

Compensation of Mutual Coupling in an Antenna Array for Direction of Arrival Estimation

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Abstract— In this paper, we propose a stable estimation method of direction of arrival (DOA) using an antenna array with mutual coupling. In a real antenna array, the mutual coupling between elements distorts the array output signal and degrades the performance of DOA estimation. Because the coupling effect is symmetric about each element, the mid-array of which the elements are affected equally is insensitive to the mutual coupling. We estimate the coupling coefficients by using this property, compensated the array output using them, and estimate the DOA using the compensated signal. The mutual coupling coefficients are gradually updated in an adaptive manner to guarantee convergence. As a result, we can get more stable or more accurate DOA estimate.

Keywords— Antenna array, ESPRIT, mutual coupling, direction of arrival

I. INTRODUCTION

The estimation of direction of arrival is widely used in global positioning system, military and mobile communication. In military communication and global positioning system, the direction of arrival estimation is used for antijamming. Also, in mobile communication, direction of arrival estimation is used to find direction of user signal for beamforming.

The most popular methods of the DOA estimation are the multiple signal classification (MUSIC) algorithm [1] and the estimation signal parameters by rotational invariance technique (ESPRIT) algorithm [2]. MUSIC algorithm is based on a subspace approach where the correlation matrix of the received array signal is divided into a signal space and a noise space. A spatial spectrum is computed from the noise subspace and the DOAs are determined from the dominant peaks in the spectrum. ESPRIT algorithm is based on the characteristics of two shifted arrays where the correlation matrices of two same type arrays shifted away have the same eigenvalues. The ESPRIT algorithm has lower complexity of DOA estimation than MUSIC algorithm [2]. However, the structure of the array is limited to the uniform linear array (ULA) in ESPRIT different from MUSIC.

The direction of arrival estimation algorithm is premised on the assumption that the antenna array has ideal properties. In practice, however, the antenna array has non-ideal properties such as mutual coupling [3], gain/phase error [4] and antenna position error [5]. Especially, the mutual coupling change the beampattern of antenna array. As a result, the performance of

DOA estimation is degraded seriously. In ESPRIT, two or more subarrays are used, and the elements of each subarray should have identical beam patterns. The different beam patterns impair the translational invariance structure between the subarrays and degrade the accuracy of DOA estimation.

The coupling effect depends on the position of the elements in the array, which results in different beam pattern of each element. However, it is a noteworthy fact that the mid-elements undergo relatively same coupling effect and the effect decreases approaching the elements of the both sides [6].

Friedlander and Weiss proposed an iterative compensation method [7]. But it is computationally expensive. So, DOA estimation results can diverge. In [8], propose a mutual coupling compensation method by using mid-array MUSIC [9].

In this paper, we propose a mutual coupling compensation method by using mid-ESPRIT. We utilize the iterative method with step-size for stable mutual coupling matrix estimation.

II. SYSTEM MODEL AND ESPRIT ALGORITHM

A. System model

To estimate one-dimensional (1D) DOA, a uniform linear array (ULA) with L -elements and M jamming sources are considered. The array output signal is given by

$$\mathbf{y} = \mathbf{A}\mathbf{s} + \mathbf{w} \quad (1)$$

where $\mathbf{y}$ is received signal, $\mathbf{A}$ is steering matrix, $\mathbf{s}$ is jamming signal and $\mathbf{w}$ is additive white Gaussian noise (AWGN). Eq. (1) is expressed in vector form as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\phi_1} & e^{j\phi_2} & \dots & e^{j\phi_M} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(L-1)\phi_1} & e^{j(L-1)\phi_2} & \dots & e^{j(L-1)\phi_M} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_M \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} \quad (2)$$

where $\phi_i = -2\pi(d/\lambda)\cos\theta_i$, d is the separation between sensors, λ is the wave length of the incident signal, θ_i is the DOA from i -th source.

The covariance matrix of the received signals is given by

$$\mathbf{R} = E[\mathbf{y}\mathbf{y}^H] = \mathbf{A}\mathbf{P}\mathbf{A}^H + \sigma^2\mathbf{I} \quad (3)$$

where $E[\cdot]$ means expectation, the covariance matrix $\mathbf{P} = E[\mathbf{s}_n\mathbf{s}_n^H]$ of the source signal is an $M \times M$ diagonal matrix, and σ^2 is the variance of noise. Form eigendecomposition of the covariance matrix, Eq. (3) becomes

$$\mathbf{R} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H \quad (4)$$

where $\mathbf{U}_s$ and $\mathbf{U}_n$ are unitary matrices defined in a signal space and a noise space, respectively, $\mathbf{\Lambda}_s$ are diagonal matrices with eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M$ being diagonal elements.

