma dt \quad (3.34)$$

$$\text{Using } \int_{\psi}^{\infty} e^{-\frac{x^2}{\mu\beta}} e^{-\lambda x} \sqrt{\pi\beta} dx = e^{\beta\lambda^2} (1 - \Psi(\lambda\sqrt{\beta} + \frac{\mu}{\sqrt{\beta}} \frac{1}{2})) \quad (3.35)$$

$$\text{Here } \beta = \frac{1}{2}, \lambda = 0$$

$$Ps(\gamma) = \sqrt{\pi/2} (1 - \Psi(\frac{\ln \psi}{\sqrt{2}} \frac{1}{2\sigma})) \quad (3.36)$$

$$Ps(\gamma) = [1 - \Psi(\frac{\ln \psi}{\sqrt{2}})] = \mu \quad (3.37)$$

$$\text{Hence } E[d] = \frac{(\lambda\pi\mu)^n}{n!} e^{-\lambda\pi\mu} \quad (3.38)$$

$$E[A_o] = \frac{E[d]}{n} (\pi r_o^2) \quad (3.39)$$

$$P_I = e^{-\lambda\pi E[A_o]} \quad (3.40)$$

$$E[A_o] = \frac{E[d]}{n} (\pi r_o^2) \quad (3.41)$$

$$P_I = e^{-\lambda\pi E[A_o]} \quad (3.42)$$

$$P_I = \exp(-[d] \lambda \pi (\frac{\lambda\pi r_o^2}{n}) E[d]) \quad (3.43)$$

$$\text{For } n = 15, \lambda = 3, P_I = 0.9842$$

$$\text{For } n = 20, \lambda = 4, P_I = 0.9914$$

Case (2): when $\mu \neq 0$

$$Ps(\gamma) = \int_{\psi}^{\infty} \frac{\varepsilon}{\sqrt{2\pi}} \frac{1}{\sigma\gamma} \exp(-(10\log_{10} \gamma - \mu)^2 / 2\sigma^2) d\gamma \quad (3.44)$$

$$\text{Let } \frac{\ln \lambda - \mu}{\sigma} = t$$

$$\ln \lambda - \mu = \sigma t$$

$$\frac{1}{\lambda} d\gamma = \sigma dt$$

$$Ps(\gamma) = \frac{\varepsilon}{\sqrt{2\pi}} \frac{1}{\sigma} \int_{\ln \psi - \mu}^{\infty} \frac{1}{\gamma} \exp \left(-\frac{t^2}{2} \right) \sigma \lambda dt =$$

$$\int_{\ln \psi - \mu}^{\infty} \exp \left(-\frac{t^2}{2} \right) dt$$

$$(3.44)$$

$$Ps(\gamma) = \frac{\varepsilon}{\sqrt{2\pi}} \sqrt{\frac{\pi}{2}} [1 - \Psi(\frac{\ln \psi - \mu}{2\sigma(\frac{1}{\sqrt{2}})})] = \frac{\varepsilon}{2} [1 -$$

$$\Psi(\frac{\ln \psi - \mu}{\sigma\sqrt{2}})] = \mu$$

$$\text{Let } E[d] = \frac{(\lambda\pi\mu)^n}{n!} e^{-\lambda\pi\mu} \text{ and } E[A_o] = \frac{E[d]}{n} (\pi r_o^2) \quad (3.45)$$

$$P_I = e^{-\lambda\pi E[A_o]}$$

$$P_I = \exp(-\lambda\pi (\frac{\lambda\pi r_o^2}{n}) E[d])$$

$$\text{For } n = 15, \lambda = 3, P_I = 0.9804$$

$$\text{For } n = 20, \lambda = 4, P_I = 0.9884$$

IV. NUMERICAL AND SIMULATION RESULTS

The numerical and simulation results are obtained from the analytical model using MATLAB. The system parameters are selected as follows: $K=10\text{dB}$, $P_{tx}=1\text{mWatt}$, $W=0.01\text{mWatt}$, $\psi=10\text{dB}$. The parameters such as m , λ , α , and σ are selected suitably. The network size for simulation is fixed to be $100\text{m} \times 100\text{m}$. We have chosen a random number of nodes according to Poisson process and the nodes are placed over the simulation area according to a random uniform distribution. Various links are then established according to the given channel model. For wireless network one of the important feature is

connectivity. The condition for nodes to be connected in a network is their deterministic distance should less than transmission radius. The analysis of node isolation probability with border effect is done for deterministic, Rayleigh and log normal shadowing channel and comparison of these channels is done with the above condition.