B. ESPRIT algorithm

For ESPRIT, we divide a ULA into two subarrays as shown in Fig. 1. The outputs of the two subarrays are given by

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{A}_1 \mathbf{s} + \mathbf{w}_1 \\ \mathbf{y}_2 &= \mathbf{A}_1 \Phi \mathbf{s} + \mathbf{w}_2 \end{aligned} \quad (5)$$

where $\Phi = \text{diag}\{e^{j\phi_1}, e^{j\phi_2}, \dots, e^{j\phi_M}\}$

After some tedious calculation, we can derive the following relationship between two unitary matrices

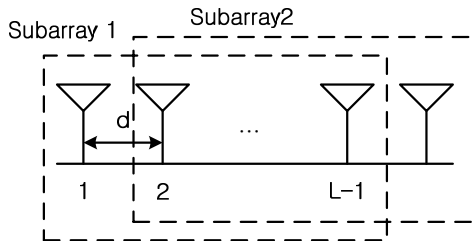


Figure 1. The antenna array structure for ESPRIT algorithm

$$\begin{aligned} \mathbf{U}_2 &= \mathbf{U}_1 \Psi \\ \Psi &= \mathbf{T}^{-1} \Phi \mathbf{T} \end{aligned} \quad (6)$$

where $\mathbf{T}$ is $M \times M$ complex matrix satisfying $\mathbf{U}_1 = \mathbf{A}_1 \mathbf{T}$ and $\mathbf{U}_2 = \mathbf{A}_2 \mathbf{T}$.

From Eq. (5), (6), we know that Φ and Ψ have the same eigenvalues $e^{j\phi_m}$ ($m=1, \dots, M$). Therefore, we can estimate the DOAs by finding the eigenvalues of Ψ . From the eigenvalues, we can estimate the DOAs as follows:

$$\theta_m = \cos^{-1} \left\{ \frac{1}{2\pi(d/\lambda)} \arg(e^{j\phi_m}) \right\} \quad (7)$$

where $\arg(x)$ means the phase of x .

III. THE PROPOSED METHOD

The output of antenna array with mutual coupling is given by

$$\mathbf{y} = \mathbf{C}\mathbf{A}\mathbf{s} + \mathbf{w} \quad (8)$$

where $\mathbf{C}$ is mutual coupling matrix. $\mathbf{C}$ is indicated by banded symmetric Toeplitz matrix [8][9].

$$\mathbf{C} = \begin{bmatrix} 1 & c_1 & c_2 & \dots & c_{p-1} & \dots & 0 \\ c_1 & 1 & c_1 & c_2 & \dots & \ddots & 0 \\ c_2 & c_1 & 1 & c_1 & c_2 & \dots & c_{p-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ c_{p-1} & \dots & c_2 & c_1 & 1 & c_1 & c_2 \\ 0 & c_{p-1} & \dots & c_2 & c_1 & 1 & c_1 \\ 0 & \dots & c_{p-1} & \dots & c_2 & c_1 & 1 \end{bmatrix} \quad (9)$$

For the mutual coupling matrix, we define the coupling coefficient vector as follows

$$\mathbf{c} = [1 \ c_1 \ c_2 \ \dots \ c_{p-1} \ \dots \ 0] \quad (10)$$

The coupling vector indicates the effect of coupling from neighboring elements, and generally the gains of coupling coefficients are $1 > |c_1| > \dots > |c_{p-1}|$.

The mutual coupling effect with the coupling length is 3 as shown in Fig. 2. The mutual coupling effect depends on the position of the elements in the array. The nearest element is most affected, and the effect becomes smaller as the distance between the elements increases. These properties degrade the performance of DOA estimation by ESPRIT algorithm especially.

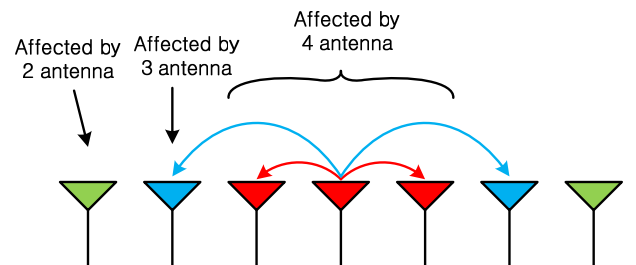


Figure 2. The effect of mutual coupling.

However, the mid-elements undergo relatively same coupling effect. In other words each element has identical beam pattern. We can estimate the accurate DOA by using these properties.

Fig. 3 shows the mutual coupling compensation method included in DOA estimation, and each step is outlined as follows:

- Step 1: DOA estimation by using the mid-array that is affected identically by mutual coupling.
- Step 2: Calculate the mutual coupling matrix (MCM) C_k [8].
- Step 3: Compensate the received signal of the full-array with the inverse of the MCM estimate.

$$\hat{y} = C_k^{-1} y \quad (8)$$

- Step 4: Apply the full array to estimate the DOA by ESPRIT algorithm.