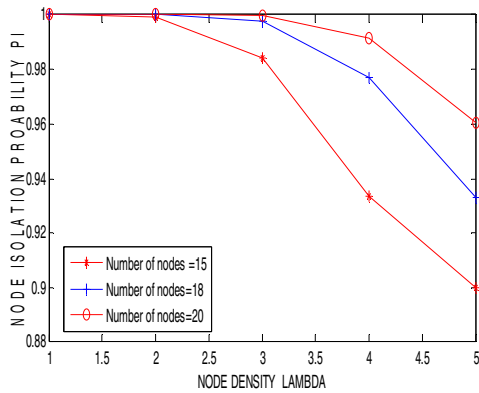


Figure 1. Node density vs. node isolation probability of deterministic channel

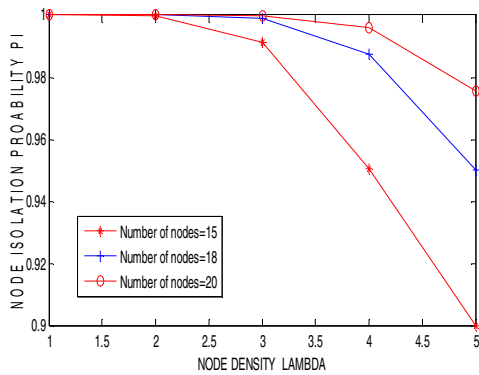


Figure 2. Node density vs. node isolation probability of Rayleigh fading channel

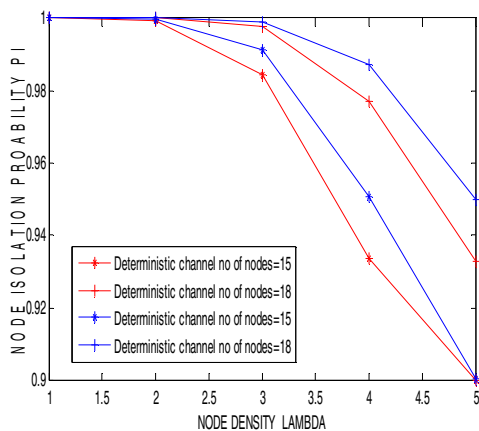


Figure 3. Comparison of deterministic and Rayleigh fading channel.

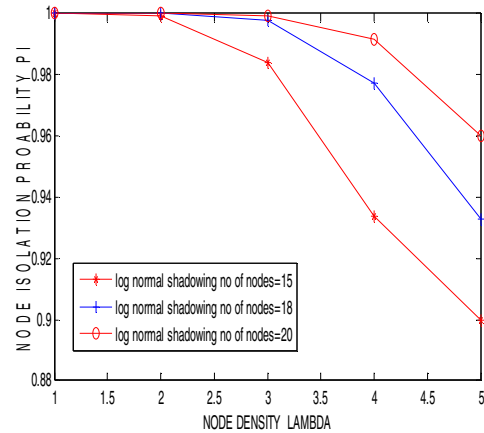


Figure 4. Node density vs. node isolation probability of log normal shadowing channel.

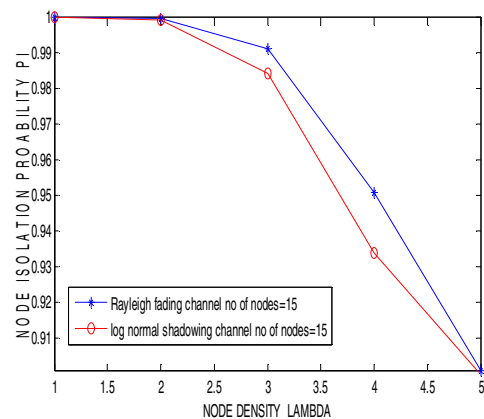


Figure 5. Comparison of Rayleigh fading and log normal shadowing channel

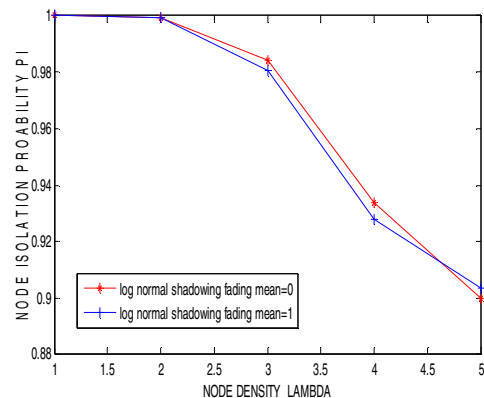


Figure 6. Comparison of log normal shadowing for different mean conditions.

All the results are verified by simulation. The node isolation probability with border effect is calculated using the parameters node density (λ)

and number of nodes (n). For $\lambda = 2, n = 15$, the node isolation probability of deterministic and Rayleigh fading channel are 0.9842 and 0.9782 respectively. Less the node isolation probability more is the connectivity in the network. So Rayleigh fading channel has better connectivity compared to deterministic channel. Further, for $\lambda = 2, n = 15$, the node isolation probability of log normal shadowing channel is 0.9992. Comparatively Rayleigh fading has better connectivity compared to log normal and deterministic channel. So the performance is better in Rayleigh fading channel.

V. CONCLUSIONS

Connectivity is one of the important parameter of wireless ad-hoc and sensor network, which needs a lot of work to be carried out from time. It is said to be there in a network only when their deterministic distance is less than transmission radius. Several research work has been done so far considering various parameters of medium as well as fading factor but there is a lack of concentration towards border effect. The effect which concerns about the nodes lying on the boundary region of considerable area of a network is called border effect. This paper analyses the scenario of border effect with several medium conditions. The analysis of node isolation probability considering border effect is done in this research work. As we increase the number of nodes, isolation probability of nodes decreasing in the context of border effect, which is opposite scenario to ideal case result. The mathematical analysis provides good understanding of node isolation with border effect.

REFERENCES

- [1] P. Gupta and P. R. Kumar, Critical power for asymptotic connectivity, IEEE Proc. of CDC, Tamper, FL, USA, 1998.
- [2] O. Dousse, P. Thiran, and M. Hasler, Connectivity in ad-hoc and hybrid networks, IEEE Proc. of INFOCOM, New York, USA, 2002.
- [3] O. Dousse and P. Thiran, Connectivity vs capacity in dense ad hoc networks, IEEE Proc. of INFOCOM, Hongkong, 2004.
- [4] O. Dousse, F. Baccelli and P. Thiran, Impact of interferences on connectivity in ad hoc networks, IEEE/ACM Tran. on Networking, Vol. 13, No. 2, pp. 425 - 436, Apr. 2005.
- [5] M. Zorzi and S. Pupolin, Outage probability in multiple access packet radio networks in the presence of fading, IEEE Tran. Veh. Tech., vol. 43, no. 3, pp. 604-610, Aug. 1994.
- [6] C. Bettstetter and C. Hartmann, Connectivity of wireless multihop networks in a shadow fading environment, ACM Proc. Of MSWiM, San Diego, CA, 2003.
- [7] D. Miorandi and E. Altman, Coverage and connectivity of ad hoc networks in presence of channel randomness, IEEE Proc. of INFOCOM, Vo. 1, 13-17 pp. 491 - 502 Mar. 2005.
- [8] M. D. Penrose, "On k-connectivity for a geometric random graph," Random Structures and Algorithms, vol. 15, no. 2, pp. 145-164, 1999.
- [9] M. D. Penrose, "Extremes for the minimal spanning tree on normally distributed points," Advances in Applied Probability, vol. 30, no. 3, pp. 628-639, 1998.
- [10] Peng-Jun Wan and Chih-Wei Yi, "Asymptotic critical transmission radius and critical neighbor number for k-connectivity in wireless ad hoc networks," in Proceedings of ACM MobiHoc '04. 2004, pp. 1-8, ACM Press.
- [11] Christian Bettstetter, "On the connectivity of Ad Hoc networks," Computer Journal, vol. 47, no. 4, pp. 432-447, July 2004.
- [12] Esa Hyttia, Pasi Lassila, and Jorma Virtamo, "Spatial node distribution in the random waypoint mobility model with applications," submitted for publication, available at <http://www.netlab.tkk.fi/julkaisut/bib/>, Mar. 2005.
- [13] Christian Bettstetter, "On the Minimum Node Degree and Connectivity of a Wireless Multihop Network," ACM Proc. of MobiHoc, Jun. 2002.
- [14] Sayandev Mukherjee and Dan Avidor, "Connectivity and Transmit-Energy Considerations between Any Pair of Nodes in a Wireless Ad Hoc Network Subject to Fading," IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, VOL. 57, NO. 2, MARCH 2008
- [15] L. Hanzo, C. H. Wong, and M. S. Yee, Adaptive Wireless Transceivers. New York: John Wiley and Sons, 2002.
- [16] A.V. Babu and Mukesh Kumar Singh, "node isolation probability of wireless adhoc networks in Nakagami fading channel," International journal of Computer Networks & Communications (IJCNC), Vol.2, March 2010



Nagesh K. Narayanaswamy obtained his Master degree in Digital electronics and communication from Visvesvaraya Technological University India, and Bachelor degree in Electronics & Communication from Visvesvaraya Technological University India. His teaching interests are Communication systems, Digital communication. His areas of research of research interest are,

Wireless communication.



Satyanarayana obtained his PhD from JNTU, India. His research interests are wireless networks and LP analysis. He has held various positions at university level. Currently he is working as Head of Electronics department at RGM College of Engineering and Technology.



Nagesh Poojary has PhD from University of Mysore in Speech Signal Processing. He has several years of industry, research and academic experience. His main research interests include speech recognition, statistical pattern recognition and image processing. He has several refereed international journal publications, book chapter and conference papers.



Chandrasekar Ramaiah obtained his Maser degree in Industrial Electronics Visvesvaraya Technological University India, and Bachelor degree in Electronics & Communication from Kuvempu University India. His areas of research interest are, Digital signal processing and Computer networks.