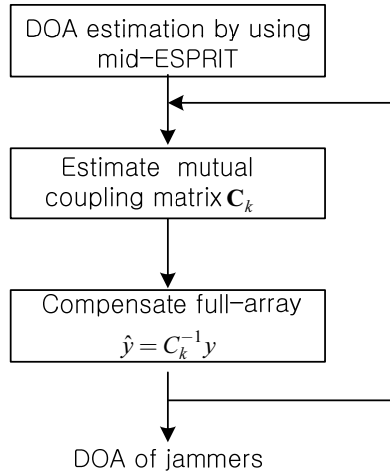


Figure 3. The flow chart of the proposed mutual coupling compensation method

In the second step, the MCM estimate is updated at each iteration. In this case, the estimated MCM can diverge with low JNR (Jammer to Noise Ratio). Therefore, we gradually update the estimate of the MCM in an adaptive manner for stable estimation. The MCM is updated drawing on difference of previous estimate and present estimate as follows:

$$C_{k+1} = (1 - \alpha)C_k + \alpha C_{k+1}^t \quad (9)$$

where C_{k+1}^t is present estimate, C_k is previous estimate and α is step size. Much simulations show that the MCM converges stably with the step size of 0.3~0.6 in a reasonable iteration number of about 10.

IV. SIMULATION RESULTS

We demonstrate the performance of proposed method through some experiments. The simulation parameters are based on GPS environment and represented in Table 1. Since we use a one dimensional ULA, we only consider azimuth with elevation of zero degree.

Fig. 4 shows the performance of proposed method with two jammers versus JNR. We repeat the foregoing 4 steps 100 times and calculate a root-mean-square (RMS) value of DOA estimation. Jammers located at $(25^\circ, 0^\circ)$ $(70^\circ, 0^\circ)$.

The proposed method reveals better performance than other cases over all JNR except JNR=0. The performance of the full-array is better than coupling-insensitive mid-array. This means, the 3 elements are inadequate to estimate 2 jammers with mutual coupling.

Fig. 5 shows the performance of proposed method with two jammers versus the number of snapshot. The JNR is 6dB and jammer position is same in previous experiment.

At the number of snapshot ≥ 100 , the performance of the proposed method is better performance than other cases. The performances of all the 3 cases change a little when the number of snapshot is greater than 1000.

Table 1. The simulation parameters

Jammer (azimuth, elevation) [deg]	$(25^\circ, 0^\circ), (70^\circ, 0^\circ)$
Jamming signal type	Continuous wave
Antenna structure	Uniform linear array (7-element array)
The number of snapshot	11,428 (One period of C/A code)
Coupling coefficient	$[1, 0.41+j0.57, 0.24-j0.18]$

V. CONCLUSIONS

In this paper, we propose a stable estimation method of the mutual coupling matrix and the DOA using it. And we demonstrate the estimation performance of DOA of jammers by applying ESPRIT algorithm as a function of the JNR and the number of snapshot.

Simulation results show that the proposed method gives a more stable and accurate estimate of the mutual coupling matrix or the DOA. Especially, the propose method shows better estimation performance under the low JNR and small snapshot.

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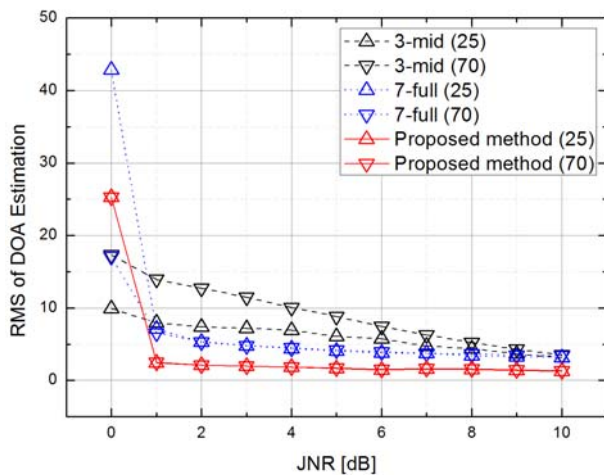


Figure 4. The performance of proposed method with 2 jammers.

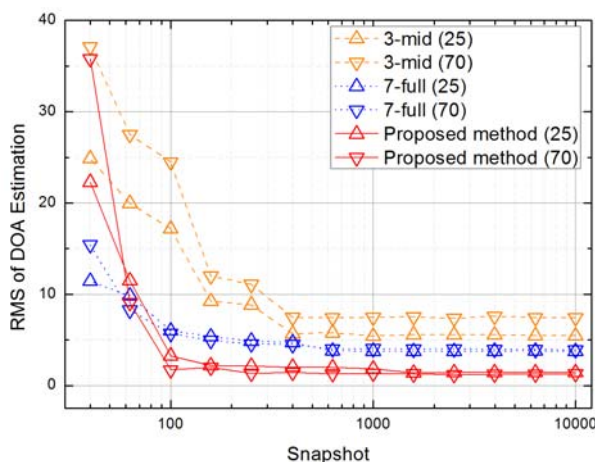


Figure 5. RMS errors of DOA estimates versus snapshot with 2 jammers.

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